Mathematics across the Curriculum

- Educational Reform as a Problem Solving Activity -

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Overview

During the past 20 years, a small but potentially powerful initiative has established itself in the mathematics education landscape: Mathematics Across the Curriculum (MAC). This curricular reform movement was designed to address a serious problem: Not only are students unable to demonstrate understanding of mathematical ideas and their applications, but also they harbor misconceptions about the meaning and purpose of mathematics.

This paper chronicles the brief history of the MAC movement. The sections of the paper correspond loosely to the typical steps one might take to solve a mathematics problem. The Problem Takes Shape presents a discussion of the social and economic forces that led to the need for increased articulation between mathematics and other fields in the American educational system. Understanding the Problem presents the potential value of exploiting these connections throughout the curriculum, and the obstacles such action might encounter. Devising a Plan provides an overview of the support systems provided to early MAC initiatives by government and professional organizations. Implementing the Plan contains a brief description of early collegiate programs, their approaches and their differences. Extending the Solution details the adoption of MAC principles to the K-12 sector and throughout the world. The paper concludes with Retrospective, a brief discussion of lessons learned and possible next steps.

The Problem Takes Shape

By the late 1980s, there was general consensus that American mathematics education was in serious trouble, both at the pre-college and college levels. The mathematical demands of society were increasing dramatically. "More than ever before, Americans need to think for a living; more than ever before, they need to think mathematically" (NRC, 1989, p. 1). How well positioned was the educational system to meet this challenge of preparing students for further academic study, work, and citizenship?

According to data released by the International Association for the Evaluation of Educational Achievement (IEA), American students were graduating high school unprepared in mathematics, in comparison with comparable students in other countries (Robitaille & Garden, 1989). In a report released in 1990 by National Center for Education Statistics (Snyder & Hoffman, 1990), although 96.0 percent of American 17-year old students demonstrated proficiency in basic operations and beginning problem solving, the proficiency levels were only 51.1 percent in moderately complex procedures and reasoning and an alarming 6.4 percent in multiple step problems and algebra. In fall 1989, approximately 68 percent of the four-year colleges and universities in the U.S. had to offer remedial courses in mathematics (Mansfield et. al., 1991). These findings cast doubt on the mathematical preparedness of American high school graduates to achieve academically.

In order to compete effectively in an increasingly global economy, America needed a well educated workforce, especially in science, mathematics, engineering and technology. In particular, during this period, mathematics-based careers were growing at twice the rate of other occupations (NRC, 1989), but more than half of the students enrolled in undergraduate math courses were studying at levels below Calculus (Albers et. al., 1992).

In order to function as competent consumers and citizens, students need to learn how to apply the mathematics that they are learning to other contexts. Math thinking is important for access to a variety of academic disciplines (Steen, 1990), but during this period students were seeing little connection between math thinking and the kind of thinking needed for other disciplines (NAEP, 1983). Furthermore, traditional college entry-level math courses were not designed to prepare students for study in other disciplines (Sons, 1995). American students were not learning mathematics or how to apply it. These facts call into question their ability to perform academically, in the workplace, or as informed citizens.

Understanding the Problem

There is a significant disparity in beliefs about the nature of mathematics between mathematicians and students of mathematics. On the one hand, to mathematicians, mathematics is the ability to view situations quantitatively, logically, and spatially (Paulos, 1991); mathematics is the art of sense-making and the search for patterns (Schoenfeld, 1992). On the other hand, to American students in the 1980s, mathematics is the application of formulas, equations, and algorithms that need to be memorized (NAEP, 1983); even those who considered mathematics to

be useful still believed that it is about memorizing and following rules (Dossey et. al., 1988; McKnight et. al., 1987). It is clear that students not only were having difficulty doing mathematics, but also harbored distorted views of the subject.

As noted earlier, many American students were not learning the mathematics they were being taught in school. Worse, even those who were proficient in mathematics often were unable toapply it to other situations. By the end of the 1980s, the difficulties individuals experienced connecting school mathematics with real-world problem solving had been well documented (Carraher et. al, 1985 Lave et. al., 1984; Lave et. al., 1989). More recently, it has been shown that university students, when asked to solve mathematical problems and analogous problems in physics, microbiology and computer science, perform considerably better on the mathematics problems than those in other contexts (Britton, 2002). Why this disconnect between the ability to do school mathematics and to apply mathematics to other areas?

It is generally accepted that knowledge is bound inextricable to context and intention. In their seminal paper, Brown, Collins, and Duguid (1989) argued, "Different activities produce different indexicalized representations, not equivalent, universal ones" (p. 36). The situated character of knowledge explains the difficulties individuals experience when they attempt to transfer knowledge from one situation to another.

