

3-D information of Object by Modified Goldstein Algorithm at Digital holography

Seon Kyu Yoon¹, Sung-Kyu Kim²

¹Imaging Media Research Center, KIST, Seongbuk-gu, Seoul 136-719, Korea

TEL:82-2-958-5794, e-mail: rhmonkey@wow.tech.ac.kr.

²Dept. of Electric Engineering, Your College, Town/City, MA 02139, U.S.A.

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Abstract

Generally many kind of phase unwrapping method are used to obtain three-dimensional feature in digital holography. Goldstein algorithm is representative method. But Goldstein algorithm has some problems. We developed a modified Goldstein algorithm that could solve the problem of Goldstein algorithm using the boundary information. Obtained three-dimensional information can be applied to 3-D contents of stereoscopic, multi-view, SMV, or holographic display.

1. Introduction

The object phase information that has been obtained from the digital holography is used to analyses the 3rd dimension information of the object. The phase information that has been obtained from the digital holography shows the phase difference(ϕ) between the reference wave and the object wave. The object phase value is always larger than 2π , however the phase difference value is between $-\pi \leq \phi < \pi$. This phase difference value is called wrapped phase. Phase unwrapping is a technique that used to reconstructing the wrapped phase data using a numerical method. Itoh's theory is mostly used method in 1st dimensioned phase unwrapping.[1] This method uses the fact that when the wrapped phase value can be differential, differentiated value of the similar interval between the wrapped phase and the object phase is same. However when some values are impossible to differentiate, residue, phase unwrapping is impossible. If residue does not exist in the 2nd dimension array, all the differentiated values are same with the object information. However, if residue does exist, then the integrated value reveals different result. To overcome this problem many different algorithms are being used. [2-6]. Out of all the algorithms, Goldstein algorithm is mostly used due to its short execution time and small

usage of the memory.[7] This algorithm connects the residues that are close to each other. This is called branch-cut, and other than branch-cut, wrapped phase can be integrated in any direction.

However, Goldstein algorithm returns incorrect result when some residue from the other object that is not connected but is located nearer than the pair of original residue.

To overcome this problem, we propose to use the boundary information that has been obtained from the wrapped phase, and use this information to identify the residue that is need to branch-cut.

By using the modified Goldstein algorithm, more accurate information of the 3rd dimension, and using the information, obtaining the 3rd dimension image through the numerical information.

2. Principle of phase unwrapping

The relationship between the wrapped phase and phase unwrapping that fulfill the minimum sampling data ratio is as follows:

$$W\{\phi(n)\} = \phi(n) = \varphi(n) - 2nk \quad (1)$$

From the formula, W is a wrapping operator, $\phi(n)$ is wrapped phase, $\varphi(n)$ is actual phase, k is a integer and n is a constant from the samples. Wrapping operator should always be between $-\pi \leq \phi < \pi$. The difference between the wrapped phase value from the Itoh's calculation and the actual phase value can be defined as formula (2).

$$\Delta\{\phi(n)\} = W\{\Delta\{W\{\phi(n)\}\}\} = W\{\Delta\{\phi(n)\}\} \quad (2)$$

$\Delta\{\phi(n)\}$ is a differentiated value of actual phase

data, $W\{\Delta\{\phi(n)\}\}$ is a differentiated value of the wrapped data and the value are same. Therefore, unless differentiation is impossible, the restored phase can be seen from the formula (3).

$$\varphi(n) = \varphi(0) + \sum_{n=0}^{m-1} W\{\Delta\{\phi(n)\}\} \quad (3)$$

Formula (3) shows that the real phase can be restored from the wrapped phase difference and the integration. This formula can be used in the 2nd dimensional data. However if there is a discontinued points, the outcome of the formula is differ, this is due to the fact that the result depends on the integrated path. The method to obtain the result that does not depend on the path has been developed. The fastest method, Goldstein algorithm, is mostly being used. The table 1 shows the number of phase unwrapping algorithm and their execution time.

Table 1. Phase unwrapping algorithm

Algorithm	Memory requirement	Executing time
Goldstein	3	30sec
Mask Cut	4	4min
Flynn	5	10min
PCG	5	4min
Multi-Grid	7	20min

3. Experimental

To show the problem of the Goldstein algorithm and the result of the modified Goldstein algorithm using the computer simulation, following 3rd dimensional object has been produced.

The object that will be used in the computer simulation and the wrapped phase data are shown as figure 1 and 2.

