Fuzzy (r, s)-pre-semicontinuous mappings
퍼지 (r, s)-pre-semicontinuous 함수

이석종[1], 김진태[2], 엄연석[3]

[1] 충북대학교 자연과학대학 수학과
sjl@cbnu.ac.kr
[2] kjtmath@hanmail.net
[3] setojs@orgio.net

Abstract

In this paper, we introduce the concepts of fuzzy (r, s)-pre-semiopen sets and fuzzy (r, s)-pre-semicontinuous mappings on intuitionistic fuzzy topological spaces in Šostak's sense. The concepts of fuzzy (r, s)-pre-semiinterior, fuzzy (r, s)-pre-semiclosure, fuzzy (r, s)-pre-semineighborhood, and fuzzy (r, s)-quasi-pre-semineighborhood are given, and several properties of these concepts are discussed. Using these concepts, the characterizations for the fuzzy (r, s)-pre-semicontinuous mappings are obtained. Also, we introduce the notions of fuzzy (r, s)-pre-semiopen and fuzzy (r, s)-pre-semiclosed mappings on intuitionistic fuzzy topological spaces in Šostak's sense, and then we investigate some of their characteristic properties.

Keywords : fuzzy (r, s)-pre-semiopen set, fuzzy (r, s)-pre-semicontinuous mapping, fuzzy (r, s)-pre-semiopen mapping, fuzzy (r, s)-pre-semiclosed mapping

1. Introduction

Chang [4] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [16], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay and his colleagues [5], and by Ramadan [15].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Çoker [6] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [8] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth topological spaces and intuitionistic fuzzy topological spaces. S. Z. Bai [2] introduced the concepts of fuzzy pre-semiopen sets and fuzzy pre-semicontinuous mappings, and S. Z. Bai and W. L. Wang [3] established some other properties of fuzzy pre-semicontinuous mappings on Chang's fuzzy toplogical spaces. S. J. Lee and Y. S. Eoum [11] considered these concepts on smooth topological spaces.

In this paper, we introduce the concepts of fuzzy (r, s)-pre-semiopen sets and fuzzy (r, s)-pre-semicontinuous mappings on intuitionistic fuzzy topological spaces in Šostak's sense. The concepts of fuzzy (r, s)-pre-semiinterior, fuzzy (r, s)-pre-semiclosure, fuzzy (r, s)-pre-semineighborhood, and fuzzy (r, s)-quasi-pre-semineighborhood are given, and several properties of these concepts are discussed. Using these concepts, the characterizations for the fuzzy (r, s)-pre-semicontinuous mappings are obtained. Also, we introduce the notions of fuzzy (r, s)-pre-semiopen and fuzzy (r, s)-pre-semiclosed mappings on intuitionistic fuzzy topological spaces in Šostak's sense, and then we investigate some of their characteristic properties.

2. Preliminaries

Definition 2.1. ([8]) Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense*(SoIFT for short) $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2)$ on X is a mapping $\mathcal{T} : I(X) \to I \otimes I$ which satisfies the following properties :

(1) $\mathcal{T}_1(\underline{0}) = \mathcal{T}_1(\underline{1}) = 1$ and $\mathcal{T}_2(\underline{0}) = \mathcal{T}_2(\underline{1}) = 0$.

(2) $\mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$ and $\mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$.

(3) $\mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i)$ and $\mathcal{T}_2(\bigcup A_i) \leq \bigvee \mathcal{T}_2(A_i)$.

The $(X, \mathcal{T}) = (X, \mathcal{T}_1, \mathcal{T}_2)$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense*(SoIFTS for short). Also, we call $\mathcal{T}_1(A)$ a *gradation of openness* of A and $\mathcal{T}_2(A)$ a *gradation of nonopenness* of A.

