A Study on Shotcrete Algorithm in Discontinuous Deformation Analysis (DDA).

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1. Introduction

Discontinuous deformation analysis (DDA) method is one of the most recently developed and powerful numerical analysis techniques. This technique has the merits of both FEM and DEM. It has the form of a global simultaneous equations derived by the principle of minimum potential energy like FEM, and it can handle large displacement of blocks like DEM.

In order that DDA becomes a more practical analyzing tool, it should be able to simulate rock supports such as rock bolt and shotcrete in rock block analysis. As for the simulation of rock bolt, Kim (1999) and Moosavi (2006) suggested advanced algorithms using connecting bars which had been developed by Shi (1984), while the simulation of shotcrete has not been published yet. Zhang et al.(2005) showed a new method in which FEM and DDA were combined together and conducted in turn one after the other. This combined method, however, allowed only small displacement and penetration in each step, so that it is difficult to solve kinematic or dynamic problems of rock blocks with large displacement by using the method.

In our research, a new approach to simulate the shotcrete using penalty spring method is studied. The penalty spring method has been generally adopted to analyse block contact or collision. When a block contacts with another block, a contact penalty spring is set up at the contact point. Displacement of both blocks after the contact are calculated by minimizing the potential energy of rock blocks accumulated by the penalty spring.

In this paper, the new shotcrete support algorithm is explained after introducing a basic theory of DDA, and finally, the suggested algorithm is verified with three simple case studies.

2. Penalty Method in DDA

In DDA, a block system is formed by contact of blocks. Assuming a block system consisting of n blocks, the simultaneous equilibrium equations can be expressed in a matrix form as follow.

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & \cdots & K_{1n} \\ K_{21} & K_{22} & K_{23} & \cdots & K_{2n} \\ K_{31} & K_{32} & K_{33} & \cdots & K_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ K_{n1} & K_{n2} & K_{n3} & \cdots & K_{nn} \end{bmatrix} \begin{pmatrix} D_1 \\ D_2 \\ D_3 \\ \vdots \\ D_n \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ \vdots \\ F_n \end{pmatrix} \tag{2.1}$$

D_i and F_i in equation (2.1) represent a 6×1 submatrix each where D_i indicates the deformation of block i while F_i means a loading on block i. Because each block has six degrees of freedom in 2D, K_{ij} in the coefficient matrix (called stiffness matrix) of equation (2.1) is a 6×6 submatrix. K_{ij}, where $i \neq j$ is defined by contact between block i and block j.

These equilibrium equations are derived by minimizing the potential energy Π produced by the forces and stresses. The potential energy produced by contact between blocks is defined by the stiffness and displacement of contact penalty spring.

$$\Pi_c = \frac{p}{2} d^2 \tag{2.2}$$

where, p and d are the stiffness and displacement of a penalty spring, respectively.

The submatrix K_{ij} contains the coefficients of an equilibrium equation of load, moment and stress exerted on both blocks in contact as follow.

$$\frac{\partial \Pi_c}{\partial d_{ri}} = 0, \qquad r = 1, \ldots, 6 \tag{2.3}$$

The differentiations

$$-\frac{\partial \Pi(0)}{\partial d_{ri}}=0, \qquad r=1,...,6 \tag{2.4}$$

are the free terms of equation (2.3) after shifting to the right side of the equation. Therefore, all terms of equation (2.4) form a 6×1 submatrix, which is added to the submatrix F_i.

3. Shotcrete Support Algorithm

3.1 Definition for adopting penalty method

Figure 3.1 shows two blocks bonded by shotcrete. As shown in the figure, block j is fixed under constraint and shotcretes are installed on 3 edges of $\overline{p_1p_2}$, $\overline{p_3p_4}$ and $\overline{p_5p_6}$. The supporting forces of the shotcrete work on both lower vertices named p_3 and p_4.

In order to adopt the penalty spring method to the shotcrete algorithm, we firstly define supported area by 'penetrating point' and 'virtual reference line'.

