2단계 공급사슬의 결합적 가격 및 재고 정책의 결정
Joint Price-Delivery Decision in a Single-Manufacturer-Single Retailer Supply Chain

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Abstract

In the traditional inventory problem, market parameters such as demand and selling price are exogenous. But incorporating these factors into the model can provide an opportunity for increasing the total profit. So we investigate the joint price-inventory policy in a supply chain consisting of a single retailer and a single manufacturer. Demand at the retailer depends on the retail price. The retailer and the manufacturer cooperate closely each other to maximize overall profit of the supply chain. The mathematical model is presented and the solution procedure is developed in order to jointly determine the optimal policy including the retail price, the production lot sizes, and the delivery frequency from the manufacturer to the retailer.

1. Introduction

In the past decades, many studies have been carried out to extend classical Economic Order Quantity(EOQ) problem to develop effective cooperative mechanisms among parties for successful management of supply chain. Among those works, our interest lies in price-inventory models which study interactions of an inventory decision with pricing policies. In the classical EOQ model, demand is assumed to be fixed and exogenous. But, in reality, demand is very sensitive to the price in today's competitive market, and many firms try to expand their market shares and maximize the profit by adjusting their prices. Therefore, it can be more effective for today's economic inventory management policies to reflect the demand variation by price adjustment. In this vein, this paper discusses how to integrate inventory and price decision problem and to determine the optimal policies in a supply chain consisting of a single retailer and a single manufacturer.

Many papers dealing with price-inventory models for retailer side have been published after Kunreuther and Richard(1971)'s research. In addition, as concerns about supply chain management increase, coordination issues among parties involved in supply chains have been discussed. Abad (1994) studied a problem of coordination between a vendor and a buyer. Parla and Wang (1994) developed a model for a single supplier and a single buyer when the buyer used a constant profit margin pricing policy. Weng (1995) discussed extensively about impacts of joint decision policies on channel coordination in a system consisting of a supplier and a buyer, and provided the managerial insights between joint coordination and quantity discount. Reyniers (2001) determined the optimal retail price and order quantity under a lot-for-lot policy for a constant price-elasticity demand function, assuming that the manufacturer's production rate is equal to the demand rate. Chen and Chen (2006) discussed a multi-product and multi-echelon supply chain in which the manufacturer produces the perishable items at finite production rates respectively. They argued that the optimal solution can be obtained when both the concave properties and Hessian matrix condition are satisfied.

Recently, we discussed about an integrated system consisting of a single manufacturer and a single retailer under linear decreasing demand function (Kim et al., 2006). This paper is on same line and extends it to the concept that manufacturer produces an integer multiple of the retailer's order size in a lot. The remainder of the paper is organized as follows. Section 2 presents the proposed mathematical model and a related analysis. In section 3, numerical examples are shown and section 4 gives summary and conclusions.

2. Model

Consider a supply chain consisting of a single manufacturer and a single retailer. The retailer places orders to the manufacturer according to an EOQ policy and the manufacturer produces the items in batches which are set as an integer multiple of the order quantity, and delivers the order quantity to the retailer. Following notations are used throughout the paper to develop the mathematical model.

c : unit production cost at the manufacturer
R : production rate at the manufacturer
S : setup cost at the manufacturer
A : ordering cost at the retailer
h_m : inventory holding cost at the manufacturer
h_r : inventory holding cost at the retailer

Demand at the retailer is assumed by a deterministic and linearly decreasing function in price (p), i.e. $D(p) = a - bp,\ a, b > 0$ and also assumed that $R \geq D(0)$. Our objective is to determine the optimal retail price and the corresponding inventory policy so as to maximize the average joint profit. To formulate the problem, the necessary decision variables are defined as follows:

p : unit selling price at the retailer
Q : order quantity at the retailer
n : delivery frequency in a single production cycle

the average joint profit can be arranged by

$$\Pi(p,Q,n) = (p-c)D(p) - \left(\frac{S}{n}+A\right)\frac{D(p)}{Q} - \frac{Q}{2}\left[h_r + h_m\left\{(n-1)-(n-2)\frac{D(p)}{R}\right\}\right] \tag{1}$$

It is not mathematically easy to derive the optimal retail price and inventory policies maximizing Eq.(1) simultaneously. Hence, a sequential procedure is followed to determine these optimal values. First, assume that n and Q are given, and then $\Pi(p,Q,n \mid Q,n)$ is a concave in p because $d^2\Pi(p,Q,n \mid Q,n)/dp^2 < 0$. Therefore, the optimal retail price for given Q, n can be uniquely determined by solving $d\Pi(p,Q,n \mid Q,n)/dp = 0$. It is obtained by

$$p^*(Q,n) = \frac{1}{2}\left[\left(\frac{S}{n}+A\right)\frac{1}{Q} + \left(\frac{a}{b}+c\right) - \frac{(n-2)h_m Q}{2R}\right] \tag{2}$$

