APPLICATION OF FUZZY LINEAR PROGRAMMING FOR TIME COST TRADEOFF ANALYSIS

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Abstract

In real world, the project managers handle conflicting goals that govern the use of resources within the stipulated time and budget with required quality and safety. These conflicting goals are required to be optimized simultaneously by the project managers in the framework of fuzzy aspiration levels. The fuzzy linear programming model proposed herein helps project managers to minimize total project costs, completion time, and crashing costs considering indirect costs, contractual penalty costs etc by practically charging them in terms of direct cost of the project. A case study of bituminous pavement under construction is considered to demonstrate the feasibility of applying the proposed model for optimization of project parameters. Consequently, the proposed model yields an efficient compromise solution and the decision maker's overall degree of satisfaction with multiple fuzzy goal values. Additionally, the proposed model provides a systematic decision-making framework, enabling decision maker to interactively modify the fuzzy data and model parameters until a satisfactory solution is obtained. The significant characteristics that differentiate the proposed model with other models include, flexible decision-making process, multiple objective functions, and wide-ranging decision information.

Keywords: Fuzzy sets, fuzzy numbers, fuzzy linear programming, flexibility, membership function.

Introduction

The construction industry is one of the largest in the world encompassing projects of all scales. In spite of being the largest there has not been a landmark evolution of an organized effort to regulate the processes to make them more efficient. Over the years, the construction processes are getting more complicated and is difficult to complete any project without the aid of systems directed at efficient management of time and cost. There is an on going need for the construction industry to expand and improve its capabilities and its scope of operations to meet changing and, in the long run, growing demands for its success.

Minimizing project completion time and cost continues to be a universally sought and highly desirable goal and these are conflicting objectives. Project managers attempt to schedule project activities at appropriate times and in proper sequence such that project completion cost and time are optimized. However, project managers are continuously facing a situation in which they must take a decision to complete the project within the stipulated time and budget with the required quality and safety. Thus, there is a need to identify the set of decisions that lead to optimum project duration and cost resulting in optimum time cost trade off analysis. Traditionally, the critical path method (CPM) and the program evaluation review technique (PERT) have been employed to solve time cost trade off problems. These techniques minimize project completion time and cost through crashing particular activities and plan, coordinate and display the necessary activities. These help the project progress to be monitored and controlled by the project managers. Also, these techniques establish a feasible and desirable relationship between the time and cost of the project by reducing the target time accounting the cost of expediting.

However, these techniques involve constructing a single objective optimization model from the minimum crash cost under a time constraint. The optimum solution gained from such a single objective programming leaves the decision maker to either accept the results unconditionally or reject them. The model does not always provide reasonable answers nor typically lead to a true understanding of and insight into the actual problem. Failure of many contractors to fully use these techniques expresses fundamental failure in these models. Field and academic research have failed to question the feasibility of these techniques. It is found that these models are neither true models nor best approximate model of the construction process because control of construction resources is more desirable than minimum calendar duration of the whole project. Therefore, minimizing cost is an objective to be considered as much as minimizing overall duration.

Increased sophistication in optimization techniques led to examine the possibility of incorporating the linear programming (LP) to trade off time and cost within the optimization framework. The model is advantageous when changes in the network logic is involved, decisions required to be made simple, and to handle a large data or alternatives. The model deals with the optimization of a linear objective function subject to a set of constraints in the form of linear in equalities and equations. The model involves the planning of activities in order to obtain an optimal solution, which reaches the specified goal best among all feasible alternatives.

The proposed model aims to optimize total project costs, completion time, and crashing costs, simultaneously considering indirect costs, contractual penalty costs etc by practically charging them to direct cost of the project. Fuzziness in the proposed model stems from the imprecise aspiration levels attained by the decision maker to all of the objective functions. These imprecise aspiration levels are quantified through the use of piecewise linear and continuous membership functions. The proposed model minimizes the membership value of all the objectives, which forms the fuzzy decisions. A practical application of the proposed model to a real-life construction project is developed and solved using LINGO optimization solver. Comparisons were made between the solutions of crisp and fuzzy models.

Literature Review

A summary of research made by various researchers related to this topic using different fuzzy optimization models are presented.

Arikan and Gungor (2001) presented a practical application of fuzzy goal programming (FGP) to a real life project network problem. A project network problem with two objectives and 65 activities is considered. LP, FLP, LMM and FGP models used LINDO computer package to solve the problem. Chen and Tsai (2001) have investigated fuzzy goal programming (FGP) problem with different important levels and preemptive priorities.

