Robust Back-Stepping Control with Polynomial-type PD input for Flexible Joint Robot Manipulators

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Key Words : Robust control, Flexible Joint, Back-stepping method, Polynomial-type PD input

Abstract

This paper proposes a robust back-stepping control with polynomial-type PD input for flexible joint robot manipulators to overcome parameter uncertainty. In the first step, a fictitious control is designed with polynomial-type PD input for the rigid link dynamic by the H-infinity control method. In second and third steps, the other fictitious control and real control are designed using saturation control and polynomial-type PD input based on the Lyapunov's second method. In each step, the designed robust inputs satisfy the L2-gain, which is equal to or less than gamma in the closed loop system. In contrast with the previous researches, the proposed method proves performance relations with PD gain from the robust gain. The performance robustness of the proposed control is verified through a 2-DOF robot manipulator with joint flexibility.

1. Introduction

Previous research has proposed various control design methods for robot manipulators with joint flexibility. The feedback linearization method was used on nonlinear static state-feedback and diffeomorphic coordinate transformations [1], [2]. And, an adaptive control method has been proposed to achieve global convergence of tracking errors to zero with all signals remaining bounded [3]. However, if measurement signals get out of their bounded value, excessive control inputs could be generated. Then, the singular perturbation method using integral manifold is proposed to overcome these limitations [4].

Recently, the integrator back-stepping method has been designed by [5], [6] as a method which provides a framework for recursive design of nonlinear and adaptive systems by achieving system stability at each step. These methods do not need feedback information of acceleration, jerk and suitable the cascaded system. However the back-stepping control becomes easily unstable under the unknown parameters. To get over the limitation, the adaptive back-stepping method using tuning function, the back-stepping control with neural network, etc., have been proposed by [7], [8].

In this paper, the dynamic system is partitioned into two series cascaded subsystems. And the proposed robust controller is designed for these subsystems using the back-stepping method. In the first step, a fictitious control input is designed such that it consists of the dynamic feed-forward input, the robust control input and the polynomial-type PD input. These robust control and polynomial-type PD input are derived by applying the nonlinear H-infinity control method. Their robust gain is lead to solving the Hamilton Jacobi Inequality (HJI). The solution to the HJI is obtained with a more tractable nonlinear matrix inequality (NLMI) method [9], [10]. Therefore, the closed-loop system with robust input is to achieve L_2-gain which has equal to or less than γ [11], [12].

In the second step, the fictitious control input of the back-stepping method is designed by the polynomial-type PD and the saturation control method [13]. The saturation control, one of nonlinear robust control methods, is designed by defining the upper- and lower-bounds of the unknown parameters. This control input minimizes the magnitude of the control in the worst case of uncertainty. Then, the designed control input satisfies the L_2-gain property in each step. Eventually, the overall system becomes an energy dissipative system.

Since the proposed method gets the acceleration value from the relation between acceleration and the link dynamics unlike the previous researches, the proposed controller does not need the measurement of angular acceleration. Also, we define the bounded function of the states and parameters in the partial differential equation. As a result, the proposed controller does not need the

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information on the desire jerk. The proposed control method needs only the feedback information of velocity and position at the link and motor side. And, PD gain is derived from the robust gain. As a result, the PD control input has directly relation with the robust performance. From the simulation result, the proposed control method has more robustness performance, stability and convergence than existed robust controller [13].

Section 2 describes the dynamic equation and the property of flexible manipulators. In Section 3, the robust back-stepping control is proposed. Section 4 shows the computer simulation results of the proposed control method, which is followed by conclusion in Section 5.

