Electron Field Emission for a Cylindrical Emitter of Single Carbon Nanotube

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Abstract

We investigated the field emission of single carbon nanotube including the anode effect by calculating the tunneling probability of an electron. The experimental results from this study were in agreement with our theoretical calculations. The constant enhancement factor was calculated using an approximation of the potential barrier.

1. Introduction

Carbon nanotubes are excellent field emitters due to their high aspect ratios, high emission currents and chemical stabilities[1-3]. The field emission properties of various carbon nanotube structures, which are good candidates for a high-quality electron source, have been well-studied[4]. Field-emission cathodes have been used in a variety of vacuum electron devices[5,6]: field emission displays, electron guns in microscopy, and cold-cathode x-ray tubes.

Field emission current-voltage characteristics have been interpreted based on the theory of electron tunneling from a planar surface, developed by Fowler and Nordheim[7], which uses a planar model of the tip with a classic image correction. In reality, carbon nanotubes have cylindrical shapes with a radius at the curvature of the tip that is smaller than 10nm. Many investigators have commented that standard F-N theory applied to planar electrodes does not provide accurate results for emitters with a nano-scale radii[8-10]. The F-N plot, however, deviates from a linear fit, with a slope that increases as the extraction voltage is lowered.

It is neccessary to investigate the field emission properties for single carbon nanotubes in order to gain an understanding of their intrinsic characteristics and how these could be applied to vacuum electron devices.

Forbes[11] proposed a model, which he described as the "hemisphere on a post." However, this model is applicable only when the distance from the cathode to the anode, d, is much greater than the protrusion length, L. Bonard's[9] model did not require d to be much greater than L, but his model was obtained by fitting.

The calculation made by our group was without restrictions on either the gap distance or the length of the carbon nano tube. And then our theoretical calculation was compared with both our empirical results and with Bonard's model.

2. Field Emission

To calculate current density, the simple model for a single carbon nanotube with height h and diameter $2R$ is considered (Fig. 1). The paths of many electrons emitted from a single nanotube may lie on the x-axis.

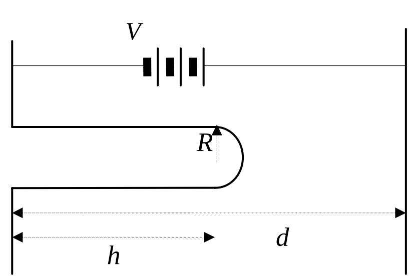


Fig.1. single carbon nanotube model

The tunneling rate of an electron with a given potential barrier $U(x)$ is given by[12]

$$\ln D = \sum \ln D_{\text{part}} \approx -\frac{2}{\hbar}\int dx \sqrt{2m|U(x)-E|} \quad (1)$$

where, E is the kinetic energy of the electron. The total potential can now be described by the equation:

$$U(x) = E_F + \phi + \frac{R\,eV\,(h+R)(d-h)}{d(d-h-R)(x+R)} - \frac{eV}{d}x - \frac{eV\,(h+R)(d-h)}{d(d-h-R)} - \frac{R\,e^2}{8\pi\varepsilon_0 x(r+2R)} \quad (2)$$

where R and h are the radius and the height of the single carbon nanotube, respectively; d is the distance between the substrate and the cathode and represents the position on the axis from the end of the tip. And V is the electric potential difference between the cathode and the anode. E_F is the Fermi energy, ϕ is the work function of the emitter material.

The potentials of the anode and the cathode are satisfied by the following boundary conditions:

$$U_{\text{app}}(x=0)=0,\;\; U_{\text{app}}(x=d-h-R)=-eV \quad . (3)$$

Then,

$$D(E) = \exp\left\{-\frac{2}{\hbar}\int_{x_1}^{x_2} dx \times\left[2m\left(E_F + \Phi + \frac{R\,eV\,(h+R)(d-h)}{d(d-h-R)(x+R)} - \frac{eV}{d}x - \frac{eV\,(h+R)(d-h)}{d(d-h-R)} - \frac{R\,e^2}{8\pi\varepsilon_0 x(r+2R)} - E\right)\right]^{1/2}\right\} . \quad (4)$$

The kinetic energy of an electron is

$$E = \frac{m{v_x}^2}{2} , \quad (5)$$

where the velocity of the electron, v_x, is the normal component of the velocity to the cathode.