Hapke and Slowinski (1996) presented a generalized heuristic method for solving a resource constrain project-scheduling problem (RCPS) with uncertain time parameters. An application of the method to an example problem, in particular, graphical presentation of fuzzy Gantt charts and resource usage profiles are explained. Kuwano (1996) proposed a fuzzy goal-programming (FGP) model as a substitute for a fuzzy multi objective linear programming (FMOLP) problem. Shipley et al. (1996) incorporated fuzzy logic, belief functions, extension principles and fuzzy probability distributions, and developed the fuzzy PERT algorithm. Chang et al. (1995) compared methods of analyzing fuzzy numbers into an efficient procedure for solving project scheduling problems.

Hapke et al. (1994) presented a fuzzy project scheduling (FPS) decision support system. Expected project completion time and maximum lateness are identified as the project performance measures and a sample problem is demonstrated for a software engineering project involving 53 activities. Lorterapong (1994) introduced a resource-constrained project scheduling method that addressed three performance objectives: (i) expected project completion time; (ii) resource utilization; an (iii) resource interruption. Nasution (1994) argued that for a given alpha-cut level of the slack, the availability of the fuzzy slack in critical path models provides sufficient information to determine the critical path. Rommelfanger (1994) presented a new method for determining starting dates and slack time in project network models with fuzzy intervals.

McCahon (1993) compared the performance of fuzzy project network analysis (FPNA) and PERT. Premchandra (1993) demonstrated the activity crashing in project networks using goal programming (GP). An 8-activity product development project network is solved and the solutions were compared with crisp linear programming (LP). DePorter and Ellis (1990) presented a project crashing model using fuzzy linear programming, LP and goal programming to determine the project crashing cost and project duration under each solution technique.

Hannan (1981) described the use of fuzzy set theory in goal programming which demonstrated how fuzzy aspirations levels of the decision maker could be quantified through the use of piece wise linear and continuous functions. Leberling (1981) presented an application of fuzzy approaches to the linear vector maximum problem. Zimmermann (1978) developed fuzzy linear programming model, which solves the linear vector maximum problems.

Fuzzy Sets Concept

The theory of fuzzy sets originated in 1965 when Prof. Lotfi A. Zadeh from California University, Berkeley, introduced an idea of fuzzy set which is a central concept of the classical set theory. According to dictionary, a fuzzy set is a generalization of the classical set, which allows for varying degrees of membership functions. This means that a value between 0 and 1, the degree of membership in the fuzzy set, is assigned to each element. The membership of an element in a given fuzzy set increases with increasing value of the degree of membership. The theory starts from this simple and natural idea allowing the theory to adapt in an environment of uncertainty and inaccuracy, which is inherent to the real world. The word fuzzy also corresponds to the theory base and it may be understood as not sharp, dim, vague or uncertain. Although the original attitude of “black-and-white" mathematicians was rejected, the theory of fuzzy sets was under theoretical investigations. Today this theory belongs to well-developed and appreciated mathematical theories.

Fuzzy Linear Programming

The classical linear programming model is defined as

Maximize Z = CX

Subject to AX $\leq$ b X $\geq$ 0 (1)

Here, $X = \langle x_1, x_2, ...x_n \rangle^T$ is a vector of variables, $A = [a_{ij}], where\ i \in N_m\ and\ j \in N_n$ is a constraint matrix, and $b = \langle b_1, b_2, ...b_n \rangle^T$ is a right hand side vector. The optimal values for these problems can be achieved by graphical method or with simplex methodology. Equation 1 is effective as far as the constraints and their coefficients are crisp, but as in many practical situations the constraints are not crisp and don’t have a precise value rendering them to be given some flexibility. Hence to incorporate these vague factors into the mathematical equations fuzzy linear programming is used. The generalized fuzzy linear programming is stated as:

Maximize $\sum_{j=1}^{n} \tilde{C}_j X_j$

Subject to $\sum_{j=1}^{n} \tilde{A}_{ij} X_j \underset{\sim}{<} \tilde{B}_i \quad (i \in N_n)$ (2)

$X_j \geq 0 \quad (j \in N_m)$

Where, $\tilde{A}_{ij}, \tilde{B}_i\ and\ \tilde{C}_j$ are fuzzy numbers and X_j are variables whose states are fuzzy numbers $(i \in N_n, j \in N_m)$. Here $\underset{\sim}{<}$ denotes the ordering of fuzzy numbers or approximately less than or equal to. The fuzziness can be in the availability of resources, coefficients of objective functions, coefficients of the constraints, or combination of these parameters.