Penetrating point is a reference point to measure displacement of supported area. The potential energy of contact springs are defined by relative displacement of penetrating point to reference line and stiffness of the spring. Penetrating points is detected when an edge of which shotcretes are installed contacts with other edges of same condition. For example, reinforce edge $\overline{p_3p_4}$ contacts with other reinforced edges $\overline{p_1p_2}$and $\overline{p_5p_6}$. Virtual reference line is set after penetrating point is defined. The reference line is normal to the edge containing the penetrating point. It is considered that the reference line is included to the block when the penetrating point contacts.

The shotcrete has 3 input parameters; normal stiffness, shear stiffness and thickness.

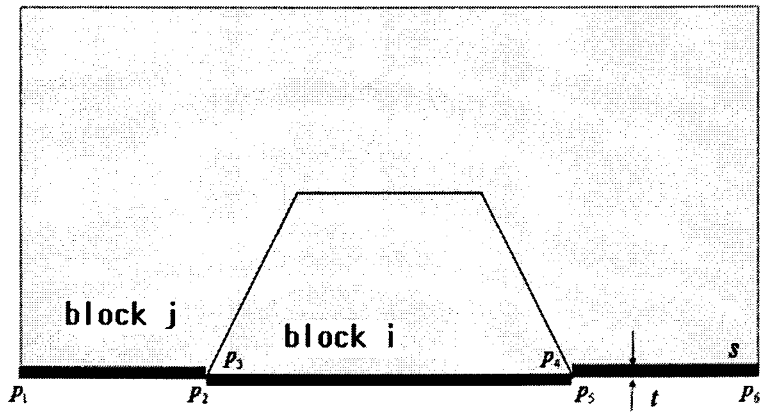


Figure 3.2 two blocks bonded by shotcrete

After executing the algorithm defining penetrating points, two penetrating points and virtual reference line is set as shown in Figure 3.2. p_3 of block i and p_5 of block j are both set as penetrating points. Because the algorithm defining penetrating points carries out at every starting point of edges in order, p_5 is set as penetrating point rather than a vertex p_4 of block i. Two virtual reference lines are set as $\overline{v_1v_2}$ and $\overline{v_3v_4}$ when penetrating points are defined. $\overline{v_1v_2}$ is belong to block j while $\overline{v_3v_4}$ is belong to block i.

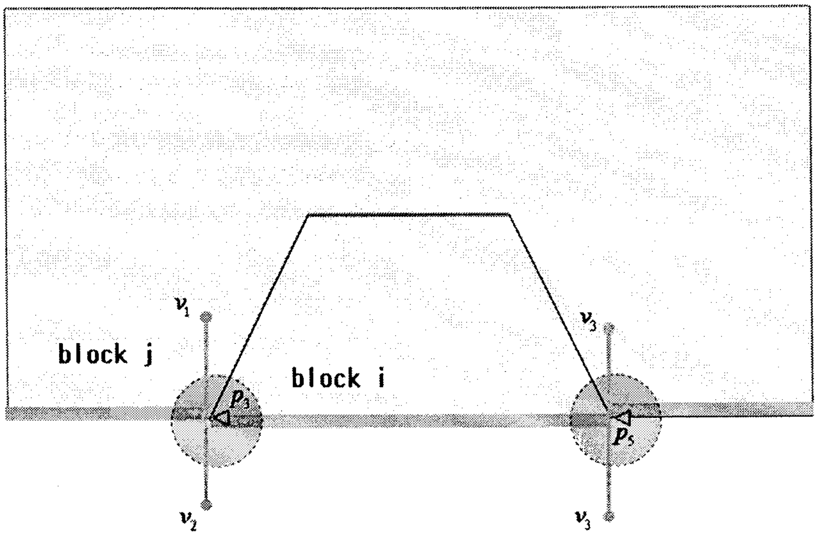


Figure 3.2 penetrating points and virtual reference lines

A set of penalty springs are installed at a point where a reference line contacts with a penetrating point. Then normal contact energy is defined by normal relative displacement of penetrating point and stiffness of normal contact spring while shear contact energy is defined by shear relative displacement of penetrating point and stiffness of shear contact spring. Both normal and shear displacements are controled by tensile force and shear force of supporting area. So the stiffness of normal and shear springs represent tensile and shear stiffness of shotcrete, respectively.

