Multi-Robot Localization based on Bayesian Multidimensional Scaling

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Abstract

This paper presents a multi-robot localization based on Bayesian Multidimensional Scaling (BMDS). We propose a robust MDS to handle both the incomplete and noisy data, which is applied to solve the multi-robot localization problem. To deal with the incomplete data, we use the Nyström approximation which approximates the full distance matrix. To deal with the uncertainty, we formulate a Bayesian framework for MDS which finds the posterior of coordinates of objects by means of statistical inference. We not only verify the performance of MDS-based multi-robot localization by computer simulations, but also implement a real world localization of multi-robot team. Using extensive empirical results, we show that the accuracy of the proposed method is almost similar to that of Monte Carlo Localization(MCL).

Keywords:

Multi-Robot Localization, Bayesian MDS

Introduction

In recent years, there has been increased activity in the area of collaborative approaches to multi-robot localization. A simple approach for cooperative localization system with no infrastructure was first proposed in [1]. A dominant modern approach is the probabilistic Monte Carlo Localization (MCL) [2] utilizing an independence property to estimate the position of the individual robots. A study [3] on the influence of different group trajectories on the accuracy of MCL showed that through appropriate cooperation, localization error decreases while the number of robots increases. Other approaches taking advantage of relative inter-robot range and bearing observations also have been proposed [4, 5, 6]. A maximum likelihood estimation-based approach is given in [7], and an Extended Kalman Filter [8] using relative observations of range and bearing is described.

In general, most of robot localization methods, which have been proposed in recent, have concentrated on a question that is ``Where am I?" in an environment with known/unknown map, however, we regarded multi-robot localization problem as a task of finding relative positions of each robot in multi-robot team. So we consider neither global localization nor map building in this paper.

We performed extensive simulations to verify which choice is good. Our experiments show that the prediction strategy is the best choice for initialization. The rest of this paper is organized as follows. Next section describes a MDS framework which is the key of our proposed method. Then, a multi-robot localization based on the proposed MDS framework will be described. The results of experiments also must be drawing. Finally, we make the conclusion and the future works of our contribution.

Multidimensional Scaling

Although MDS has its origins in psychometrics and was originally proposed to help understand people's judgments of the similarity of members of a set of objects, it has found several applications in diverse fields as marketing, sociology, physics, political science, biology, and engineering. MDS is a generic term that includes many different specific types. These can be classified according to whether the data are qualitative or quantitative, the number of similarity matrices, the nature of the MDS model, and the implementation of the algorithm to solve the MDS problem.

Classical metric MDS

The task of classical MDS is to find the coordinates of p dimensional points $\mathbf{x}_r (r = 1, \ldots, n)$ from a Euclidean distance matrix. The Euclidean distance between $\mathbf{x}_r$ and $\mathbf{x}_s$ is defined as

$$d_{ij}^2 = (\mathbf{x}_i - \mathbf{x}_j)^T (\mathbf{x}_i - \mathbf{x}_j). \quad (1)$$

Let the inner product of $\mathbf{x}$ is $\mathbf{B}$, where $[\mathbf{B}]_{ij} = b_{ij} = \mathbf{x}_i^T \mathbf{x}_j$. Then the Euclidean distance d_{ij}^2 can be represented by $\mathbf{B}$ as

$$\begin{aligned} d_{ij}^2 &= \mathbf{x}_i^T\mathbf{x}_i + \mathbf{x}_j^T\mathbf{x}_j - 2\mathbf{x}_i^T\mathbf{x}_j \\ &= b_{ii} + bjj - 2b_{ij}. \end{aligned} \tag{2}$$