Substituting $p^*(Q,n)$ into Eq. (1), the average joint profit function can be re-arranged as

$$\Pi_1(Q,n) \equiv \Pi(p^*(Q,n),Q,n) = -\frac{Q}{2}\left(h_r + h_m(n-1)\right) + \frac{b}{4}\left\{\left(\frac{S}{n}+A\right)\frac{1}{Q} - \left(\frac{a}{b}-c\right) - \frac{(n-2)h_m Q}{2R}\right\}^2 \tag{3}$$

Next step is to determine the optimal order quantity Q^* assuming that n is given. But since its concavity with respect to Q is not guaranteed, we need a further consideration to obtain Q^*. Differentiating Eq. (3) with respect to Q, we obtain

$$\frac{d\Pi_1(Q,n \mid n)}{dQ} = -\left(\frac{S}{n}+A\right)^2\frac{b}{2Q^3} + \left(\frac{S}{n}+A\right)\frac{(a-bc)}{2Q^2} - \frac{1}{2}\left[h_r + h_m\left\{(n-1) - \frac{(a-bc)}{2R}(n-2)\right\}\right] + \frac{bh_m^2}{8R^2}(n-2)^2 Q \tag{4}$$

Since the characteristic of $\Pi_1(Q,n \mid n)$ is changed according to the value of n, we analyze it about two distinctive cases of $n = 2$ and $n \geq 3$, respectively. The lot-for-lot case of $n = 1$ is referred to Kim et al.(2006)

2.1 $n = 2$ case

The signs in coefficients of the Eq. (4) change two times after putting $n = 2$. Hence, it is known that $d\Pi_1(Q,2)/dQ = 0$ has either none or two positive solutions in accordance with Descartes' Rule. Also, since the solution of $d^2\Pi_1(Q,2)/dQ^2 = 0$ is uniquely set and $\lim_{Q\to 0^+}\Pi_1(Q,2) = \infty$, $\lim_{Q\to\infty}\Pi_1(Q,2) = -\infty$, we can know that Eq. (3) is a convex-concave function in this case. But we cannot decide the optimal order quantity yet because of $\lim_{Q\to 0^+}\Pi_1(Q,2) = \infty$, therefore we need additional information to obtain optimal order quantity.

It is obvious that $p^*(Q,2)$ must satisfy two conditions, $p^*(Q,2) \geq c$ and $D\{p^*(Q,2)\} \geq 0$. It is easily verified that $p^*(Q,2) \geq c$ always holds for any positive Q. Substituting $p^*(Q,2)$ in Eq. (2) into $D\{p^*(Q,2)\} \geq 0$, following inequality restricting the feasible range of Q can be obtained.

$$Q_L \equiv \left(\frac{S}{2}+A\right)\Big/\left(\frac{a}{b}-c\right) \leq Q \tag{5}$$

From Eq. (5) Q_L can be set as a lower bound for a selection of optimal order quantity. Furthermore, we see that the total profit function $\Pi_1(Q,n)$ and its first derivative in Eq. (8) have negative value at Q_L. Hence, we can know that two positive solutions of

$d\Pi_1(Q,2)/dQ = 0$, Q_1 and Q_2, satisfy the inequality $Q_L < Q_1 < Q_2$ if they exist. Therefore, if $d\Pi_1(Q,2)/dQ = 0$ has two positive solutions and $\Pi_1(Q_L,2) \le \Pi_1(Q_2,2)$, it is obvious that $Q^* = Q_2$ is the optimal solution maximizing $\Pi_1(Q,2)$. Otherwise, $Q^* = Q_L$ is the optimal order quantity maximizing $\Pi_1(Q,2)$ but in this case any order quantity giving the positive average joint profit cannot be existed since $\Pi_1(Q^*,2) = -(h_r + h_m)Q_L/2 < 0$.

2.2 $n \ge 3$ case

Similarly to $n = 2$ case, from $p^*(Q,n \mid n \ge 3) \ge c$ and $D\{p^*(Q,n \mid n \ge 3)\} \ge 0$ the interval of order quantity Q is

$$Q_L \equiv \frac{-\gamma + \sqrt{\gamma^2 + 4\alpha\beta}}{2\beta} \le Q \le \frac{\gamma + \sqrt{\gamma^2 + 4\alpha\beta}}{2\beta} \equiv Q_U$$

where, (6)

$\alpha = S/n + A$, $\beta = h_m(n-2)/2R$, $\gamma = a/b - c$.