The derivation of the potential energy of shotcrete are almost same with derivation of block contact energy (Shi, 1984). The submatrices derived by shotcrete potential energy are derived in the following chapter.

3.2 Submatrices of a Contact Spring

Assume that there is a spring between a point P_1 and a reference line P_2P_3. (x_i, y_i) and (u_i, v_i) denote coordinates and the displacement increment of point P_i, respectively. Then we have the distance d_n from point P_1 to line P_2P_3 as equation (3.1)

$$d_n = \frac{\Delta}{l} = \begin{vmatrix} 1 & x_1+u_1 & y_1+v_1 \\ 1 & x_2+u_2 & y_2+v_2 \\ 1 & x_3+u_3 & y_3+v_3 \end{vmatrix} \tag{3.1}$$

where length of reference line is set as unit length.

$$\begin{vmatrix} 1 & x_1+u_1 & y_1+v_1 \\ 1 & x_2+u_2 & y_2+v_2 \\ 1 & x_3+u_3 & y_3+v_3 \end{vmatrix} = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} + \begin{vmatrix} 1 & u_1 & y_1 \\ 1 & u_2 & y_2 \\ 1 & u_3 & y_3 \end{vmatrix} + \begin{vmatrix} 1 & x_1 & v_1 \\ 1 & x_2 & v_2 \\ 1 & x_3 & v_3 \end{vmatrix} + \begin{vmatrix} 1 & u_1 & v_1 \\ 1 & u_2 & v_2 \\ 1 & u_3 & v_3 \end{vmatrix} \tag{3.2}$$

The last term of equation (3.2) is a second order infinitesimal which can be neglected. Then from (3.2) we have

$$d_n \approx S_0 + \begin{vmatrix} 1 & u_1 & y_1 \\ 1 & u_2 & y_2 \\ 1 & u_3 & y_3 \end{vmatrix} + \begin{vmatrix} 1 & x_1 & v_1 \\ 1 & x_2 & v_2 \\ 1 & x_3 & v_3 \end{vmatrix} \tag{3.3}$$

where S_0 is a first term of (3.2)

$$\begin{aligned} d_n = S_0 &+ u_1(y_2-y_3) + v_1(x_3-x_2) \\ &+ u_2(y_3-y_1) + v_2(x_1-x_3) \\ &+ u_3(y_1-y_2) + v_3(x_2-x_1) \end{aligned} \tag{3.4}$$

$$\begin{aligned} d_n = S_0 &+ ((y_2-y_3) \ \ (x_3-x_2))[T_i(x_1, y_1)][D_i] \\ &+ ((y_3-y_1) \ \ (x_1-x_3))[T_j(x_2, y_2)][D_j] \\ &+ ((y_1-y_2) \ \ (x_2-x_1))[T_j(x_3, y_3)][D_j] \end{aligned} \tag{3.5}$$

where $[T_i(x_i, y_i)]$ and $[D_i]$ are deformation matrix and displacement variables of block i.

let

$$\begin{aligned} [H_i] &= [Y_i(x_1, y_1)]^T \begin{pmatrix} y_2-y_3 \\ x_3-x_2 \end{pmatrix} \\ [C_j] &= [T_j(x_2, y_2)]^T \begin{pmatrix} y_3-y_1 \\ x_1-x_3 \end{pmatrix} + [T_j(x_3, y_3)]^T \begin{pmatrix} y_1-y_2 \\ x_2-x_1 \end{pmatrix} \end{aligned} \tag{3.6}$$

then

$$d_n = [H_i]^T[D_i] + [G_j]^T[D_j] + S_0 \tag{3.5}$$

The stiffness of normal penalty spring is p, then the strain energy moving the stiff spring of shotcrete a distance of d_n results in

$$\begin{aligned} \Pi_{ns} &= \frac{p}{2}d^2 \\ &= \frac{p}{2}([H]^T[D] + [G]^T[D] + S_0)^2 \\ &= \frac{p}{2}([D]^T[H][H]^T[D] + [D]^T[G][G]^T[D] + 2[D]^T[H][G]^T[D] \\ &\quad + 2S_0[D]^T[H] + 2S_0[D]^T[G] + S_0^2) \end{aligned} \tag{3.6}$$