Definition 2.2. ([10, 13, 14]) Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is called

(1) a *fuzzy (r, s)-continuous* mapping if $f^{-1}(B)$ is a fuzzy (r, s)-open set in X for each fuzzy (r, s)-open set B in Y,

(2) a *fuzzy (r, s)-open* mapping if $f(A)$ is a fuzzy (r, s)-open set in Y for each fuzzy (r, s)-open set A in X,

(3) a *fuzzy (r, s)-semicontinuous* mapping if $f^{-1}(B)$ is a fuzzy (r, s)-semiopen set in X for each fuzzy (r, s)-open set B in Y,

(4) a *fuzzy (r, s)-semiopen* mapping if $f(A)$ is a fuzzy (r, s)-semiopen set in Y for each fuzzy (r, s)-open set A in X,

(5) a *fuzzy (r, s)-semiclosed* mapping if $f(A)$ is a fuzzy (r, s)-semiclosed set in Y for each fuzzy (r, s)-closed set A in X,

(6) a *fuzzy (r, s)-precontinuous* mapping if $f^{-1}(B)$ is a fuzzy (r, s)-preopen set in X for each fuzzy (r, s)-open set B in Y,

(7) a *fuzzy (r, s)-preopen* mapping if $f(A)$ is a fuzzy (r, s)-preopen set in Y for each fuzzy (r, s)-open set A in X,

(8) a *fuzzy (r, s)-preclosed* mapping if $f(A)$ is a fuzzy (r, s)-preclosed set in Y for each fuzzy (r, s)-closed set A in X.

3. Fuzzy (r, s)-pre-semiopen sets

Now, we define the notion of fuzzy (r, s)-pre-semiopen sets on intuitionistic fuzzy topological spaces in Šostak's sense, and then we investigate some of their properties.

Definition 3.1. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be

(1) *fuzzy (r, s)-pre-semiopen* if
$$A \subseteq \mathrm{sint}(\mathrm{cl}(A, r, s), r, s),$$

(2) *fuzzy (r, s)-pre-semiclosed* if
$$\mathrm{scl}(\mathrm{int}(A, r, s), r, s) \subseteq A.$$

Theorem 3.2. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then the following statements are equivalent :

(1) A is a fuzzy (r, s)-pre-semiopen set.

(2) A^c is a fuzzy (r, s)-pre-semiclosed set.

Theorem 3.3. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS and $(r, s) \in I \otimes I$. Then the following statements are true.

(1) If $\{A_i\}$ is a family of fuzzy (r, s)-pre-semiopen sets in X, then $\bigcup A_i$ is fuzzy (r, s)-pre-semiopen.

(2) If $\{A_i\}$ is a family of fuzzy (r, s)-pre-semiclosed sets in X, then $\bigcap A_i$ is fuzzy (r, s)-pre-semiclosed.

Definition 3.4. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$,

(1) the *fuzzy (r, s)-pre-semiinterior* is defined by
$$\mathrm{psint}(A, r, s) = \bigcup\{B \in I(X) \mid B \subseteq A,\ B \text{ is fuzzy } (r, s)\text{-pre-semiopen}\},$$

(2) the *fuzzy (r, s)-pre-semiclosure* is defined by
$$\mathrm{pscl}(A, r, s) = \bigcup\{B \in I(X) \mid A \subseteq B,\ B \text{ is fuzzy } (r, s)\text{-pre-semiclosed}\}.$$

Theorem 3.5. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then we have

(1) $\mathrm{psint}(A, r, s)^c = \mathrm{pscl}(A^c, r, s)$,

(2) $\mathrm{pscl}(A, r, s)^c = \mathrm{psint}(A^c, r, s)$.

Theorem 3.6. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then we have

(1) psint(pscl(psint(pscl(A, r, s), r, s), r, s), r, s) =psint(pscl(A, r, s), r, s),

(2) pscl(psint(pscl(psint(A, r, s), r, s), r, s), r, s) =pscl(psint(A, r, s), r, s).

4. Fuzzy (r, s)-pre-semicontinuous mappings

We define the notions of fuzzy (r, s)-pre-semicontinuous, fuzzy (r, s)-pre-semiopen, and fuzzy (r, s)-pre-semiclosed mappings on intuitionistic fuzzy topological spaces in Šostak's sense, and then we investigate some of their characteristic properties.

Definition 4.1. Let A be an intuitionistic fuzzy set and $x_{(\alpha,\beta)}$ an intuitionistic fuzzy point in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is called

(1) a *fuzzy (r, s)-pre-semineighborhood* of $x_{(\alpha,\beta)}$ if there is a fuzzy (r, s)-pre-semiopen set B in X such that $x_{(\alpha,\beta)} \in B \subseteq A$,

(2) a *fuzzy (r, s)-quasi-pre-semineighborhood* of $x_{(\alpha,\beta)}$ if there is a fuzzy (r, s)-pre-semiopen set B in X such that $x_{(\alpha,\beta)} \mathrm{q} B \subseteq A$.