A useful metaphor is the switch. Students behave as if they have a math switch. An individual student enters a math class and her math switch turns on. She is now ready to think about mathematical problems, learn mathematics concepts and procedures, and communicate mathematically; in short, she has a mathematical perspective or frame of mind. As she leaves the classroom, the math switch turns off. She enters another class, and although the topic being discussed may have a connection to what was being discussed in her math class, she is unable to make the connection because her math switch is off. Rather than Boolean, the switch may be rheostatic, with many gradations of activation possible in different situations. The question then is how to keep this switch as activated as possible?

Heibert and Carpenter (1992) argued that transfer cannot be assumed but must be supported with appropriate practice. Moreover, experience with different problem situations can facilitate the abstraction of mathematical concepts and promote transfer (Druckman & Bjork, 1994). There appears to be a mutually reinforcing system at work, involving mathematical ideas and applied problem solving. When students study mathematical applications to other disciplines in their mathematics classes, they "learn to construct powerful representation of mathematical principles and to appreciate their applicability to a variety of disciplines (Cerreto et al., 1997, p. 387)."

Studying mathematical ideas while in other courses "enables students to develop more robust understandings of important concepts in these disciplines and provides additional experience with mathematical concepts. (p. 387)." The resulting intellectual synergy has a powerful potential.

This then is the raison d'etre of MAC: to provide students with a wealth of experiences in a variety of contexts in order to promote transfer of mathematical knowledgeand help students realign their definitions of mathematics.

Devising a Plan

Despite the fact that mathematics across the curriculum provided a tremendously powerful potential solution to the problems outlined above, there were many obstacles to overcome. In addition to the challenges encountered by any attempt at interdisciplinary reform (compartmentalization of knowledge, lack of reward structures, etc.), integrating mathematics into other areas presented special barriers.

For example, it would appear that MAC programs might emanate naturally from the numerous successful WAC implementations (e.g., Walvoort et. al, 1997). By the end of the 1980s, the writing-across-the-curriculum (WAC) movement was firmed established in the American higher and secondary education systems. A survey conducted in 1987 showed that approximately 38 percent of the over 100 institutions of higher education that replied had some sort of a WAC program in place, and at over half of them, the program had been in place for at least three years (Russell, 1991). Certainly both writing and mathematics are broadly based academic perspectives; they both use powerful languages to describe phenomena. What differences between writing and mathematics, especially in higher education cultures, inhibited the development of MAC programs then?

First, writing is much more prevalent than "mathing." Everyday life is filled with opportunities to write (e-mail, kitchen notes, etc.). Contrast this with the amount of time people do math around the house, and the case for writing as a more natural act is easily made.

Second, academics write often and well, and they value good writing. From the composition of course syllabi, to the preparation of manuscripts for publication, to the completion of committee reports, college professors and K-12 teachers spend a large chunk of their time engaged in writing. The effectiveness of this writing is highly correlated with their career success. In contrast, except for those in mathematics, science, and engineering, many instructors do not engage in significant mathematical behavior, nor do they depend on their mathematical

achievement for professional advancement.

Third, academics know good writing when they see it. Besides reading and reviewing professional works, most instructors read (and react to) their students' written work regularly. Again, except in a small number of courses, the same cannot be said about mathematics.

Finally, academics require their students to write well. Most instructors expect their students to be able to communicate effectively in written form often final course grades are based, in large part on students' written work. Rarely are students expected to demonstrate mathematical proficiency in non-mathematical courses.

Thus, although the parallels between WAC and MAC, on the surface, seem to be compelling, there are notable differences that render the development of MAC programs considerably more problematic. In fact, integrating mathematics effectively into closely related fields, science and engineering, has proven to be challenging (Winkel, 1999). Imagine the difficulties that would be encountered by those who attempted to infuse mathematics into less related fields. With this powerful inertia in play, external forces were required.

In order to overcome the many obstacles faced by those who would advocate MAC, professional and governmental action was absolutely necessary. A few "true believers" spread out on college campuses across the country could not provide sufficient energy to support the development of a MAC movement. The first half of the next decade saw the convergence of precisely the required elements.

From 1989 to 1995, mathematics professional organizations, national and local, K-12 and college, along with governmental groups, laid the groundwork for the MAC movement. Their work in developing standards, advocating positions, and publishing important works calling for dramatic educational reform provided the necessary justification for individuals desiring to establish MAC programs.

In 1989, three seminal publications called for mathematics education reform. The National Council of Teacher of Mathematics (NCTM, 1989) published the *Curriculum and Evaluation Standards for School Mathematics*, which provided a set of standards that would guide efforts to revise and improve school mathematics curricula and to evaluate the success of mathematics reform. In the same year, the Mathematical Association of America (MAA) released *Reshaping College Mathematics* (Steen, 1989), which described a new undergraduate mathematics curriculum. Finally, through the release of *Everybody Counts* (NRC, 1989), the National Research Council issued a call for changes in mathematics instruction from kindergarten through graduate school.