By centering the coordinator matrix $\mathbf{X}$ to origin that($\sum_{i=1}^{b} b_{ij} = 0$) and summing Eq. (2) over i, over j, and over i and j, we find that

$$\begin{aligned} &\frac{1}{N}\sum_{i=1}^{N} d_{ij}^2 = \frac{1}{N}\sum_{i=1}^{N} b_{ii} + b_{jj}, \\ &\frac{1}{N}\sum_{j=1}^{N} d_{ij}^2 = b_{ii} + \frac{1}{N}\sum_{j=1}^{N} b_{jj}, \\ &\frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N} d_{ij}^2 = \frac{2}{N}\sum_{i=1}^{N}\sum_{j=1}^{N} b_{ii}. \end{aligned} \tag{3}$$

By combining Eq. (2) and (3), we find that

$$\begin{aligned} b_{ij} &= -\frac{1}{2}(d_{ij}^2 - d_{i\cdot}^2 - d_{\cdot j}^2 + d_{\cdot\cdot}^2) \\ &= (a_{ij} - a_{i\cdot} - a_{\cdot j} + a_{\cdot\cdot}), \end{aligned} \tag{4}$$

where $a_{ij} = -\frac{1}{2} d_{ij}^2$.

We can rewrite $\mathbf{B} = \mathbf{HAH}$ by double centering for the matrix of $[a_{ij}] = \mathbf{A}$, where the centering matrix $\mathbf{H} = \mathbf{I} - N^{-1}\mathbf{1}\mathbf{1}^T$. Since the matrix $\mathbf{B}$ is symmetric, positive semi definete, and $rank(\mathbf{B}) = rank(\mathbf{X}^T\mathbf{X}) = rank(\mathbf{X}) = p$, it can be decomposed as

$$\mathbf{B} = \Gamma\Lambda\Gamma^T, \tag{5}$$

where $\Lambda = diag(\lambda_1, \cdots, \lambda_p)$ is the diagonal matrix of $\mathbf{B}$, which corresponds to the eigen vector $\Gamma = (\gamma_1, \cdots, \gamma_p)$. Hence, it can be represent the coordinators of $\mathbf{X} \in \mathbf{R}^p$ space as

$$\mathbf{X} = \Gamma\Lambda^{1/2}. \tag{6}$$

Dealing with incomplete data

Classical MDS finds the exact coordinates from the distance matrix when its rank is full, but it is difficult to obtain all distance information among the points. We propose an efficient MDS using an approximation method (called Nyström approximation), which has been applied to Gram kernel matrix or Euclidean distance matrix completion problem. The Nystrom approximation can build a kernel matrix $\mathbf{K}$ by using the sub matrix from the distance matrix $\mathbf{D}$. Here, we have two matrices $\mathbf{K}$ and $\mathbf{D}$ that can be partitioned into four sub matrices as

$$\mathbf{D} = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix} \text{and } \mathbf{K} = \begin{bmatrix} E & F \\ F^T & G \end{bmatrix}, \tag{7}$$

where A and E have the size of $m \times m$, B and F have the size of $m \times (N-m)$, and C and G have the size of $(N-m) \times (N-m)$, respectively. The Nyström approximation permits the computation of the coordinates $\mathbf{x}_i$ using only the information in matrices E and F. Assuming $\mathbf{K}$ is positive semi-definite, it should be represented by the dot products of columns of the matrices $\mathbf{X}$ and Y as

$$\mathbf{K} = \begin{bmatrix} \mathbf{X}^T\mathbf{X} & \mathbf{X}^T\mathbf{Y} \\ \mathbf{Y}^T\mathbf{X} & \mathbf{Y}^T\mathbf{Y} \end{bmatrix}. \tag{8}$$

Identifying the sub matrices in Eq. (8) with those in thoes in Eq. (7) yields

$$\begin{aligned} E &= \mathbf{X}^T\mathbf{X}, \\ F &= \mathbf{X}^T\mathbf{Y}, \end{aligned} \tag{9}$$

where E is identical to $\mathbf{B}$ of the classical MDS . Thus, the eigen decomposition of E is $E = \Gamma\Lambda\Gamma^T$, and then obtain the coordinators by Eq.(6). The coordinates corresponding to F can be derived by solving the linear system as

