FUZZY GOAL PROGRAMMING FOR CRASHING ACTIVITIES IN CONSTRUCTION INDUSTRY

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Abstract

Many contracting firms and project managers in the construction industry have started to utilize multi objective optimization methods to handle multiple conflicting goals for completing the project within the stipulated time and budget with required quality and safety. These optimization methods have increased the pressure on decision makers to search for an optimal resources utilization plan that optimizes simultaneously the total project cost, completion time, and crashing cost by considering indirect cost, contractual penalty cost etc., practically charging them in terms of direct cost of the project which is fuzzy in nature. This paper presents a multiple fuzzy goal programming model (MFGP) that supports decision makers in performing the challenging task. The model incorporates the fuzziness which stems from the imprecise aspiration levels attained by the decision maker to these objectives that are quantified through fuzzy linear membership function. The membership values of these objectives are then maximized which forms the fuzzy decision. The problem is solved using LINGO 8 optimization solver and the best compromise solution is identified. Comparison between solutions of MFGP, fuzzy multi objective linear programming (FMOLP) and multiple goal programming (MGP) are also presented. Additionally, an interactive decision making process is developed to enable the decision maker to interact with the system in modifying the fuzzy data and model parameters until a satisfactory solution is obtained. A case study is considered to demonstrate the feasibility of the proposed model for optimization of project network parameters in the construction industry.

Keywords: Fuzzy sets, multi objective optimization, membership function, fuzzy goal programming, fuzzy decision

Introduction

Construction time-cost optimization problems are viewed as one of the most important aspects of construction decision-making. Time and cost are the two main concerns of construction project. In construction industry, project manager and contractors usually use previous experience to estimate project duration and cost. Typically, a project is broken down into activities to which resources can be assigned and durations and costs are estimated. The activities are linked according to work sequences to form a network. Critical path method (CPM) techniques are used to analyze the network to identify critical path(s) and project duration. In general, the more resources assigned to an activity, the less time it takes to complete, but cost is usually higher. This trade-off between time and cost gives decision makers both challenges and opportunities to work out the best construction plan which optimizes time and cost to complete a project. In addition, the project managers need to accelerate a project to meet a dead line issued by the owner because of delay of previous activity. Adjustments are needed to level the resource of the individual activities in the project network to optimize the project duration at minimum cost.

Heuristic and mathematical methods are the two major approaches use to solve the time-cost optimization problems in the construction project. Although, these techniques are available since 1960s, the construction industry has not widely accepted in their day-to-day planning tasks. Some project managers, trained in operations research and computer applications are experimenting with mathematical models using linear programming, integer programming, dynamic programming etc., to provide better solutions. However, formulating constraints and objective functions requires considerable effort and is prone to errors.

Most practitioners in construction industry agree that time-cost optimization is an important issue, and is ignored in the analysis. In the real world, time-cost optimization problems are of multi dimensional with multiple objectives and conflicting each other, which are required to be optimized simultaneously in an uncertain environment. These problems deal with uncertainties associated with the goals, model parameters, and the quality of information data. The major uncertainty factors include workmen factors, construction factors, controlling factors, and resource management factors, which defy quantification Project managers expresses their wish to have an easy – to-use tool to model these uncertainties to provide an optimal balance of time and cost based on their personal experience, intuition, judgment, aspiration and knowledge. This paper develops an efficient algorithm to yield accurate solutions and easy-to-use tool for real practice. Fuzziness in the proposed model stems from the imprecise aspiration levels attained by the decision maker to all of the three objectives. These imprecise aspiration levels are quantified through the use of piecewise linear and continuous membership functions. The objective is to minimize the membership value of the objectives, which forms the fuzzy decisions. A practical application of the proposed model to a real-life project network problem is formulated and solved using LINGO 8 computer package.

Objectives

The objective of the paper is to probe and develop methods and procedures for

- to formulate an interactive multiple fuzzy goal programming model to support multi objective optimization.
- to quantify the fuzzy aspiration levels to represent decision maker's fuzzy goals by allowing fuzziness derived from the experts in the field.
- to develop the membership functions for each fuzzy goals.
- to transform the crisp single objective minimization model to a single-goal MFGP (max L) problem
- to optimize simultaneously the total project cost, completion time, crashing cost and the decision makers overall degree of satisfaction.
- to obtain the fuzzy crash time tolerance, corrected duration, start and finish times, float times of all the activities.

- to execute and modify the interactive decision making process until a satisfactory solution is obtained.