2. DYNAMIC OF FLEXIBLE ROBOT MANIPULATORS

The dynamics of flexible joint robot manipulators is represented by

$$M(q_1)\ddot{q}_1 + C(q_1,\dot{q}_1)\dot{q}_1 + G(q_1) + K(q_1 - q_2) = 0 \tag{1}$$

$$J\ddot{q}_2 + K(q_2 - q_1) = u \tag{2}$$

where $q_1 \in R^n$ and $q_2 \in R^n$ are the link and motor angles, respectively. $M(q_1)$, $C(q_1,\dot{q}_1)$, $G(q_1)$, K, J are the link inertia, the coriolis/centrifugal term, the gravity term, the diagonal matrix representing joint stiffness, the diagonal matrix representing motor inertia, respectively, and u is the motor torque. Since model uncertainty exists in the dynamics, the robust control is need for the recursive design. The manipulator dynamics, (1) and (2) is a cascaded system and thus can be transformed to

$$K^{-1}\left[M(q_1)\ddot{q}_1 + C(q_1,\dot{q}_1)\dot{q}_1 + G(q_1) + Kq_1\right] = x_1 \tag{3}$$

$$\dot{x}_1 = x_2 \tag{4}$$

$$\dot{x}_2 = J^{-1}\left[u - K(x_1 - q_1)\right]. \tag{5}$$

The system in (1-5) has following properties [14]:

Property 1: The link inertia matrix $M(q_1)$ is symmetric, positive definite, and both $M(q_1)$ and $M^{-1}(q_1)$ are uniformly bounded as follows:

$\alpha_M \geq \|M(q_1)\|$ and $\alpha_I \geq \|M^{-1}(q_1)\|$ where α_M and α_I are positive constants.

Property 2: If suitably chosen, $C(q_1,\dot{q}_1)$ is uniformly bounded such that $\alpha_C\|\dot{q}_1\| \geq \|C(q_1,\dot{q}_1)\|$ where α_C is a positive constant.

Property 3: The gravitational term $G(q_1)$ is uniformly bounded such that $\alpha_G \geq \|G(q_1)\|$ where α_G is a positive constant.

3. ROBUST BACK-STEPPING CONTROL

3.1 Concept of Back-Stepping Design

The controller will be designed in the following order. At first, fictitious control law α_1 is designed for x_1, which is not actual control input of the manipulator. The next virtual control law α_2 is designed for x_2. Finally, the fictitious control law for actual torque input u is designed in the last procedure. Let's define the error in control, or the difference between the state variables and their fictitious controls:

$$e_1 \triangleq x_1 - \alpha_1, \tag{6}$$

$$e_2 \triangleq x_2 - \alpha_2 \tag{7}$$

where α_1 and α_2 represent fictitious control laws.

3.2 The First Fictitious Robust Control

To use the H-infinity control theory, the new state s is defined as

$$s = \dot{e} + \Lambda_1 e = \dot{q}_1 - \{\dot{q}_{1d} - \Lambda_1(q_1 - q_{1d})\} = \dot{q}_1 - v \tag{8}$$

where Λ_1 is a positive diagonal matrix. The robust back-stepping control is designed with (3) firstly. In (3), x_1 is the feedback input and the fictitious control input, α_1, is designed for x_1. Note that (3) is just the manipulator dynamics with rigid joints. Therefore, the control input would be

$$\begin{aligned}\alpha_1 = \hat{K}^{-1}\left[\hat{M}(q_{1d})\ddot{q}_{1d} + \hat{C}(q_{1d},\dot{q}_{1d})\dot{q}_{1d} + \hat{G}(q_{1d})\right]\\ + q_1 + u_{pd} + u_r\end{aligned} \tag{9}$$

where $\hat{M}$, $\hat{C}$, $\hat{G}$ and $\hat{K}$ are the matrixes with estimated parameter values, u_r, $u_{PD} \in R^m$ are the robust input, the polynomial-type PD input. With (3) and (9), (6) is transformed to

$$\begin{aligned}e_1 = \hat{K}^{-1}\hat{M}(q_{1d})(\dot{s} + \Lambda_1\dot{e}) + \hat{K}^{-1}\hat{C}(q_{1d},\dot{q}_{1d})(s + \Lambda_1 e)\\ - u_r - u_{pd} + w\end{aligned} \tag{10}$$

where disturbance, w, is $w = \bar{M}\ddot{q}_1 + \bar{C}\dot{q}_1 + \bar{G}$ with $\bar{M} \triangleq K^{-1}M(q_1) - \hat{K}^{-1}\hat{M}(q_{1d})$, $\bar{C} \triangleq K^{-1}C(q_1,\dot{q}_1) - \hat{K}^{-1}\hat{C}(q_{1d},\dot{q}_{1d})$, $\bar{G} \triangleq K^{-1}G(q_1) - \hat{K}^{-1}\hat{G}(q_{1d})$.