$$\mathbf{Y} = \mathbf{X}^{-T}F = \Gamma^T\Lambda^{-1/2}F. \tag{10}$$

The equality of the approximation is proportional to $\| G - F^T E^{-1} F \|$. Matrices E and F must be derived only from the sub-matrices A and B (in the distance matrix $\mathbf{D}$). The centering formulas provide E and F as

$$E_{ij} = -\frac{1}{2}\left(A_{ij}^2 - e_i \frac{1}{m}\sum_r A_{rj}^2 - e_j \frac{1}{m}\sum_s A_{is}^2 + \frac{1}{m^2}\sum_{r,s} A_{rs}^2 \right), \tag{11}$$

$$F_{ij} = -\frac{1}{2}\left(B_{ij}^2 - e_i \frac{1}{m}\sum_r B_{rj}^2 - e_j \frac{1}{m}\sum_r A_{ir}^2 \right), \tag{12}$$

where e_i is the vector of all ones.

Dealing with uncertainty

We must consider the uncertainty of observations, which is inevitable from the observed distance obtaind from the noisy sensors. To over come this problem, we utilize the idea of the statistical inference for MDS. We assume the possible distributions for the noisy observation δ are normal, $\delta_{ij} \sim N(d_{ij}, \sigma^2)$, where d_{ij} is the true distance, and σ^2 is the variance of the distribution. Usually, the true distance is unknown, but the variance of the distribution can be identified by empirical or by referring to the specification of sensors. We can denote the matrix form of the problem as

$$D = \begin{bmatrix} d_{11}^2 & \cdots & d_{1N}^2 \\ \vdots & d_{ii} & \vdots \\ d_{N1}^2 & \cdots & d_{NN}^2 \end{bmatrix}, \Delta = \begin{bmatrix} \delta_{11} & \cdots & \delta_{1N} \\ \vdots & \delta_{ii} & \vdots \\ \delta_{N1} & \cdots & \delta_{NN} \end{bmatrix},$$

$$\Sigma = \begin{bmatrix} \sigma_{11}^2 & \cdots & 0 \\ \vdots & \sigma_{ii}^2 & \vdots \\ 0 & \cdots & \sigma_{NN}^2 \end{bmatrix}. \quad (13)$$

By using the Bayesian formulation of the relations between Δ and D, we know that

$$P(\Delta \mid D) = \frac{P(D \mid \Delta)P(\Delta)}{\int P(D \mid \Delta)}. \quad (14)$$

We compute the maximum a posterior(MAP) of $P(D \mid \Delta)$ as

$$lnP(D \mid \Delta) = -\frac{1}{2} ln\left[(2\pi)^2 \mid \Sigma \mid\right] - \frac{1}{2}(D-\Delta)^T \Sigma^{-1}(D-\Delta). \quad (15)$$

In this work, we use a simple *'Metropolis Hasting'* algorithm in [14] to obtain the solution of the Eq. (15). Then, the estimated distance matrix can be obtained by

$$\arg\max_{\mathbf{D}} lnP(D \mid \Delta). \quad (16)$$

MDS-Based Localization

This section illustrates a detailed explanation of the proposed localization technique. Table I summarizes MDS-based localization algorithm. We assumed each robot is equipped with a range sensor such as a laser finder, sonar, or camera to identify distance to other robots. We also assume that each robot is equipped with a set of inertial sensors (e.g. a compass and odometer) to compute its own motion.

Table I. MDS-based localization algorithm.

Initialization :
Given a noisy distance matrix D for N points.   Generate the distance matrix D by observations using Eq. ((16)).   Permutate the m items $(m < N)$ to be the first rows and columns of these matrices.
Nyström approximation:
Compute E and F from $\mathbf{D}$ by Eq. ((11)) and Eq. ((12)). Approximate the full matrix $\mathbf{K}$ with $\overline{\mathbf{K}}$.
Get the coordinates :
Eigen decomposition : $E = \Gamma \Lambda \Gamma^T$.   Solve Eq. (6) with Γ and Λ. $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}2, ..., \mathbf{x}_N]$: the current coordinates.