Minimizing Π_{ns} by taking the derivatives, four 6×6 submatrices and two 6×1 submatrices are obtained and added to $[K_{ii}]$, $[K_{jj}]$, $[K_{ji}]$, $[K_{ij}]$, $[F_i]$ and $[F_j]$ respectively. The derivatives of Π_{ns} are as follows.

$$\begin{array}{llll} p[H][H]^T & \rightarrow & [K_{ii}], & i = 1,\cdots,n \\ p[H][G]^T & \rightarrow & [K_{ij}], & i,j = 1,\cdots,n \\ p[G][H]^T & \rightarrow & [K_{ji}], & i,j = 1,\cdots,n \\ p[G][G]^T & \rightarrow & [K_{jj}], & j = 1,\cdots,n \\ -pS_0[H] & \rightarrow & [F_i], & i = 1,\cdots,n \\ -pS_0[G] & \rightarrow & [F_j], & j = 1,\cdots,n \end{array} \tag{3.7}$$

Submatices of shear penalty spring can be derived by similar method of normal penalty spring. The only difference comes from the definition of shear displacement of penetrating point. Assuming that penetrating point moved from P_o to P_1 in a step, shear displacement d_s can be defined as follows.

$$\begin{aligned} d_s &= \overrightarrow{P_0P_1} \cdot \overrightarrow{P_2P_3} \\ &= \left((x_1+u_1)-(x_0+u_0) \quad (y_1+v_1)-(y_0+v_0)\right)\begin{pmatrix}(x_3+u_3)-(x_2+u_2) \\ (y_3+v_3)-(y_2+v_2)\end{pmatrix} \end{aligned} \tag{3.8}$$

neglecting second order infinitesimal

$$d_s \approx S_s + (x_3 - x_2 \quad y_3 - y_2)\begin{pmatrix} u_1 - u_0 \\ v_1 - v_0 \end{pmatrix}, \tag{3.9}$$

$$d_s \approx S_s + (x_3 - x_2 \quad y_3 - y_2)\begin{pmatrix} u_1 \\ v_1 \end{pmatrix} - (x_2 - x_3 \quad y_2 - y_3)\begin{pmatrix} u_0 \\ v_0 \end{pmatrix}$$

where

$$S_s = (x_1 - x_0 \quad y_1 - y_0)\begin{pmatrix} x_3 - x_2 \\ y_3 - y_2 \end{pmatrix} \tag{3.10}$$

$$[N_i] = [T_i(x_1, y_1)]\begin{pmatrix} x_3 - x_2 \\ y_3 - y_2 \end{pmatrix}, \tag{3.11}$$

$$[M_j] = [T_i(x_0, y_0)]\begin{pmatrix} x_2 - x_3 \\ y_2 - y_3 \end{pmatrix}$$

then from (3.11) we have

$$d_s = [N_i]^T[D_i] + [M_j]^T[D_j] + S_s \tag{3.12}$$

Let stiffness of shear penalty spring is f, then the strain energy of moving the stiff spring of shotcrete a distance of d_s is

$$\begin{aligned} \Pi_{ss} &= \frac{f}{2}d_s^2 \\ &= \frac{f}{2}([N]^T[D] + [M]^T[D] + S_s)^2 \\ &= \frac{f}{2}([D]^T[N][N]^T[D] + [D]^T[M][M]^T[D] + 2[D]^T[N][M]^T[D] \\ &\quad + 2S_s[D]^T[N] + 2S_s[D]^T[M] + S_s^2) \end{aligned} \tag{3.13}$$