For applying the nonlinear H-infinity control method, (10) represents the generalized form such as

$$\dot{s} = A(q_{1d},\dot{q}_{1d})(s + \Lambda_1 e) + B(q_{1d})(u_r + u_{PD} + w + e_1) + \Lambda_1\dot{e} \tag{11}$$

where $A(q_{1d},\dot{q}_{1d}) = \hat{M}^{-1}(q_{1d})\hat{C}(q_{1d},\dot{q}_{1d})$, $B(q_{1d}) = \hat{M}^{-1}$

$(q_{1d})\hat{K}$. With performance vector z, the generalized nonlinear system can be described as

$$\dot{s} = A(s+\Lambda_1 e_1) + Bw + B(u_r + u_{PD} + e) + \Lambda_1 \dot{e} \tag{12}$$

$$z = Hs + Du_r, \quad H^T D = 0,\ D^T D > 0 \tag{13}$$

where H and D are the constant matrices of suitable dimensions. To show that real control achieves stability about the equilibrium point, a positive definite Lyapunov function of

$$V_1 = s^T Ps, \quad \text{with} \quad P > 0 \tag{14}$$

is defined. Derivative Lyapunov function is

$$\begin{aligned}\dot{V}_1 &= 2s^T P(As + Bw + Bu_r) \\ &\quad + 2s^T P(Bu_{pd} + Be_1 + \Lambda_1 \dot{e} - A\Lambda_1 e).\end{aligned} \tag{15}$$

When matrix D is not a square matrix, a non-singular square matrix R satisfying $D^T D = R^T R$ is defined. Introducing $\gamma^2\|w\|^2 - \|z\|^2$ to the above equation yields

$$\begin{aligned}\dot{V}_1 &= \gamma^2\|w\|^2 - \|z\|^2 + s^T\{P^T A + A^T P + 1 + H^T H \\ &\quad + (1/\gamma^2)P^T BB^T P + A^T A - P^T B(R^T R)^{-1}B^T P\}s \\ &\quad + \left\|Ru_r + R^{-T}B^T Ps\right\|^2 - \gamma^2\left\|w - (1/\gamma^2)B^T Ps\right\|^2 \\ &\quad - \|P\Lambda_1\dot{e} + s\|^2 - \|P\Lambda_1 e + As\|^2 + 2s^T PBe_1 \\ &\quad + \|P\Lambda_1\dot{e}\|^2 + \|P\Lambda_1 e\|^2 - 2k_{d1}\dot{e}^2 - 2k_{p1}e^2\end{aligned} \tag{16}$$

Let's assume HJ inequality

$$\begin{aligned}W_1 &\triangleq P^T A + A^T P + (1/\gamma^2)P^T BB^T P + 1 \\ &\quad + A^T A + H^T H - P^T B[R^T R]^{-1}B^T P \le 0\end{aligned} \tag{17}$$

is satisfied. Then, the control input that satisfies the L_2-gain property becomes $u_r = -[R^T R]^{-1}B^T Ps.$ (18)

And the following inequality is satisfied if $k_{p1}, k_{d1} \ge \frac{1}{2}\lambda_{\max}(\Lambda_1^T P^T P\Lambda_1)$ So, the inequality satisfied as

$$W_2 \triangleq \|P\Lambda_1\dot{e}\|^2 + \|P\Lambda_1 e\|^2 - 2k_{d1}\dot{e}^2 - 2k_{p1}e^2 \le 0 \tag{19}$$

Therefore, the polynomial-type PD control input as like that $u_{pd} = B^{-1}P^{-1}s(k_{i1}\dot{e}^2 + k_{p1}e^2).$ (20)