Objectives

The objectives of the paper is to probe and develop methods for

- Setting up the crisp multi objective goal programming problem.
- Constructing intervals for fuzzy aspiration levels using information getting directly from the decision maker, and the intervals determined from the linear programming solutions.
- Formulating the piecewise linear and continuous membership function for each fuzzy goal.
- Transforming the crisp problem into an equivalent single-goal mathematical (max L), FMOLP problem.
- Executing and modifying the interactive decision-making process using appropriate optimization solver until a satisfactory solution is obtained.

Multiple Goal Programming (MGP)

Multiple goal programming (MGP) model is developed by identifying and defining the variables, formulating the constraints and the objective functions. This allows single objective minimization using linear programming model to optimize each objective function in succession. The first objective function is considered to minimize the total project cost (Z_1) using LP-1 model. The second objective function is considered to minimize the completion time (Z_2) using LP-2 model. The last objective function is considered to minimize the crashing cost (Z_3) using LP-3 model. The complete MGP model is stated as below.

Minimizes total project costs

$$\min\ Z_1 = \sum_i \sum_j C_{D_{ij}} + I_1 \sum_i \sum_j k_{ij} Y_{ij} + [C_I + m(E_n - T_o)] + I_2 [C_P + h(E_n - T_o)], \tag{3}$$

where, the terms $\sum_i \sum_j C_{D_{ij}} + I_1 \sum_i \sum_j k_{ij} Y_{ij}$ are used to calculated total direct costs. The total direct costs includes the total normal (direct) cost and the total crash (direct) cost, determined using additional direct resources including overtime, personnel and equipment; $[C_I + m(E_n - T_o)]$ are indirect costs including those of administration, depreciation, financial and other variable overhead costs that can be avoided by reducing total project time; $I_2[C_P + h(E_n - T_o)]$ are contractual penalty costs incurred if a project continues beyond a specified date under normal conditions.

Minimizes total completion time

$$\min\ Z_2 = E_n - E_0 \tag{4}$$

Minimize total crashing costs

$$\min\ Z_3 = I_1 \sum_i \sum_j k_{ij} Y_{ij} \tag{5}$$

Constraints

Constraints on the time between event i and event j

$$E_i + t_{ij} - E_j \le 0 \quad \forall i, \forall j \tag{6}$$

$$t_{ij} = D_{ij} - Y_{ij} \quad \forall i, \forall j$$

Constraints on the crash time for activity (i, j)

$$I_1 Y_{ij} \ge 0 \quad \forall i, \forall j \tag{7}$$

$$Y_{ij} \le D_{ij} - d_{ij} \quad \forall i, \forall j$$

Constraints on project start time and total completion time

$$E_1 = 0 \tag{8}$$

Constraints on the total budget

$$z_1 \le b \quad \forall i, \forall j \tag{9}$$

Constraints on choosing of decision alternatives

$$I_1 + I_2 = 1 \quad I_1, I_2 = 0 \text{ or } 1 \tag{10}$$

Non-negativity constraints on decision variables:

$$t_{ij}, Y_{ij}, E_i, E_j \ge 0 \quad \forall i, \forall j \tag{11}$$

Fuzzy Multi Objective Linear Programming (FMOLP)

Fuzzy multi objective linear programming (FMOLP) model is formulated similarly to that of a MGP model. In the FMOLP model the constraints remain the same as that of MGP, but three additional constraints are added to incorporate the project total cost (Z_1), completion time (Z_2) and the crashing cost (Z_3) objective functions respectively. These constraints are developed using a linear fuzzy membership function of Zimmerman (1978) and fuzzy decision making of Bellman and Zadeh (1970). In order to develop these constraints, the MGP model is solved once for each of the objectives using LP-1, LP-2 and LP-3 models respectively. From these solutions, aspired levels of achievement, the least acceptable level of achievement and the degradation allowance for each objective are determined. Finally, the objective function of the FMOLP model is transformed to a single goal optimization max (L) problem to maximize the membership value of all the three objective functions simultaneously. The complete FMOLP model is stated as below.