At each optimization step, the algorithm can be initialized. We implement three strategies for this:

- Random -- If we have no prior about the environment and the dynamics of each robot, coordinates obtained at random are set as initial point.

$$X^{(0)}(t) \sim N(\mu, \sigma^2)$$

- Previous -- When the movement of each robot is comparably small or the iteration time step is short, it may be desirable to set initial locations to the previous coordinates.

$$X^{(0)}(t) = X(t-1)$$

- Prediction -- The system is initialized with the predicted value of the current pose by applying a motion model to the previous pose.

$$X(t) = X(t-1) + \rho X(t)$$

Experiments and Results

We performed several experiments to validate the proposed multi-robot localization. Starting in a basic environment, there are 6 robots marked $R0$ to $R5$. We assumed that all inter-robot distances are available and motion dynamics can be obtained via odometry on each robot which has Gaussian noise($\sigma = 0.2$) as the measurement error. Robots are made to move randomly for 1200 time steps. As noted in Section , we applied three strategies for initialization at each optimization step. Fig. 1 shows the plot of convergence for each policy. The figure tell us that initializing with the prediction procedure, which is generated by adding the previous coordinates to the motion information, is more than 3 times faster than random initialization. The previous procedure was more than 2 times faster than the random case.

Table II. Relative location error and convergence speed

	Random	Previous	Prediction
Number of iterations	40.92	18.20	12.46
σ	5.15	3.75	2.87
Relative errors	0.0156	0.0143	0.0134
σ	0.0023	0.0026	0.0029

To check the accuracy of the proposed distance mapping with the three initialization strategies, we evaluated the mean of the relative errors between the real position and the computed position in 2D coordinate space. The error is

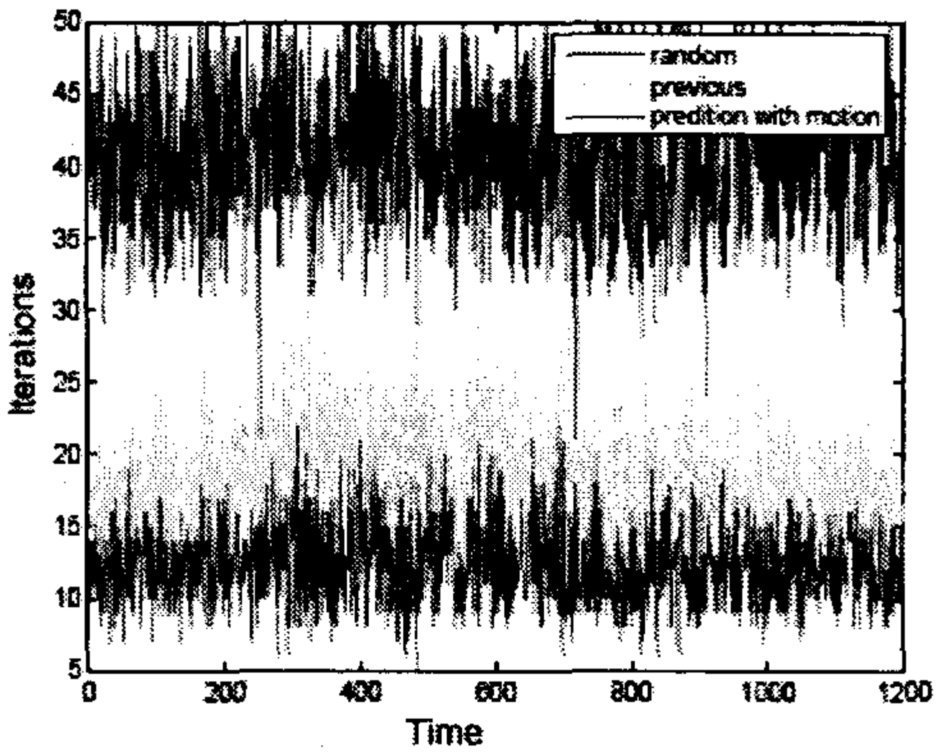

Fig. 1 Comparison of the convergence.