Literature Review

Zadeh developed fuzzy set theory in 1965, for decision making involving fuzzy information. Since then more than 5,000 publications have highlighted the concept and diversified use of fuzzy set theory. A research survey since 1978 to 2004 by various authors has highlighted the importance of fuzzy set theory, fuzzy system modeling, fuzzy linear programming, fuzzy multi objective linear programming, fuzzy multi objective goal programming, etc. for various planning models. Table 1 summarizes the papers presented by authors in various areas with different model classifications, and model attributes. Judging from the state of research as depicted in Table 1 it is observed that research on fuzzy construction project scheduling has been published over the last 30 years.

Table 1 Survey of Literature Review in the field of Fuzzy Programming

Author (s)	Model classification	Model attributes
Wang and Liang (2004)	Fuzzy goal programming	13 activity network is solved with three objectives of minimizing project cost, time and crashing cost.
Arikan and Gungor (2001)	Fuzzy goal programming	65 activity network is solved with two objectives of optimizing completion time and crashing costs.
Wang and Fang (2001)	Fuzzy linear programming	solves the aggregate production planning (APP) problem with multiple objectives
Chen and Tsai (2001)	Fuzzy goal programming	focuses on the reformulation of FGP using an additive model the problem with preemptive priorities.
Kuwano (1996)	Fuzzy goal programming	solves the FMOLP considering the coefficients as fuzzy triangular possibilistic variables.
Hapke et al. (1994)	Fuzzy project scheduling decision support system	53 activity network for resource allocation in software development.
Rommelfanger (1994)	Fuzzy network analysis	14 activity network is solved to calculate the slack times in project network models with fuzzy intervals.
Premchandra (1993)	Goal programming	activity crashing in project networks.
DePorter and Ellis (1990)	Fuzzy linear programming	10 activity network is solved to minimize completion time and crashing cost simultaneously
Tiwari et al. (1987)	Fuzzy goal programming	an additive model by employing the usual edition as an operator to aggregate the fuzzy goals.
Mjelde (1986)	Fuzzy linear programming	fuzzy resource allocation.
Leberling (1981)	Fuzzy goal programming	introduces linear and non-linear membership function to a fuzzy programming problem of several objectives.
Hannan (1981)	Fuzzy goal programming	compares vector maximum methods, goal programming and interactive techniques.
Zimmermann (1978)	Fuzzy linear programming	applies FLP approach to vector maximum problem.

Research on multi objective optimization with two objectives of minimizing the total crashing cost and completion time in the project network has been increased in the last few years. Research on optimization of additional objective of minimizing total project cost, along with the crashing cost and completion time is not found. Hence, efforts are made in this paper to demonstrate how fuzzy multi objective goal programming model assists the decision maker in modeling the objective of minimizing total project cost for project scheduling problems when multiple goal values are to be precisely identified using a real time case study.

Fuzzy Multi Objective Optimization

Many real world decision-making problems are multi dimensional and have multiple objectives, which are often non-commensurable and conflicting with each other. These multiple objectives need to be optimized simultaneously. One-way of handling these problems are to choose one of the goals and treat the other goals as constraints to ensure that some minimal "*satisfying*" levels of the other goals are achieved. The linear programming model assumes that all variables are non-negative and the constraints are binding with one objective function. In addition, the decision makers may need to formulate the goals and constraints, which are in vague and in linguistics terms due to uncertainty or imprecision in their state of knowledge. This uncertainty or imprecision may occur in several different components within the decision making process. One possible method to solve such complex decision problems is fuzzy multi objective optimization (FMO), more specifically the fuzzy goal programming. Mathematically, fuzzy goal programming problems are expressed as

$$\max \quad [\tilde{f}_1(x),\ \tilde{f}_2(x),\ \ldots,\ \tilde{f}_k(x)] \qquad (1)$$

$$\text{Subject to } \tilde{A}x \le \tilde{b}$$

$$x \ge 0$$

Where x is the vector of variables and $\tilde{b}$ is a vector of fuzzy right hand side. The membership function of fuzzy goal is defined as:

$$\mu_{gi}(x) \begin{Bmatrix} 1, & f_i(x) > f_i^+(x) \\ 1-\dfrac{f_i^+(x)-f_i(x)}{f_i^+(x)-f_i^-(x)}, & f_i^-(x) \le f_i(x) \le f_i^+(x) \\ 0, & f_i(x) < f_i^-(x) \end{Bmatrix} \qquad (2)$$