$$\text{Maximize } \lambda \tag{12}$$

Subject to:

$$\lambda \le \sum_i \sum_j C_{D_{ij}} + I_1 \sum_i \sum_j k_{ij} Y_{ij} + [C_I + m(E_n - T_o)] + I_2[C_P + h(E_n - T_o)] - \left(\frac{Z_g^u - Z_1}{Z_g^u - Z_g^l} \right)$$

$$\lambda \le \left(E_n - E_1\right) - \left(\frac{Z_g^u - Z_1}{Z_g^u - Z_g^l} \right);$$

$$\lambda \le I_1 \sum_i \sum_j k_{ij} Y_{ij} - \left(\frac{Z_g^u - Z_1}{Z_g^u - Z_g^l} \right);$$

Other constraint equations are same as that of MGP model shown from Equations 6 to 11 above

Case Study

A Single lane bituminous highway project is considered for the case study. The project meets the traffic demands and is realigned at a number of locations to improve the geometric conditions. The activities are analyzed for its duration and cost. The project consists of 65 activities and 53 events.

Multiple objectives for model of the case study are formulated using and the respective constraints are developed. The individual problem for each objective has been solved using LINGO8 optimization solver. The minimum total project cost is identified as Rs.4,62,44,840 ($9,24,896.8) by converting the other objectives such as total completion time and crashing cost as constraints. The total completion time is obtained as 267 days and total crashing cost as 1,01,157.8 ($2,023.15). Further, solving the model for minimizing total completion time, the other two objectives i.e. total project cost and crashing cost are converted to constraints and the optimal solution values are obtained as 236 days, total project cost as Rs.5,78,03,830 ($11,56,076.6) and total crashing cost as Rs.1,29,78,630 ($2,59,572.6). Similarly solving for the next objective, i.e. the objective of minimizing total crashing cost the optimal solution are obtained for the total project cost as Rs.4,63,56,350 ($9,27,127) and total completion time as 272 days and total crashing cost is zero.

The problem has been reanalyzed considering the above results into fuzzy linear programming model. The model uses the definite intervals and the total project cost is obtained as Rs.4,83,73,670 ($ 9,67,473.4), total completion time as 244.80 days, total crashing cost as Rs.31,74,017 ($ 63,480.34) with a satisfaction value of 0.7554. This indicates that all the objectives of the model are satisfied with a minimum of 75.54%. The objectives of the above models are shown in Fig. 1 in graphical form.

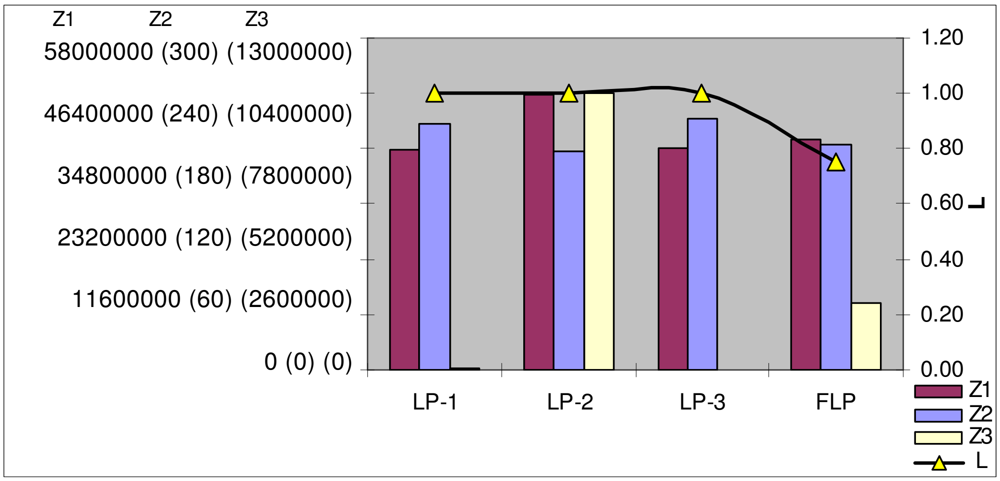

Figure 1 Comparison of models

The solution obtained using definite intervals determine the ability to incorporate the flexibility into the model using fuzzy linear programming. However, the decision maker can change the aspirations levels for obtaining project specific objectives. The model is validated and the result obtained does not change from one decision maker to another with defined objectives in the specified range. The sensitivity analysis of the obtained results is also performed.