Minimizing Π_{ss} by taking the derivatives, four 6×6 submatrices and two 6×1

submatrices are obtained and added to $[K_{ii}], [K_{jj}], [K_{ji}], [K_{ij}], [F_i]$ and $[F_j]$ respectively. the derivatives of Π_{ss} are as follows.

$$
\begin{array}{lccl}
f[N][N]^T & \rightarrow & [K_{ii}], & i=1,\cdots,n \\
f[N][M]^T & \rightarrow & [K_{ij}], & i,j=1,\cdots,n \\
f[M][N]^T & \rightarrow & [K_{ji}], & i,j=1,\cdots,n \\
f[M][M]^T & \rightarrow & [K_{jj}], & j=1,\cdots,n \\
-f\,S_s[N] & \rightarrow & [F_i], & i=1,\cdots,n \\
-f\,S_s[M] & \rightarrow & [F_j], & j=1,\cdots,n
\end{array}
\qquad (3.14)
$$

3.3 Shotcrete Failure

Contact panalty spring in DDA contacts algorithm is determined to be installed or to be removed by Open-close iteration. In our research, meanwhile, shotcrete penalty spring does not need iteration, but needs failure condition. The failure of the installed shotcrete penalty spring is determined by tensile force and shear force acting on it.

Tensile force and shear forces acting on installed shotcrete are denoted F_n and F_s. Measuring the displacement of penetrating point, We have

$$F_s = p_s \bullet d_s, \quad F_n = p_n \bullet d_n \qquad (3.15)$$

where d_s, d_n, p_n, and p_s are shear displacement, normal displacement, normal spring stiffness and shear spring stiffness, respectively.

The shotcrete failure occurs when these forces exceed the resistant capacity of shotcrete. The resistant capacity of shotcrete is defined by tensile and shear strength and shotcrete thickness. Failure condition, therefore, can be defined as follows.

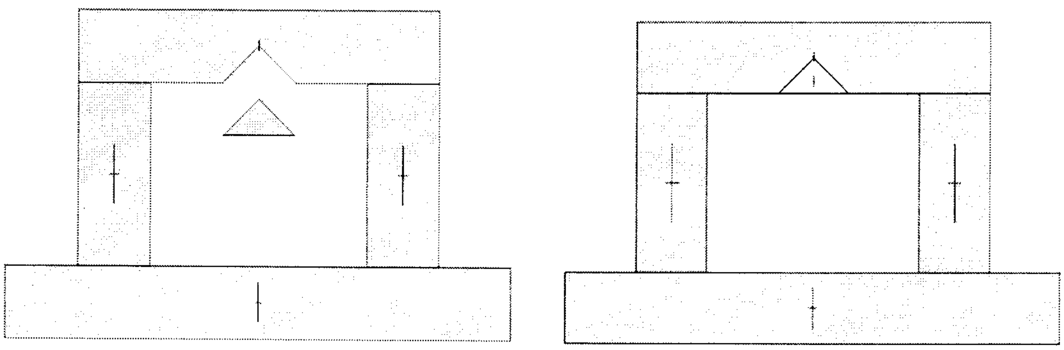
$$|p_n \bullet d_n| > \sigma_T t \quad \text{or} \quad p_s \bullet d_s > \sigma_s t \qquad (3.16)$$

where, σ_n is tensile strength of shotcrete, σ_n is shear strength of shotcrete and t is thickness of shotcrete.

If the failure condition satisfies (3.16) in a step, then the shotcrete penalty spring will be removed from next step on.

4. Validation

Firstly, new shotcrete algorithm was verified by using simple key block as shown in Fig. 4.1. In this model, the triangular key block is generated under a upper block.