$$\mu_{Cj}(x) = \begin{cases} 1, & (Ax)_j < b_j \\ 1-\dfrac{(Ax)_j - b_j}{p_j} & b_j \le (Ax)_j \le b_j + p_j \\ 0, & (Ax)_j > b_j + p_j \end{cases} \qquad (3)$$

where f_i^+(x) and f_i^-(x) represents the positive ideal solution and negative ideal solution, respectively. In this case, Equation 3 can be transformed to λ expression method as follows

$$\max_x \lambda$$

$$\text{subject to } \lambda \le 1 - \frac{f_i(x)-f_i^-(x)}{f_i^+(x)-f_i^-(x)}, i = 1,2\ldots,k \qquad (4)$$

$$\lambda \le 1 - \frac{(Ax)_j - b_j}{p_j}, j = 1,2,\ldots,m;$$

$$Ax \le b$$

$$x \ge 0$$

It can also employ max-min method to transfer Equation 1 as follows:

$$\max_{x} \min_{i,j} \lambda \qquad (5)$$

Subject to $Ax \leq b$
$x \geq 0$

Multiple Goal Programming (MGP)

Multiple goal programming (MGP) model is developed by identifying and defining the variables, formulating the constraints and the objective functions. This allows single objective minimization using linear programming model to optimize each objective function in succession. The first objective function is considered to minimize the total project cost (Z_1) using LP-1 model. The second objective function is considered to minimize the completion time (Z_2) using LP-2 model. The last objective function is considered to minimize the crashing cost (Z_3) using LP-3 model. The complete MGP model is stated as below.

Minimizes total project costs

$$\min\ Z_1 = \sum_i \sum_j C_{D_{ij}} + I_1 \sum_i \sum_j k_{ij} Y_{ij} + [C_I + m(E_n - T_o)] + I_2[C_P + h(E_n - T_o)], \qquad (6)$$

where, the terms $\sum_i \sum_j C_{D_{ij}} + I_1 \sum_i \sum_j k_{ij} Y_{ij}$ are used to calculated total direct costs. The total direct costs includes the total normal (direct) cost and the total crash (direct) cost, determined using additional direct resources including overtime, personnel and equipment; $[C_I + m(E_n - T_o)]$ are indirect costs including those of administration, depreciation, financial and other variable overhead costs that can be avoided by reducing total project time; $I_2[C_P + h(E_n - T_o)]$ are contractual penalty costs incurred if a project continues beyond a specified date under normal conditions.

Minimizes total completion time

$$\min\ Z_2 = E_n - E_0 \qquad (7)$$

Minimize total crashing costs

$$\min\ Z_3 = I_1 \sum_i \sum_j k_{ij} Y_{ij} \qquad (8)$$

Constraints

Constraints on the time between event i and event j

$$E_i + t_{ij} - E_j \leq 0 \quad \forall i, \forall j \qquad (9)$$

$$t_{ij} = D_{ij} - Y_{ij} \quad \forall i, \forall j$$

Constraints on the crash time for activity (i, j)

$$I_1 Y_{ij} \geq 0 \quad \forall i, \forall j \qquad (10)$$

$$Y_{ij} \leq D_{ij} - d_{ij} \quad \forall i, \forall j$$

Constraints on project start time and total completion time

$$E_1 = 0 \qquad (11)$$

Constraints on the total budget

$z_1 \leq b \quad \forall i, \forall j$ (12)

Constraints on choosing of decision alternatives

$I_1 + I_2 = 1 \quad I_1, I_2 = 0$ or 1 (13)

Non-negativity constraints on decision variables:

$t_{ij}, Y_{ij}, E_i, E_j \geq 0 \quad \forall i, \forall j$ (14)

Fuzzy Multi Objective Linear Programming (FMOLP)

Fuzzy multi objective linear programming (FMOLP) model is formulated similarly to that of a MGP model. In the FMOLP model the constraints remain the same as that of MGP, but three additional constraints are added to incorporate the project total cost (Z_1), completion time (Z_2) and the crashing cost (Z_3) objective functions respectively. These constraints are developed using a linear fuzzy membership function of Zimmerman (1978) and fuzzy decision making of Bellman and Zadeh (1970). In order to develop these constraints, the MGP model is solved once for each of the objectives using LP-1, LP-2 and LP-3 models respectively. From these solutions, aspired levels of achievement, the least acceptable level of achievement and the degradation allowance for each objective are determined. Finally, the objective function of the FMOLP model is transformed to a single goal optimization max (L) problem to maximize the membership value of all the three objective functions simultaneously. The complete FMOLP model is stated as below.