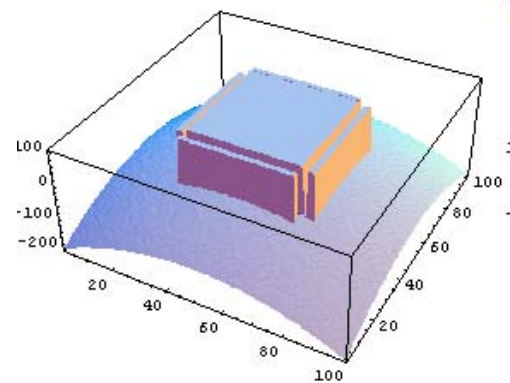


Fig. 1. Object.

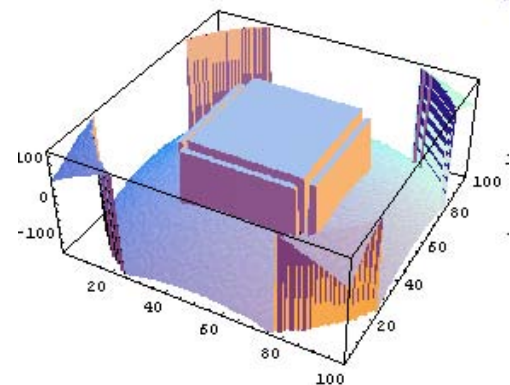


Fig. 2. Phase wrapped data.

The produced object has half sphere as the background. At the center of the half sphere, there is a large cube. Around the side of the cube, there is thin cube with a gap between them; the gap size is 1 pixel.

4. Results and discussion

The result of the Goldstein algorithm using the computer simulation can be seen in figure 3. As it can be seen from the figure, the corner of the resulted object shows different outcome with the original object.

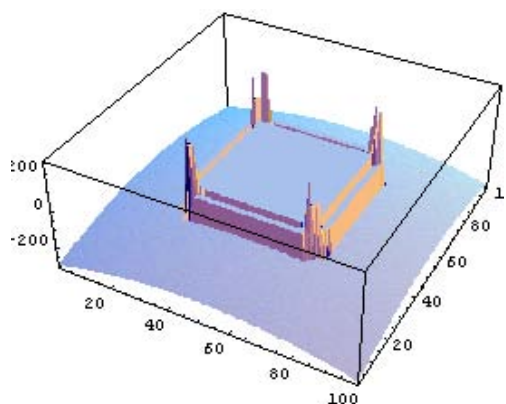


Fig. 3. Phase unwrapping data of Goldstein algorithm.

The calculated boundary information that will be used in the modified Goldstein algorithm is shown in figure 4.

As it can be seen from the figure 4, the boundary information include the residue, the residue from the boundary information is caused by the difference in the identity object.

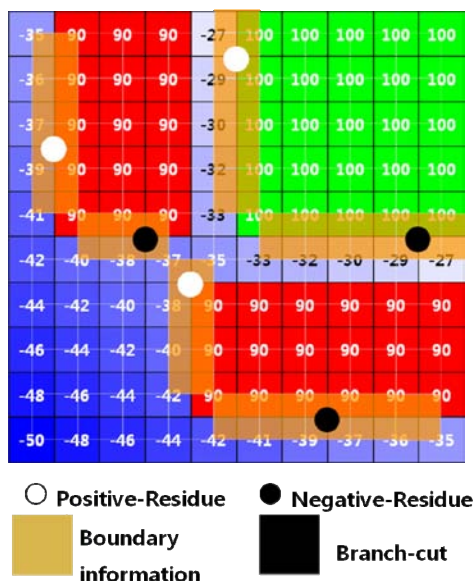


Fig. 4. Data and domain of Wrapped phase, residue, and boundary information.

The coordinate system of the boundary information and the residue must be identical. Figure 5 shows the result of the modified algorithm. The result of the modified Goldstein algorithm shows that the unwrapped objects are separated from each other unlike the Goldstein algorithm. So the modified

Goldstein algorithm returns more accurate result. The reason why two algorithm returns different result is shown on the figure 6, the direction of the branch-cut is different. This means that the coupled residue should be connected. Goldstein algorithm uses the distance between the residues, so some time the residue from the same object cannot be connected with each other.

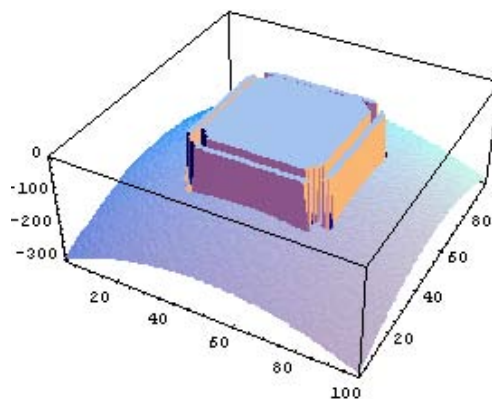


Fig. 5. Phase unwrapping data of Goldstein algorithm.