In 1994, the MAA released *Quantitative Reasoning for College Graduates: A Complement to the Standards* (Sons, 1995), which outlined the mathematics expectations for all college students pursuing bachelor's degrees, and, in the following year, the American Mathematical Association of Two-Year Colleges published *Crossroads in Mathematics* (AMATYC, 1995), which articulated standards for introductory college mathematics, based on the NCTM model.

Several mathematics reform initiatives were implemented during the late 1980s and early1990s, powered by these engines. At the undergraduate level, for example, calculus reform played a vitally important role during this period (Tucker & Leitzel, 1994). While supporting mathematics education reform in general, these professional and governmental actions also provided a basis for the establishment of a MAC movement.

One of the four major conclusions contained in the MAA report noted above is relevant to the present discussion. The report concludes that "Colleges and universities should expectevery college graduate to be able to apply simple mathematical methods to the solution of real world problems" (Sons, 1995, p. 1). The report also criticized the current introductory-level, required mathematics offerings as inadequate in preparing students to carry out basic estimates of costs and consequences, understand and deconstruct the statistical basis of popular works, and utilize related skills that are important in other disciplines. Although this recommendation was made in the context of revising mathematics course curriculum, the notion that all students must be able to solve real problems recognizes the importance of applying mathematical knowledge to other areas. This fact, combined with the mathematical knowledge transfer problem described earlier, helped those who were interested in establishing MAC programs to make their case:

Of course, the impetus to promote quantitative literacy, the leadership to define its elements effectively, and the energy to sustain its objectives will have to reside in the mathematical community. But mathematics must permeate the undergraduate experience the same way it permeates modern society: MATHEMATICS ACROSS THE CURRICULUM! (p. 17.)

The same cry could be heard from the American Mathematical Association of Two-year Colleges: "Just as the 'writing across the curriculum movement'addresses the need for students to write frequently in order to improve as verbal thinkers, a 'mathematics across the curriculum movement' is needed so that students develop as mathematical thinkers." (AMATYC, 1995, p. 43)

By the mid-1990s, most of the pieces required to set in motion a MAC movement were in place. Educators, professional organizations, and governmental agencies were expressing profound dissatisfaction with current state of affairs in mathematics education, and many were calling for significant reform. However, one essential ingredient was missing: monetary support.

In 1994, the Division of Undergraduate Education of the National Science Foundation (NSF, 1995), through itsCourse and Curriculum Development Program, awarded a series of seven grants under the new category, Mathematical Sciences and Their Applications Throughout the Curriculum. This initiative, totaling approximately \$19 million over then next nine years, provided fuel for the engines of MAC projects throughout the country. During the same time, the NSF and other sources provided smaller grants to support fledgling MAC programs, through their existing funding programs.

Implementing the Plan

The second half of the 1990s was one of experimentation with various MAC models. A cursory look at some of the winners of the NSF's Mathematical Sciences and Their Applications Throughout the Curriculum program and other early programs demonstrates the breadth of approaches institutions have taken. However, most of the projects share certain goals:

- · Improving instructional practices in mathematics
- · Fostering faculty development, both in mathematics and in other disciplines
- Promoting interdisciplinary teaching
- · Improving students' performance in mathematics and its applications
- Positively changing student attitudes about mathematics
- · Changing the infrastructure and culture of institutions regarding MAC
- Promoting the development of consortia representing colleges and universities with shared interests regarding MAC

The nature and focus of these early MAC programs, not surprisingly, was in large part influenced by the institutions' missions. At Dartmouth, a well known private liberal arts college, a primary focus was bringing together interdisciplinary teams, mathematicians and those in other disciplines, to develop curricular materials (Dartmouth, 2005). The project resulted in 16 new courses, some in mathematics and other in humanistic disciplines, as well as new modules for an additional 13 courses. Many of the interdisciplinary activities developed under this initiative are, as of this writing, still available on Dartmouth's website.

In contrast, MAC took on a very different form at Rensselaer Polytechnic Institute, given their dissimilar educational mission. At Rensselaer, faculty members developed an extensive library of hypertext tools in engineering and science applications of mathematics and made them available

on the Web (Rensselaer, 2005). In addition, the Website includes resources for instructors wishing to use the materials. MAC at Rensselaer, a technically oriented institution, emphasized mathematical connections to closely allied fields, whereas the program at Dartmouth, a liberal arts school, reached a wider set of disciplines.

Particular implementations of MAC were influenced also by their existing curriculum. For example, the program at the University of Nebraska (NSF, 1995) involved the development of a core curriculum in mathematics, science, and engineering. In contrast, the Richard Stockton College of New Jersey, a 1996 awardee of an NSF Institution-wide Reform grant for its Quantitative-Reasoning-Across-the-Disciplines (QUAD) program had a curricular structure that emphasized student choice. A core curriculum would not be feasible, given the school's culture. Instead, its program focused on the development or revision of a very large number of courses (over 70, at the program's inception) that emphasized quantitative reasoning, in mathematics and in other disciplines, from which students could choose (Cerreto et. al. 1997).