Theorem 4.2. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS and $(r, s) \in I \otimes I$. Then an intuitionistic fuzzy set A in X is fuzzy (r, s)-pre-semiopen if and only if A is a fuzzy (r, s)-pre-semineighborhood of $x_{(\alpha,\beta)}$ for each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in A.

Theorem 4.3. Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then $x_{(\alpha,\beta)} \in \mathrm{pscl}(A, r, s)$ if and only if $B \mathrm{q} A$ for any fuzzy (r, s)-quasi-pre-semineighborhood B of $x_{(\alpha,\beta)}$.

Definition 4.4. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is called

(1) a *fuzzy (r, s)-pre-semicontinuous* mapping if $f^{-1}(B)$ is a fuzzy (r, s)-pre-semiopen set in X for each fuzzy (r, s)-open set B in Y,

(2) a *fuzzy (r, s)-pre-semiopen* mapping if $f(A)$ is a fuzzy (r, s)-pre-semiopen set in Y for each fuzzy (r, s)-open set A in X,

(3) a *fuzzy (r, s)-pre-semiclosed* mapping if $f(A)$ is a fuzzy (r, s)-pre-semiclosed set in Y for each fuzzy (r, s)-closed set A in X.

The definition of fuzzy (r, s)-pre-semicontinuity can be restated in terms of fuzzy (r, s)-pre-semiclosure and fuzzy (r, s)-pre-semiinterior.

Theorem 4.5. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then the following statements are equivalent :

(1) f is fuzzy (r, s)-pre-semicontinuous.

(2) For each fuzzy (r, s)-closed set B in Y, $f^{-1}(B)$ is a fuzzy (r, s)-pre-semiclosed set in X.

(3) For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r, s)-open set B in Y such that $f(x_{(\alpha,\beta)}) \in B$, there is a fuzzy (r, s)-pre-semiopen set A in X such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$.

(4) For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r, s)-neighborhood B of $f(x_{(\alpha,\beta)})$, $f^{-1}(B)$ is a fuzzy (r, s)-pre-semineighborhood of $x_{(\alpha,\beta)}$.

(5) For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r, s)-neighborhood B of $f(x_{(\alpha,\beta)})$, there is a fuzzy (r, s)-pre-semineighborhood A of $x_{(\alpha,\beta)}$ such that $f(A) \subseteq B$.

(6) $f(\mathrm{pscl}(A, r, s)) \subseteq \mathrm{cl}(f(A), r, s)$ for each intuitionistic fuzzy set A in X.

(7) $\mathrm{pscl}(f^{-1}(B), r, s) \subseteq f^{-1}(\mathrm{cl}(B, r, s))$ for each intuitionistic fuzzy set B in Y.

(8) $f^{-1}(\mathrm{int}(B, r, s)) \subseteq \mathrm{psint}(f^{-1}(B), r, s)$ for each intuitionistic fuzzy set B in Y.

Theorem 4.6. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a bijective mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then the following statements are equivalent :

(1) f is fuzzy (r, s)-pre-semicontinuous.

(2) For each fuzzy (r, s)-closed set B in Y, $f^{-1}(B)$ is a fuzzy (r, s)-pre-semiclosed set in X.

(3) For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r, s)-open set B in Y such that $f(x_{(\alpha,\beta)}) \in B$, there is a fuzzy (r, s)-pre-semiopen set A in X such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$.

(4) For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r, s)-neighborhood B of $f(x_{(\alpha,\beta)})$, $f^{-1}(B)$ is a fuzzy (r, s)-pre-semineighborhood of $x_{(\alpha,\beta)}$.

(5) For each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r, s)-neighborhood B of $f(x_{(\alpha,\beta)})$, there is a fuzzy (r, s)-pre-semineighborhood A of $x_{(\alpha,\beta)}$ such that $f(A) \subseteq B$.

(6) $f(\mathrm{pscl}(A, r, s)) \subseteq \mathrm{cl}(f(A), r, s)$ for each intuitionistic fuzzy set A in X.