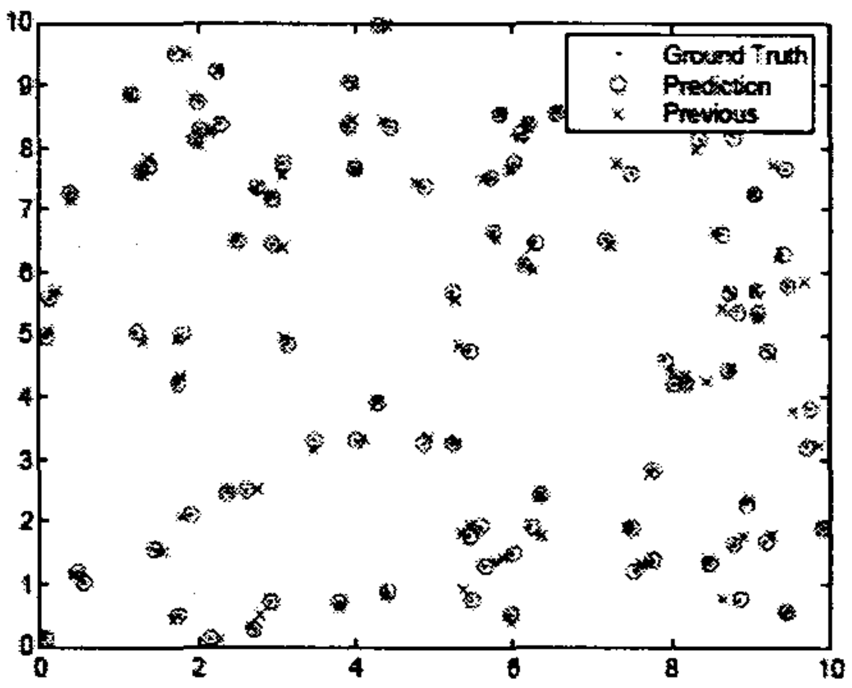

Fig. 2 The embedding results for 100 robots; initialized by prediction and previous.

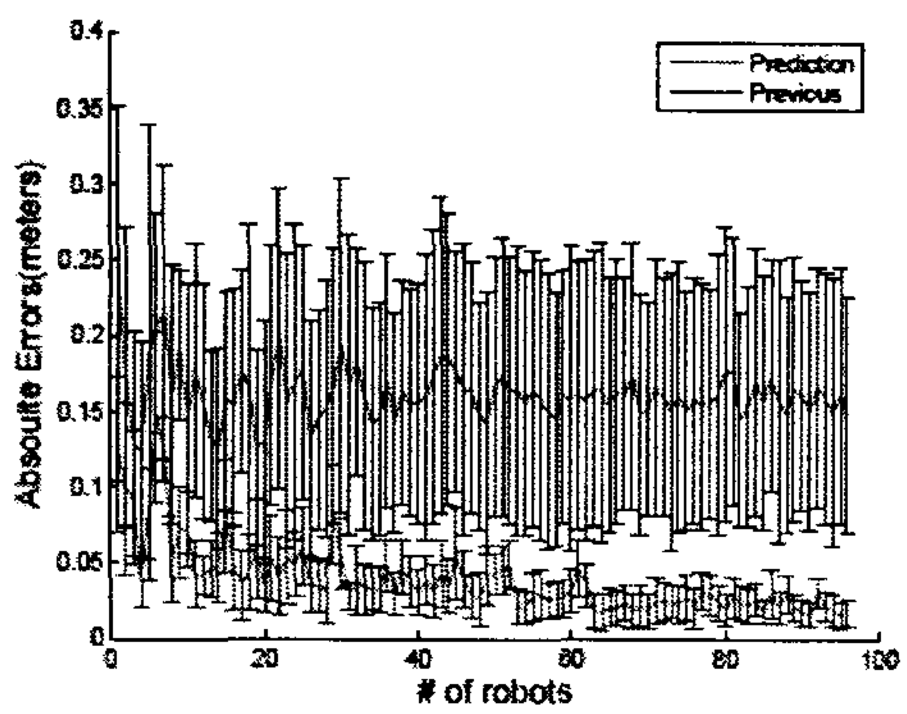

Fig. 3 Absolute errors.