Using the phase unwrapping result, stereoscopic display data has been produced. The coordinate system to display the phase unwrapped result has been defined depending on the amount of the fusion at the binocular image. Using the coordinate system the binocular image has been created. The figure 7 shows what the each eye sees and the outcome of the two images. Because the phase unwrapping result is to show the 3rd dimension information, so depending on the display method, different format can be made.

Modified Goldstein algorithm shows that the total execution has been increased by 10% compare to Goldstein algorithm.

This result shows that some of the problems from the Goldstein algorithm can be solved using the boundary information that has been obtained from the wrapped phase. The total execution time has not been increased dynamically so the main strength of the Goldstein algorithm has been partly maintained. Compare to Goldstein algorithm, hardware space will be required to save the boundary information, but compare with other algorithms, this will not have large influence.

5. Conclusion

By using the boundary information, partly keeping the main strength of the Goldstein algorithm, short execution time and small memory usage, the problem of the Goldstein algorithm can be overcome. By using the phase information, the 3-D content has been produced so the unknown sample can visualize more effectively.

6. References

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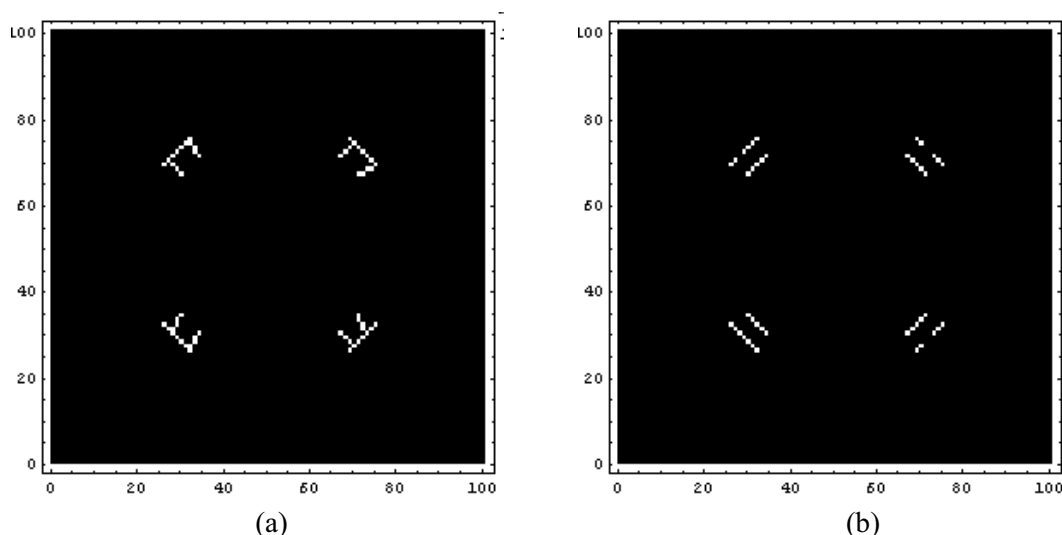


Fig. 6. Branch-cut data : (a) Goldstein algorithm, (b) modified Goldstein algorithm.

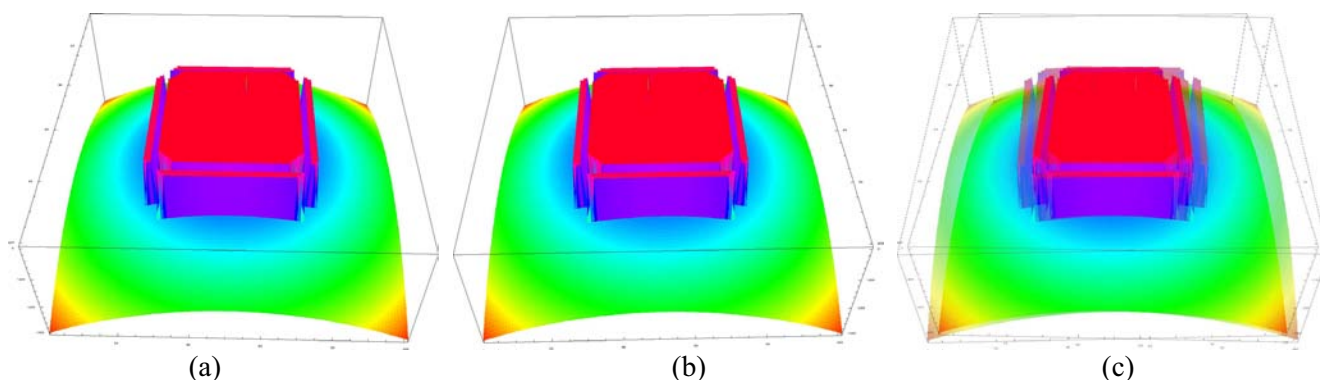


Fig. 7. Binocular image, Parallax image (a) Light eye image, (b) Left eye image, (c) Parallax image.