Scenario 1

Removing total crashing costs (Z_3), and by considering only total project costs (Z_1) and total completion time(Z_2) the model is solved and the optimal solution values are obtained as Z_1 =Rs.4,93,49,620 ($9,86,992.4), Z_2 = 241 days and Z_3 = Rs.0.000 ($0.000). Removing Z_2 (total completion time), and by considering only Z_1 (total project costs) and Z_3 (total crashing costs), the model is solved and the optimal solution values are obtained as Z_1 = Rs.4,62,92,350 ($ 9,25,847) and Z_3 = Rs.33,198.81($663.98). Removing Z_1 (total project costs) and by considering only Z_2 (total completion time) and Z_3 (total crashing costs), the model is solved and the optimal solution values are obtained as Z_2 = 245.24 days and Z_3= Rs.30,79,341 ($61,586.82).

Scenario 2

Sensitivity analysis is performed on various decision parameters of the case study. For simplicity, a select activity in the project network is considered by allowing fuzziness of -20% to +20% on incremental crashing cost. The optimal solution values are obtained as follows: With a fuzziness of +10% the total project cost is Rs.51,794.99 ($1,035.89), completion time is 245.28 days, crashing cost is Rs.30,88,414 ($61,768,28) with satisfaction level of 0.8016. Similarly, with a fuzziness of +20% the total project cost is Rs.56,727.85($1,134.55), completion time is 245.36, crashing cost is Rs.31,06,091 ($ 62,121.82) with satisfaction level as 0.8005. The results show that the incremental crashing costs affect the objective values of the activity. This finding implies that the decision maker must consider the cost-time slope in practical construction optimization decision problems. The decision maker must also improve the efficiency of internal management, to reduce the cost of capital and thus reduce the incremental crashing costs associated with each activity. Figure 2 depicts the changes in the objective values of Scenario 2.

Conclusions

In decision-making problems, existence of fuzzy parameters and multi objectives make the description of problems difficult by using traditional mathematical programming. Especially in crashed project network problems, the conflicting objectives of minimum completion time, total project cost and total crashing costs are required to be optimized simultaneously by the decision maker in the frame work of fuzzy aspiration levels. The optimal solution values of MGP and the proposed fuzzy linear programming model is compared. It is observed that the % change in the total cost ranges from 11.20% to 15.00%. The change definitely has an impact in the duration of the project and crashing cost. Hence, the average change of the models is calculated. The average change ranges from 3.30% to 10.10%. Hence, it is evident that even though the % change in the total project cost of LP-1 is more than fuzzy linear programming model, because of the % change in completion time, and % change in crashing cost, the average change of the objectives is 3.30% which is less than 4.30% as for fuzzy linear programming. Hence, it can be clearly concluded that the fuzzy linear programming model is suitable for practical applications.

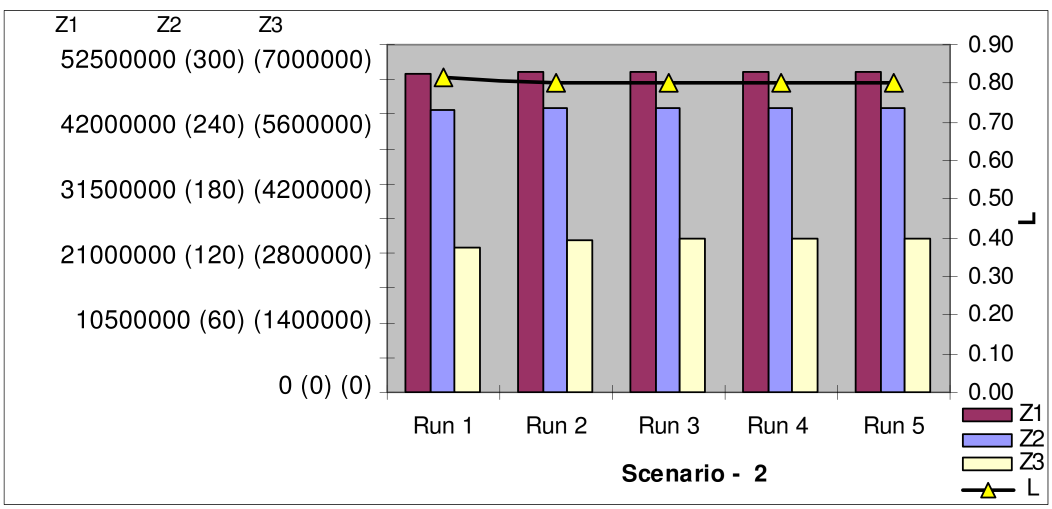

Figure 2 Results of Scenario -2

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