(a) case2: 0.01m thick shotcrete (b) case1: 0.02m thick shotcrete

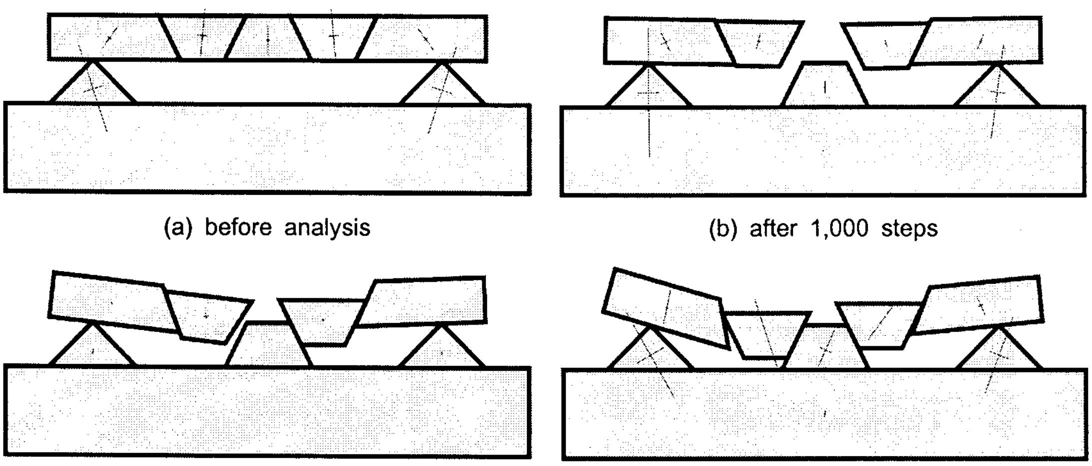
Figure 4.1 Sing key block test

By the weight of key block itself, the block must be loaded at least 0.3 MPa to be supported. Then we installed shotcrete which has its tensile and shear strenth as 10 MPa/m. Figure 4.1 shows two different results with different shotcrete thickness. Fig. 4.1(a) shows the result where 0.01m-thick-shotcrete is installed, while Fig. 4.1(b) shows the result where 0.02m-thick-shotcrete is installed. Load acting on shotcrete is 0.15 MPa, which is caused by weight of the overlying key block. The resistant capacity of shotcrete is 0.1 MPa in Fig. 4.1(a) while 0.2 MPa in Fig. 4.1(b). Both two results agree with predicted results. By further tests with increasing shotcrete thickness, the key block was dropped when the thickness was over 0.015m. Therefore, we can conclude that the installed shotcrete works and fails correctly. Table 4.1 shows some input parameters.

Table 4.1 Input parameters for key block test

Unit mass	2.5 t/m^3	Normal spring stiffness	10 GN/m
Young's modulus	10 GPa	Shear spring stiffness	5 GN/m
Poisson's ratio	0.24	tensile strength of shotcrete	10 MPa
Friction angle of joints	35°	shear strength of shotcrete	10 MPa
cohesion of joints	0	Initial time step	0.01 sec

Second example used is discontinuous bar consists of 5 blocks. It is broken as Fig. 4.2, when shotcrete is not installed under it. Input parameters are same as Table 4.1 except for the strengths of shotcrete. The length and height of bar is 1m and 10cm respectively. The crosses at center of blocks presents principal stresses of the block. To prevent the collapse of the bar, shotcrete is installed over bottom of the bar with sufficient thickness and stiffness. Then, the bar was kept intact as shown in Fig. 4.3. Principal stresses of each blocks were approximately same with predicted one.