Maximize λ (15)

Subject to:

$$\lambda \leq \sum_i \sum_j C_{D_{ij}} + I_1 \sum_i \sum_j k_{ij} Y_{ij} + [C_I + m(E_n - T_o)] + I_2[C_P + h(E_n - T_o)] - \left(\frac{Z_g^u - Z_1}{Z_g^u - Z_g^l} \right)$$

$$\lambda \leq (E_n - E_1) - \left(\frac{Z_g^u - Z_1}{Z_g^u - Z_g^l} \right); \lambda \leq I_1 \sum_i \sum_j k_{ij} Y_{ij} - \left(\frac{Z_g^u - Z_1}{Z_g^u - Z_g^l} \right);$$

Other constraint equations are same as that of MGP model shown from Equations 9 to 14 above

Multiple Fuzzy Goal Programming (MFGP)

A multiple fuzzy goal programming model is developed similar to that of a multiple goal programming model (MGP). In MFGP model, the constraints remain the same as MGP, but three additional constraints are added to incorporate the project total cost (Z_1), completion time (Z_2) and the crashing cost (Z_3) objective functions respectively. These constraints are developed by incorporating the fuzziness, which stems from the imprecise aspiration levels attained by the decision maker (DM) reflecting the relative flexibility among them. The aspiration levels are then quantified through the use of linear fuzzy membership function of Zimmerman (1978) and fuzzy decision making of Bellman and Zadeh (1970). In order to develop these constraints, the MGP model is solved for each of the objective in succession using linear programming (LP-1, LP-2 and LP-3) models. From these solutions, aspired levels of achievement, the least acceptable level of achievement, and the degradation allowance for each objective is determined by allowing a tolerance interval of -20% to +20% derived directly from the decision maker. Finally, the objective function of the MFGP model is transformed to a single goal optimization max (L) problem to maximize the membership value of all the three objective functions simultaneously. In addition, the model estimates the crash time tolerance, corrected duration, start and finish times, float times of all the project activities in a fuzzy environment. The complete MFGP model is stated as below.

Maximize λ *; Subject to:* (16)

$$\lambda \le (z_g^u - z_g)/(z_g^u - z_g^l) \quad \forall g .$$

Case Study

The case study focuses on the development of a mathematical multi objective fuzzy goal programming model to solve the widening of existing single lane bituminous pavement under construction to two lanes and simultaneous strengthening and realigning of the pavement for meeting the traffic demands. The existing pavement will have to be realigned at a number of locations in order to improve the geometric and avoid congested towns and villages. The entire project is broken into a number of activities and the activities listed in the sequential order. The normal and crash estimates are also identified. The aim of this study is to minimize simultaneously total project costs, total completion time, and total crashing costs, with reference to indirect costs, contractual penalty costs by practically them in terms of direct cost of the project. The relevant data for the case study are: the normal direct cost of the project is Rs. 4,03,09,867; total budget is Rs 6,93,64,596; fixed indirect costs is Rs 20,15,493; saved daily variable indirect costs is Rs 2,223 per day (@ 1.5% of 4,03,09,867/272 days); fixed contractual penalty costs is Rs 40,30,986 (@ 10% of 4,03,09,867); daily variable contractual penalty costs is Rs 40,309 per day (@ 0.1% of 4,03,09,867/272 days) and, the project completion time under normal conditions is 272 days.

Results of Optimization

The project network with 65 activities and 53 events is solved using MGP, FMOLP and MFGP by LINGO 8 optimization solver and the results are shown in Table 2.

Table 2 Results of Optimization

Item	LP-1	LP-2	LP-3	FMOLP	The proposed MFGP model
Objective function	Min Z1	Min Z2	Min Z3	Max L	Max L
L	100%	100%	100%	0.7554	0.8022
Z_1	Rs 4,62,44,840 $ 9,24,896.8	Rs 5,78,03,830 $ 11,56,076.6	Rs 4,63,56,350 $ 9,27,127	Rs 4,83,73,670 $ 9,67,473.4	Rs 4,82,97,830 $ 9,65,956.6
Z_2	267	236	272	244.8	245.24
Z_3	Rs 1,01,157.8 $ 2,023.15	Rs 1,29,78,630 $ 2,59,572.6	O	Rs 31,74,017 $ 63,480.34	Rs 30,79,341 $ 61,586.82