Extending the Solution

By the end of the 1990s, the MAC movement had been established firmly in the American higher education community. Dozens of colleges and universities — two-year, four-year, public private — had initiated programs designed to infuse mathematics across the curriculum. By the mid-1990s, presentations on MAC began to appear at the annual joint meetings of the MAA/AMS (Cerreto, 1997, for ex.). Several other regional and national conferences focused on MAC or included MAC presentations. Many of those involved in MAC initiatives produced relevant educational materials, and publishers began to make them available for broad dissemination (For ex., Key, 2005).

Although this paper focuses on MAC at the college level, the movements were carried out in parallel in the K-12 educational sector. The 1995 National Council of Teachers of Mathematics yearbook was dedicated to MAC issues (NCTM, 1995). National as well as state standards included an emphasis on MAC (NCTM, 2000, for ex.). A variety of MAC materials were developed and distributed (Thorson, 2002).

MAC has transcended the borders of the United States. The learning area statement in mathematics in the curriculum framework released in 1998 by the Curriculum Council of Western Australia (CCWA, 1998) contains a section devoted to MAC principles, titled *Links Across the Curriculum*. The first page of the mathematics section includes a statement that is indicative of

the council's commitment to MAC. "Mathematics plays a key role in the development of students' numeracy and assists learning across the curriculum" (p. 177). In England in 2001, after initiating a national numeracy strategy, The Department for Education and Skills published a curriculum framework for students in grades 7 through 9 (DES, 2001). Its guide contains a statement on the significance of quantitative reasoning.

Retrospective

The foregoing brief history of the MAC movement included both itspotential to result in improved mathematical preparation for all students and the difficulties in carrying out thismathematics education reform. What lessons might we learn from the experiences? Besides continued systemic support, what other characteristics appear to be associated with effective programs? Following is a list of certain identifiable elements that, while not all essential for success, appear to be shared by many successful implementations.

- True believers. At most sites, there were individuals or small groups of "movers and shakers" who were passionate about instituting mathematics education reform. These agents of change were willing to spend long hours garnering internal and external resources, meeting with other instructors, students, and administrators, building consensus, and carrying out other tasks necessary to foster the development of a program through its fledgling period (typically three to five years).
- Ongoing administrative structure and support, especially for personnel development.
 Without strong, ongoing, multi-level institutional support, MAC initiatives will likely achieve
 only limited success. Instructors in mathematics and in other disciplines need opportunities
 to learn from one another. Funds must be earmarked for appropriate training, material
 development, and administrative expenses. Rewards to those involved must be made clear.
- Broad-based involvement throughout the institution. Most successful MAC programs
 have involved many of the educational stakeholders in their development and
 implementation phases. Mathematics instructors must work with experts in other fields to
 effectively communicate the connections between quantitative reasoning and other
 disciplines. Administrative and parental involvement is also a necessity.
- A program that is consistent with the institution's history, mission and culture. As stated earlier, successful programs reflect the values of their institutions. Decisions regarding program policy and procedures, size, etc. must be appropriate to the institution.

- A MAC program at a research oriented university might look very different from one at a small middle school, for example.
- Clearly labeled courses and articulated requirements. Labeling courses as MAC courses
 provides instructors with "permission" to cover mathematical ideas in their courses and helps
 students approach the courses within a frame of mind that is ready to accept mathematical
 connections to other disciplines. Instituting graduation requirements for all students sends an
 important message about the program's importance using a currency all students can
 appreciate.
- A thorough assessment plan. In order to determine the efficacy of any program, especially a reform minded one, a clear assessment plan must be in place. The programs mentioned in this paper have resulted instudies to determine the impact of their programs on student attitudes and achievement (For example, Korey, 1999) and on faculty beliefs about the meaning of mathematics and how to teach it (Cerreto et. al., 2002).

This paper is intended to document the development of the MAC movement, following a mathematics problem solving model. Of course, just as new, related problems often arise after we have completed the solution of a current mathematics problem, so too, many questions remain regarding the future of MAC. Although preliminary assessments have been favorable, no broad-based evaluation of the impact of MAC has been conducted. To what extent has the promise of increased student understanding of mathematics and its connections to other disciplines been realized? What can be done to overcome logistical obstacles preventing instructors from working together in real schools settings? Are changes in institutional culture and relationships among academics merely transitory? Is the development of a strong base of curricular materials forthcoming? In other words, will MAC reach a level of educational permanence, or ultimately be discarded as another interesting, but unmanageable instructional fad?

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