(7) $\text{pscl}(f^{-1}(B), r, s) \subseteq f^{-1}(\text{cl}(B, r, s))$ for each intuitionistic fuzzy set B in Y.

(8) $f^{-1}(\text{int}(B, r, s)) \subseteq \text{psint}(f^{-1}(B), r, s)$ for each intuitionistic fuzzy set B in Y.

(9) $\text{int}(f(A), r, s) \subseteq f(\text{psint}(A, r, s))$ for each intuitionistic fuzzy set A in X.

Theorem 4.7. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then the following statements are equivalent :

(1) f is a fuzzy (r, s)-pre-semiopen mapping.

(2) $f(\text{int}(A, r, s)) \subseteq \text{psint}(f(A), r, s)$ for each intuitionistic fuzzy set A in X.

(3) $\text{int}(f^{-1}(B), r, s) \subseteq f^{-1}(\text{psint}(B, r, s))$ for each intuitionistic fuzzy set B in Y.

Theorem 4.8. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then the following statements are equivalent :

(1) f is fuzzy (r, s)-pre-semiclosed.

(2) $\text{pscl}(f(A), r, s) \subseteq f(\text{cl}(A, r, s))$ for each intuitionistic fuzzy set A in X.

Theorem 4.9. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a bijective mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then the following statements are equivalent :

(1) f is fuzzy (r, s)-pre-semiclosed.

(2) $\text{pscl}(f(A), r, s) \subseteq f(\text{cl}(A, r, s))$ for each intuitionistic fuzzy set A in X.

(3) $f^{-1}(\text{pscl}(B, r, s)) \subseteq \text{cl}(f^{-1}(B), r, s)$ for each intuitionistic fuzzy set B in Y.

References

[1] K. T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems **20** (1986), 87–96.

[2] Shi-Zhong Bai, *Fuzzy pre-semiopen sets and fuzzy pre-semicontinuity*, Proc. ICIS'92 (1992), 918–920.

[3] Shi-Zhong Bai and Wan-Liang Wang, *Fuzzy non-continuous mappings and fuzzy pre-semi-separation axioms*, Fuzzy Sets and Systems **94** (1998), 261–268.

[4] C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl. **24** (1968), 182–190.

[5] K. C. Chattopadhyay, R. N. Hazra, and S. K. Samanta, *Gradation of openness : Fuzzy topology*, Fuzzy Sets and Systems **49** (1992), 237–242.

[6] D. Çoker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy Sets and Systems **88** (1997), 81–89.

[7] D. Çoker and M. Demirci, *On intuitionistic fuzzy points*, NIFS. **1** (1995), 79–84.

[8] ———, *An introduction to intuitionistic fuzzy topological spaces in Šostak's sense*, BUSEFAL **67** (1996), 67–76.

[9] I. M. Hanafy, *Intuitionistic fuzzy functions*, International Journal of Fuzzy Logic and Intelligent Systems **3** (2003), 200–205.

[10] Eun Pyo Lee, *Semiopen sets on intuitionistic fuzzy topological spaces in Šostak's sense*, J. Fuzzy Logic and Intelligent Systems **14** (2004), 234–238.

[11] Seok Jong Lee and Youn Suk Eoum, *Fuzzy r-pre-semineighborhoods and r-pre-semicontinuous maps*, International Journal of Fuzzy Logic and Intelligent Systems **1** (2001), no. 1, 62–68.

[12] Seok Jong Lee and Jin Tae Kim, *Fuzzy (r, s)-irresolute maps*, International Journal of Fuzzy Logic and Intelligent Systems **7** (2007), no. 1, 49–57.

[13] Seok Jong Lee and Eun Pyo Lee, *Fuzzy (r, s)-semicontinuous mappings on intuitionistic fuzzy topological spaces in Šostak's sense*, J. Fuzzy Logic and Intelligent Systems **16** (2006), 108–112.

[14] Seung On Lee and Eun Pyo Lee, *Fuzzy (r, s)-preopen sets*, International Journal of Fuzzy Logic and Intelligent Systems **5** (2005), 136–139.

[15] A. A. Ramadan, *Smooth topological spaces*, Fuzzy Sets and Systems **48** (1992), 371–375.

[16] A. P. Šostak, *On a fuzzy topological structure*, Suppl. Rend. Circ. Matem. Janos Palermo, Sr. II **11** (1985), 89–103.