Using the initial solution of MGP model for each objective function, the objective values interval $[z_g^l, z_g^u]$ and their equivalent membership values of the decision-making in the interval [0, 1] are specified. As a result of the interactive relationship between decision maker and the analyst, the lower and upper bounds are determined as $[l_1, u_1]$ = [Rs 4,62,44,840, Rs 5,78,03,830]; $[l_2, u_2]$ = [236 days, 272 days] and $[l_3, u_3]$ = [Rs 0.000, Rs 1,29,78,630] for z_1, z_2 and z_3 respectively from the solutions of the MGP model. The complete FMOLP model of the case study problem can then be formulated as per Equation 15. The compromise optimization plan for the case study with the FMOLP model is obtained as $z_1 =$ Rs.4,83,73,670 ($ 9,67,473.4), $z_2 = 244.80$ days, $z_3 =$Rs 31,74,017 ($ 63,480.34) and the overall degree of satisfaction with the decision maker's multiple fuzzy goals is 0.7554. Similarly, using the solution of MGP aspired levels of achievement, the least acceptable level of achievement, and the degradation allowance for each objective is determined by allowing a tolerance interval of -20% to +20% derived directly from the decision maker. The complete MFGP model of the case study problem can then be formulated as per Equation 16. The compromise optimization plan for the case study with the MFGP is obtained as $z_1 =$ Rs 4,82,97,830 ($ 9,65,956.6), $z_2 = 245.24$ days, $z_3 =$Rs 30,79,341 ($ 61,586.82) and the overall degree of satisfaction with the decision maker's multiple fuzzy goals is 0.8022. Furthermore, the decision maker may try to modify interactively the piecewise linear membership grades of the fuzzy goals and related parameters until a satisfactory solution is obtained.

Analysis and Discussions

The actual implementation of the multiple fuzzy goal programming model is made, by considering various alternatives and analyzing the sensitivity of decision parameters to variations of relevant conditions, of the case study. The implementation is adapted to the following scenarios.

Scenario 1:

1-1: Removing Z3 (total crashing costs), consider only Z1 (total project costs) and Z2 (total completion time) simultaneously.
1-2: Removing Z2 (total completion time), consider only Z1 (total project costs) and Z3 (total crashing costs), simultaneously.
1-3: Removing Z1 (total project costs) consider only Z2 (total completion time) and Z3 (total crashing costs) simultaneously. Table 3 presents the results of implementing Scenario 1.

Scenario 2:

Sensitivity analysis of incremental crashing costs for each activity under the conditions of the preceding case study example. For, simplicity, consider only the incremental crashing costs for activity (32,40). Table 4 presents the results of implementing Scenario 2. Significant management implications for the practical application of the proposed model are as follows:

- The overall degree of decision making satisfaction (L) with goal values z_1 = Rs 4,82,97,830 ($ 9,65,956.6), z_2 = 245.24 days, and z_3 Rs 30,79,341 ($ 61,586.82) was initially generated as being 0.8022.
- The comparison for scenarios 1-1 to 1-3 demonstrates the interaction of trade-offs and conflicts among dependent objective functions and presented in Table 3. These solutions indicate that a fair difference and interaction exists in the trade-offs and conflicts among dependent objective functions. Different combinations of arbitrary objective function may influence the objective and L values. Accordingly, the proposed MFGP model meets the requirements of the practical application since it can minimize the total project costs, total completion time, and total crashing costs.

Table 3 Results of Scenario 1

Item	Scenarios 1-1	Scenarios 1-2	Scenarios 1-3
L	0.8656	0.9978	0.8022
Z1	Rs 4,93,49,620 $ 9,86,992.4	Rs 4,62,92,350 $ 9,25,847	--
Z2	241	--	245.24
Z3	--	Rs 33,198.81 $ 663.98	Rs 30,79,341 $ 61,586.82

Figure 1 Results of Scenario -2

- The results of Scenario 2 show that the incremental crashing costs for each activity affect the objective values. This finding implies that the decision maker must consider the cost-time slope in practical construction scheduling decision problems. The decision maker can also improve the efficiency of internal management, to reduce the cost of capital and thus reduce the incremental crashing costs associated with each activity. Figure 1 depicts the changes in the objective values of Scenario 2. Figure 2 depicts the comparison of the models.