Additionally, the following information is also obtained.

$$\Pi_1(Q_L, n \mid n \ge 3) = -\frac{Q_L}{2}\left(h_r + h_m(n-1)\right) \quad (7)$$

$$\Pi_1(Q_U, n \mid n \ge 3) = -\left(\frac{S}{n} + A\right)\frac{(a-bc)}{Q_U} - \frac{Q_U}{2}\left[h_r + h_m\left\{(n-1) - (n-2)\frac{(a-bc)}{R}\right\}\right] \quad (8)$$

Since the signs in coefficients of the Eq. (4) change three times, $d\Pi_1(Q,n \mid n \ge 3)/dQ = 0$ has either one or three positive solutions. Let these be Q_1, Q_2 and Q_3 with a single solution case. And since $\lim_{Q \to 0^+} \Pi_1(Q,n \mid n \ge 3) = \infty$, $\lim_{Q \to 0+} d\Pi_1(Q,n \mid n \ge 3)/dQ = -\infty$, $\lim_{Q \to \infty} \Pi_1(Q,n \mid n \ge 3) = \infty$, $\lim_{Q \to \infty} d\Pi_1(Q,n \mid n \ge 3)/dQ = \infty$, we can suppose the shape of function easily.

Therefore, if there is any Q_2 satisfying $Q_L < Q_2 < Q_U$ and $\Pi_1(Q_2,n \mid n \ge 3)$ is larger than $\Pi_1(Q_L,n \mid n \ge 3)$ and $\Pi_1(Q_U,n \mid n \ge 3)$, it is obvious that Q_2 is the optimal solution. Otherwise, optimal solution is decided by Q_L or Q_U which yields higher value of $\Pi_1(Q,n \mid n \ge 3)$. But in this case, the positive average joint profit cannot be achieved since $\Pi_1(Q_L,n \mid n \ge 3)$ and $\Pi_1(Q_U,n \mid n \ge 3)$ have the negative values as shown in Eqs. (7), (8). Therefore, If $d\Pi_1(Q,n \mid n \ge 3)/dQ = 0$ has three different solutions and the middle value Q_2 yields the positive profit, we can decide this into nontrivial optimal solution.

Up to now, we showed how we determine the optimal order quantity under given n. Remaining one is to determine the optimal delivery frequency n^* but it is too difficult to compute analytically. Therefore, we compute n^* numerically. Since n^* is a positive integer and cannot be relatively a large number, it can be determined easily through an exhaustive search without any computational difficulty.

3. Numerical example

In this section, numerical test is carried out to observe the selection of decision variables. Example data set for observation is shown in Table 1.

[Table. 1] Basic data set for analysis

c	R	S	A	h_m	h_r	a	b
50	3,500	100	10	5	10	3,000	10

The optimal policies about this parameter set are described in Table 2. In this example, the average joint profit is maximized when n is equal to 5.

[Table. 2] The optimal values

Delivery frequency (n)	Order quantity (Q^*)	Price (p^*)	Avg. joint Profit (Π)
1	152.54	175.42	154,451.42
2	99.88	175.30	154,750.90
3	77.04	175.25	154,845.93
4	63.83	175.23	154,881.22
5	55.11	175.21	154,890.96
6	48.87	175.20	154,887.64
7	44.16	175.20	154,876.89
8	40.46	175.19	154,861.64
9	37.48	175.19	154,843.52
10	35.01	175.19	154,823.56

To show the feasibility of numerically derived n^*, the relationship between n and average joint profit $\Pi(p^*,Q^*,n)$ is observed. As seen in Figure 1, we can suppose that the joint profit values by n will have a shape of concave function and be maximized at a unique point.

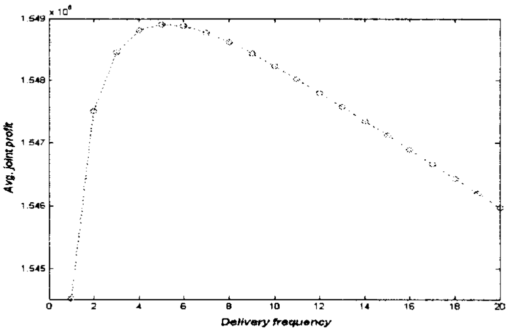

Figure 1. delivery frequency and avg. joint profit

4. Summary and Conclusions

This paper discussed a price-inventory policy in a supply chain consisting of a single retailer and a single manufacturer. The demand was assumed by a linear decreasing function in price. In the proposed model, we tried to maintain the concept for manufacturer to be discriminated from wholesaler. An analytical solution procedure was discussed to determine the optimal policy.

In further research, we need to consider the more realistic contractual problem because the full integration of two different firms is very complicated problem. For example, revenue sharing or quantity discount problem can be considered. Also, it is of interest to extend the model for other demand function such as a constant price elasticity demand function.

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