(a) before analysis (b) after 1,000 steps

(c) after 3,000 steps (c) after 4,000 steps

Figure 4.2 Result of discontinuous bars without shotcrete

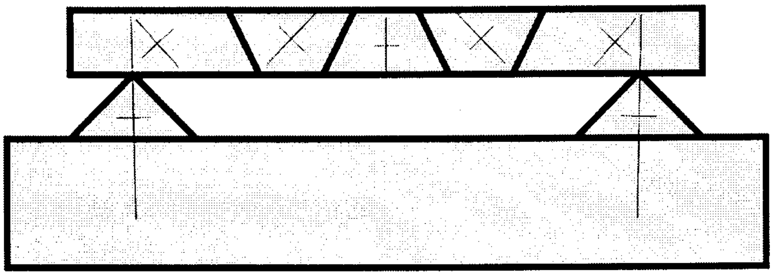


Figure 4.3 Result of discotinuous bars with shotcrete

Lastly we applied the new shotcrete algorithm to a simple tunnel geometry. The tunnel geometry is as shown in Fig. 4.4. Fig. 4.5(a) shows that The tunnel loops was collapsed without any support. Then we supported tunnel by installing shotcrete on its loop and sidewall. Fig. 4.5(b) is DDA result when we installed shotcrete on the sidewall. We could find that key block on the loop was droped while sidewall was supported by the shotcrete. Fig. 4.5(c) is the DDA result when the shotcrete was installed both on the loop and sidewall. As shown in the figure, the tunnel section was supported safely by shotcrete.

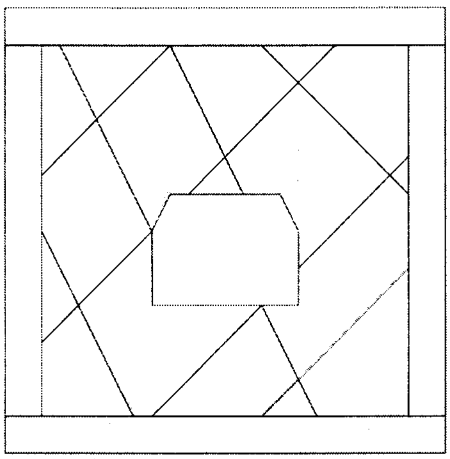
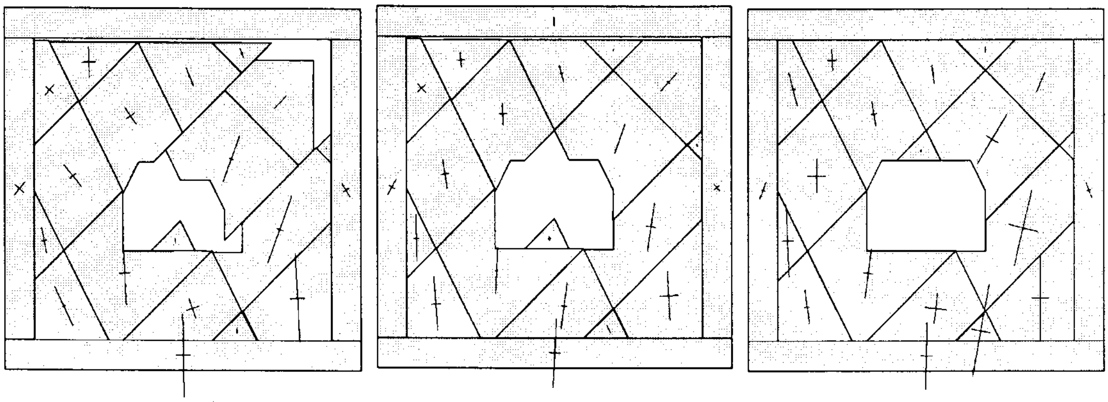


Figure 4.4 The Tunnel geometry for DDA analysis



(a) case1 (b) case2 (c) case3

Figure 4.5 DDA result of the tunnel analysis

5. Conclusions

In this paper, shotcrete support algorithm using penalty spring method was developed. By defining penetrating point and virtual reference line, we could derive submatrices of shotcrete penalty spring which were added to global matrix. Though mathematical process was similar with contact penalty springs, the shotcrete penalty spring needs its failure condition rather than open-close iteration. The penalty spring works when an edge on which shotcrete was installed contacts with other reinforced edge. Also, it fails when tensile force or shear force exceeds its resistant capacity. As verified by 3 simple case studies, the shotcrete algorithm worked well when it was installed on any edges, and removed successfully when it failed. From this study, shotcrete support algorithm can contribute to make DDA more practical analyzing tool for blocky system models.

Acknowledgement

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