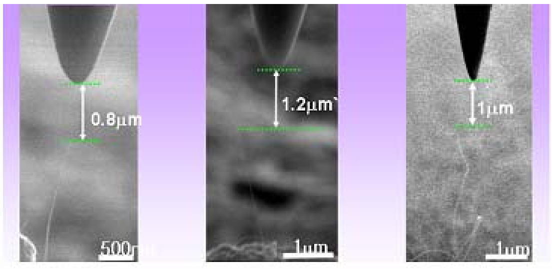
It is not easy to estimate the tunneling probability for this potential. Even if it is done, the $\ln I/V^2$ vs $1/V$ plot is not linear. To calculate the tunneling probability, the Simpson integral formula in C programming language was used.

The tunneling current density, then, may be given by:

$$J(V,T) = e\int_0^\infty v_x\, N(v_x)\, D(E)\, dv_x = \frac{em^2k_BT}{2\hbar^3\pi^2}\int_0^\infty v_x \ln\left\{1+\exp\left[-\left(\frac{m{v_x}^2}{2}-E_F\right)/k_BT\right]\right\} D(E)\, dv_x = \frac{emk_BT}{2\hbar^3\pi^2}\int_0^\infty \ln\{1+\exp[-(E-E_F)/k_BT]\} D(E)\, dE \quad . \quad (6)$$

3. Results and discussion

In order to get the low-density vertical carbon nanotube, screen-printed carbon nanotube, fabricated by mixing carbon nanotube with organic binder, were used. The mixed carbon nanotubes were fixed on ITO substrate and heated to 350℃. A single nanotube was created using a piezoelectric nanomanipulator (MM3A) with a sharp tungsten tip. A sharp, chemically etched tungsten tip with a tip radius of curvature of approximately 100nm was mounted to the nano-motor to serve as an anode. The nano-motor permitted precise movement on the order of 1nm along all three axes. The tungsten tip approached the top of a single carbon nanotube and the distance between the tungsten tip and the carbon nanotube was controlled within a few micrometers using the nano-motor, as shown in Fig.2. By gradually varying the voltage, the current-voltage characteristic curves were obtained (Fig. 3).



(a) (b) (c)

Fig.2. The SEM images of carbon nanotubes.

(a) $R = 3\text{nm}$, $d = 2.37\mu\text{m}$, and $h = 1.57\mu\text{m}$
(b) $R = 11\text{nm}$, $d = 3.98\mu\text{m}$, and $h = 2.78\mu\text{m}$
(c) $R = 17\text{nm}$, $d = 4.32\mu\text{m}$, and $h = 3.32\mu\text{m}$

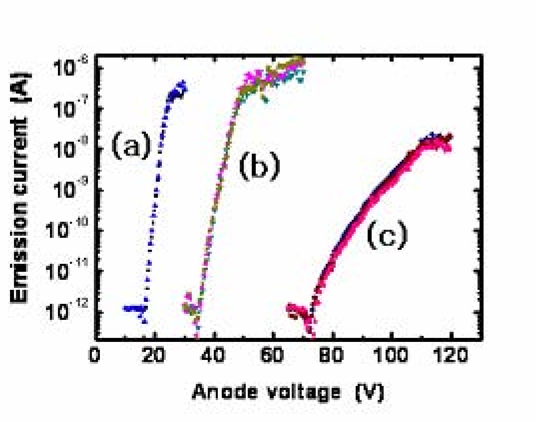


Fig.3. Emission current-voltage characteristics of the carbon nanotubes

4. Results and discussion

From the equation of tunneling current density, The graph of $\ln I/V^2$ vs $1/V$ is represented in Fig. 4.