Table 4 Results of Scenario 2

	Run 1	Run 2	Run 3	Run 4	Run 5
k_{3240}	0	Rs 49,328.57 $ 9,86.57	Rs 51,794.99 $ 1,035.89	Rs 54,261.42 $ 1,085.22	Rs 56,727.85 $ 1,134.55
L	0.8137	0.8022	0.8016	0.8011	0.8005
Z_1	Rs 4,80,85,960 $ 9,61,719.2	Rs 4,82,97,830 $ 9,65,956.6	Rs 4,83,08,570 $ 9,66,171.4	Rs 4,83,19,300 $ 9,66,386	Rs 4,83,29,480 $ 9,66,589.6
Z_2	244.47	245.24	245.28	245.32	245.36
Z_3	Rs 29,00,241 $ 58,004.82	Rs 30,79,341 $ 61,586.82	Rs 30,88,414 $ 61,768.28	Rs 30,97,487 $ 61,949.74	Rs 31,06,091 $ 62,121.82

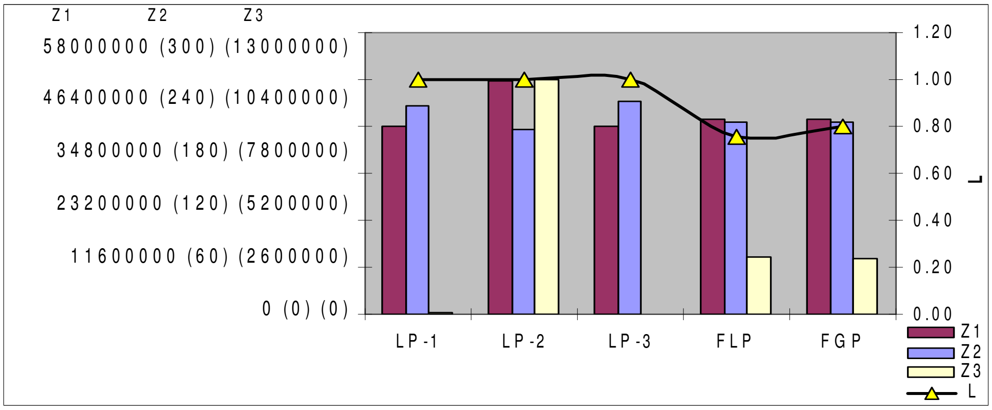

Figure 2 Comparison of models

Conclusions

In decision-making problems, existence of fuzzy parameters and multi objectives make the description of problems impossible by using traditional mathematical programming. Especially in crashed project network problems, the conflicting objectives of minimum completion time, total project cost and total crashing costs are required to be optimized simultaneously by the decision maker in the frame work of fuzzy aspiration levels. From Table 5, it is observed that the % change in the total cost ranges from 11.20% to 15.00%. The change definitely has an impact on the duration of the project and crash cost. Hence, the average change of the models is calculated. The average change ranges from 3.30% to 10.10%. Therefore, it is evident that even though the % change in the total cost of MFGP is more than FMOLP, because of the % change in time, and % change in crash cost, the weighted change is 4.11% which is less than 4.37% as for FMOLP. Hence, it can be clearly concluded that the MFGP model is the best.

Table 5 Analysis of the results

Total normal direct cost Rs. 40,309,867

Total completion time 272.00 Days

	MFGP	FMOLP	LP1	LP2	LP3
Total project cost	Rs.48,297,830	Rs.48,373,670	Rs.46,244,840	Rs.57,803,830	Rs.46,356,350
Total completion time	245.24 Days	244.80 Days	267.00 Days	236.00 Days	272.00 Days
Total crashing cost	Rs.3,079,341	Rs.3,174,017	Rs.101,158	Rs.12,978,630	Rs.0.000
Direct cost (Calculated)	Rs.45,218,489	Rs.45,199,653	Rs.46,143,682	Rs.44,825,200	Rs.46,356,350
L Value	0.8022	0.7554	1.0000	1.0000	1.0000
% Change in total project cost	12.20 %	12.10 %	14.50 %	11.20 %	15.00 %
% Change in completion time	-9.80 %	-10.00 %	-1.80 %	-13.24 %	0.00 %
% Change in total crashing cost	7.60 %	7.90 %	0.30 %	32.20 %	0.00 %
% Average change.	3.30 %	3.30 %	4.30 %	10.10 %	5.00 %
% Weighted change	4.11 %	4.37 %	4.30 %	10.10 %	5.00 %

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