defined as $RelErr = \frac{\| \delta(P) - \delta(\hat{X}) \|}{(N-1)^2}$, where $\delta(P)$, $\delta(\hat{(X)})$ are distance matrices for the real position.

The relative errors are compared in Table II. Even though it seems like their accuracy have little difference in the terms of the relative error, the random initialization has some problems. When it is required to reconstruct the absolute position, the random initialization needs more than two or three robots to have known positions. The number of robots might not affect the performance of absolute localization. The plot of the average error to compare MCL-based with MDS-based multi-robot localization is shown in Fig. 4 as well. As a result of the comparison, the average error of MCL-based localization is 0.41 and that of MDS-based localization is 0.39. Thus, we can assure that the accuracy of both localization methods are almost same.

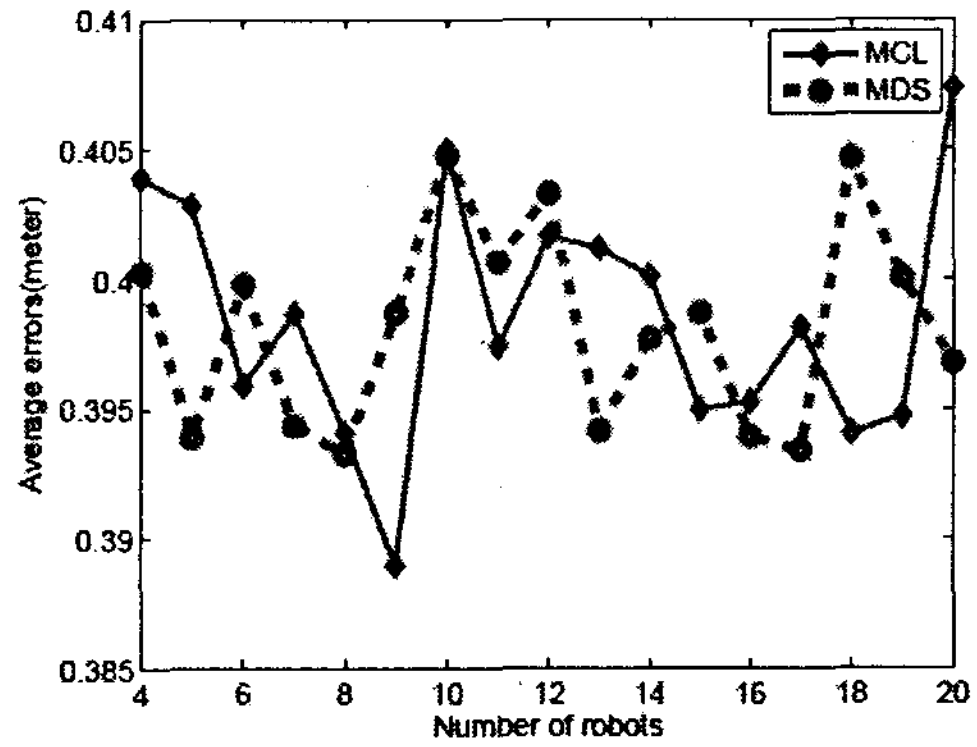

Fig. 4 Comparison of the absolute errors; between MCL and proposed method for multi-robot localization.