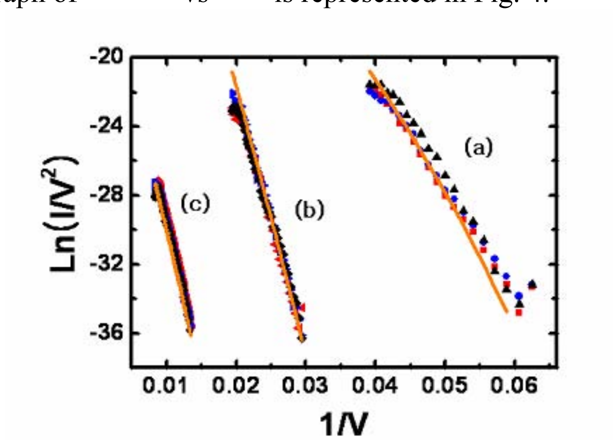


Fig. 4. $\ln I/V^2$ vs $1/V$ for three carbon nanotubes

According to Fig. 4. the curvature of the F-N plot decreases an increase in the radius of the carbon nanotube. As the radius of carbon nanotube increases, the slope of the F-N plot becomes steeper.

In this study, the constant enhancement factor was described, although the actual slope of the F-N plot was not constant. When fitting empirical data, it is convenient to use the value of gamma, γ. To calculate γ, the potential can be approximated as,

$$U_{\text{app}}(x) = \frac{ReV(h+R)(d-h)}{d(d-h-R)(x+R)} - \frac{eV}{d}x - \frac{eV(h+R)(d-h)}{d(d-h-R)}$$

$$\approx \frac{eV(h+R)(d-h)}{d(d-h-R)}\left(1 - \frac{x}{R} + \frac{x^2}{R^2}\right) - \frac{eV}{d}x - \frac{eV(h+R)(d-h)}{d(d-h-R)}$$

$$\approx \frac{eV}{d}x\left[\frac{(h+R)(d-h)}{(d-h-R)} + 1\right] \quad (7)$$

In the case of field emission between two planar electrodes, the potential is

$$U_{\text{app}} = -\gamma\frac{eV}{d}x \quad (8)$$

By comparing the two potentials given in equations (7) and (8), the constant enhancement factor γ can be approximated by:

$$\gamma \approx \frac{(h+R)(d-h)}{(d-h-R)R} + 1 \quad (9)$$

Using the F-N equation[7]

$$I = A\frac{1.5\times10^{-6}}{\phi}\left(\frac{V}{d}\right)^2 \gamma^2 \exp\left[\frac{10.4}{\sqrt{\phi}}\right]\exp\left[-\frac{6.44\times10^9\phi^{1.5}d}{\gamma V}\right]$$
(10)

Bonard[9] obtained γ by fitting

$$\gamma = 1.2\left(2.5 + \frac{h}{r}\right)^{0.9}\left[1 + 0.013\left(\frac{d-h}{d}\right)^{-1} - 0.033\left(\frac{d-h}{d}\right)\right].$$
(11)

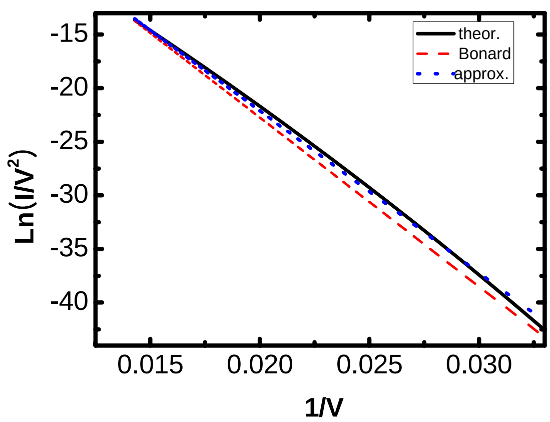


Fig. 5. Theoretical (solid line), approximation of our model(dotted line), and Bonard model(dashed line). In all cases, the dimensions of the tip are $R = 11\text{nm}$, $d = 3.98\mu\text{m}$, and $h = 2.78\mu\text{m}$.

In the case of $d - h >> R$, γ can be approximated by:

$$\gamma \approx \frac{h}{R} + 2 \qquad (12)$$

and

$$\gamma \approx \frac{d}{d-h-R} + 1, \qquad (13)$$

when $d - h \approx R$.

5. Summary

A simple model was constructed for the electron field emission of carbon nanotubes. The current density was calculated from this model using an empirical analysis. The F-N plot was slightly curved in this study. In addition, the slope was dependent on the radius of the carbon nanotube.

Using this simple model to describe the potential barrier at the hemisphere tip, the current density for an individual carbon nanotube was obtained. By using empirical methods, this model can be used in practical applications of carbon nanotube emitters.

6. References

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