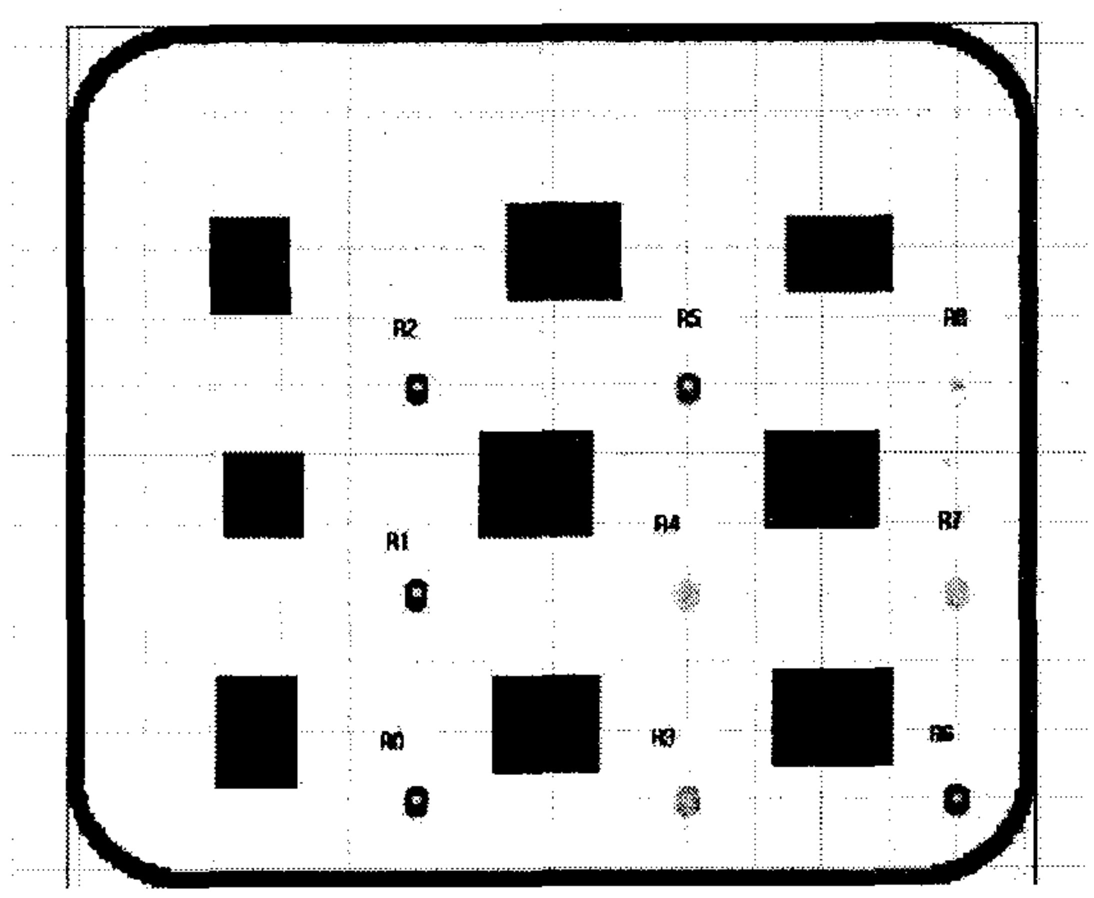

Fig. 5 The player/stage simulation environment with 9 robots.

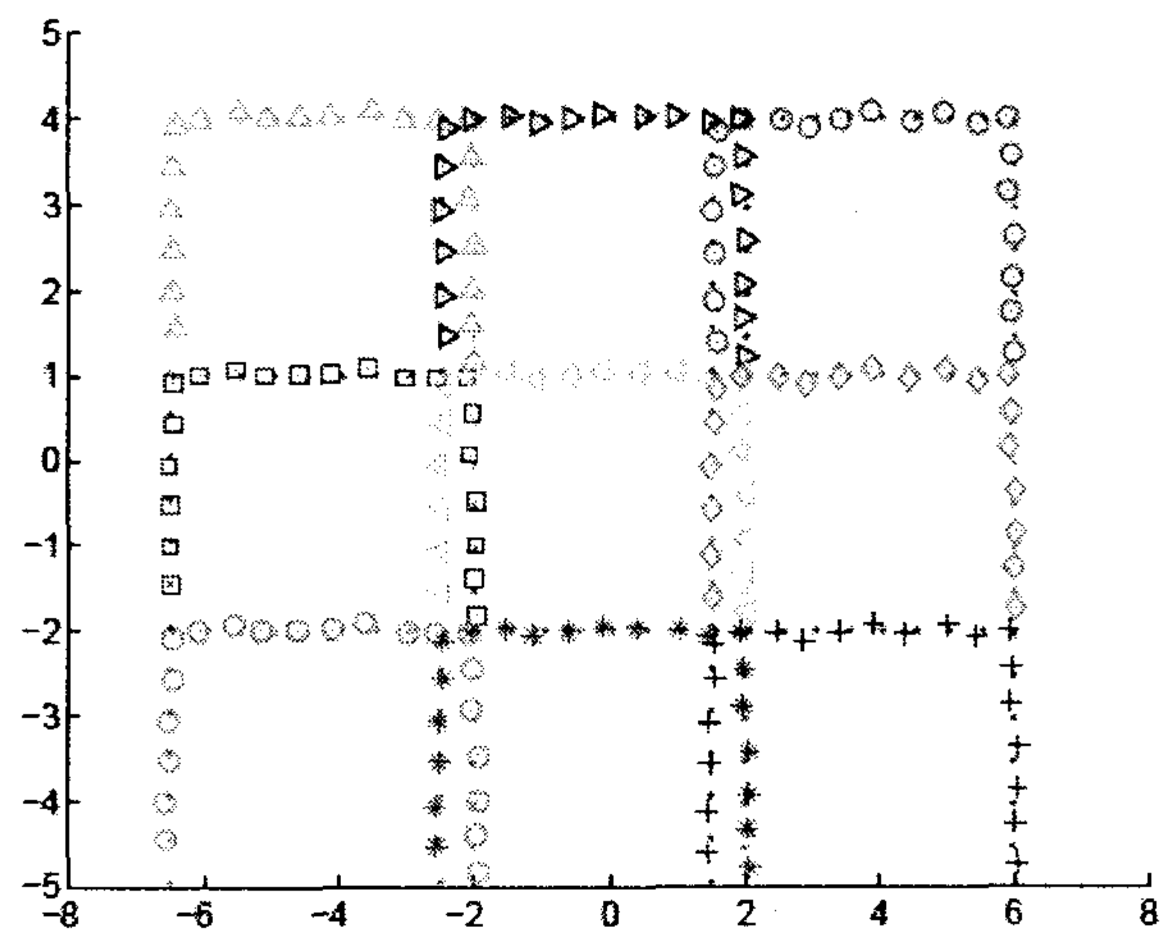

Fig. 6 The trajectory of the ground truth (represented as dots '.') and the embedding by the proposed method (represented as symbols).

To better reflect a real situation, we performed experiments on the Player/Stage simulator[15] with 9 robots. We configured a world with complex corridors causing occlusions at most time step. Fig. 5 shows the simulated environment. Each robot has a laser finder which can observe other objects within 5 meters range in an 180 degree angle, a odometry device to calculate its motion, and a fiducial bar-code to be identified by other robots.

Discussion and Conclusion

We do not focus on the global localization and map building, but on the relative multi-robot localization. We make two contributions. First, we propose the use of Multi Dimensional Scaling (MDS) for multi-robot localization. Second, by using the Nyström approximation and the Bayesian formulation, we propose the robust MDS on incomplete and noisy data. We take advantage of the motion information of robots to help the optimization procedure. In addition we verify the performance of both MCL-based and MDS-based multi-robot localization are almost same. We should extend the proposed MDS to solve simultaneous localization and mapping (SLAM) problem. Now, our current work does not worry about the orientation of robots and motion model. For solving the data association procedure in the SLAM problem, we need to include the motion model into the proposed MDS framework and devise an estimation method of the current robot's pose.

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