For the robust and polynomial-type PD control input, we obtain the robust gain P from the HJ inequality. To obtain the solution to (17), it is transformed to a nonlinear matrix inequality (NLMI) using Schur complement. Then the HJ inequality becomes

$$\begin{bmatrix} P^T A + A^T P + H^T H + (1/\gamma^2)P^T BB^T P + 1 + A^T A & P^T B \\ B^T P & R^T R \end{bmatrix} \le 0. \tag{21}$$

Solving NLMI yields the convex optimization problem. If the matrices forming the NLMI are bounded, only a finite number of LMIs [15] are to be solved. If the robust gain P that satisfies the inequality exists and $e_1 = 0$, the derivative Lyapunov function becomes

$$\dot{V}_1 \le \gamma^2\|w\|^2 - \|z\|^2 \tag{22}$$

Thus, the closed loop system becomes dissipative [11]. And, with $w = 0$, the system becomes asymptotically stable.

3.3 The Second Fictitious Robust Control

In the next step, the second fictitious control input, α_2, is designed for x_2. In order to derive a control law for x_2, signal e_1 is differentiated:

$$\dot{e}_1 = x_2 - \tfrac{\partial\alpha_1}{\partial q_1}\dot{q}_1 - \tfrac{\partial\alpha_1}{\partial\dot{q}_1}\ddot{q}_1 - \tfrac{\partial\alpha_1}{\partial q_{1d}}\dot{q}_{1d} - \tfrac{\partial\alpha_1}{\partial\dot{q}_{1d}}\ddot{q}_{1d} - \tfrac{\partial\alpha_1}{\partial\ddot{q}_{1d}}q_{1d}^{(3)} \tag{23}$$

From (3)

$$\ddot{q}_1 = M^{-1}(q_1)\left[-C(q_1,\dot{q}_1)\dot{q}_1 - G(q_1) + K(x_1 - q_1)\right]. \tag{24}$$

Differentiating α_1 with respect time and substituting $\ddot{q}_1$ with the right-hand-side of (24),

$$\begin{aligned}\dot{\alpha}_1 &= +\tfrac{\partial\alpha_1}{\partial q_1}\dot{q}_1 + \tfrac{\partial\alpha_1}{\partial q_{1d}}\dot{q}_{1d} + \tfrac{\partial\alpha_1}{\partial\dot{q}_{1d}}\ddot{q}_{1d} + \tfrac{\partial\alpha_1}{\partial\ddot{q}_{1d}}q_{1d}^{(3)} \\ &\quad + \tfrac{\partial\alpha_1}{\partial\dot{q}_1}M^{-1}(q_1)\left[-C(q_1,\dot{q}_1)\dot{q}_1 - G(q_1) + K(x_1 - q_1)\right]\end{aligned} \tag{25}$$

Thus, (23) becomes

$$\dot{e}_1 = x_2 - \dot{\alpha}_1(q_1, \dot{q}_1, q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d}, q_{1d}^{(3)}, x_1). \tag{26}$$

For control input α_2, combining (7) and (26) yields

$$\dot{e}_1 = x_2 - \dot{\alpha}_1 = e_2 + \alpha_2 - \dot{\alpha}_1 \tag{27}$$

Differentiating the Lyapunov function of

$$V_2 = V_1 + e_1^T Ke_1, \tag{28}$$

results in

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + 2e_1^T K\dot{e}_1 = \dot{V}_1 + 2e_1^T K(e_2 + \alpha_2 - \dot{\alpha}_1) \\ &= \gamma^2\|w\|^2 - \|z\|^2 - \gamma^2\left\|w - (1/\gamma^2)B^T Ps\right\|^2 + s^T W_1 s + W_2 \\ &\quad - \|P\Lambda_1\dot{e} + s\|^2 - \|P\Lambda_1 e + As\|^2 + 2e_1^T K(\Delta_1 + e_2 + \alpha_2)\end{aligned} \tag{29}$$

where $\Delta_1 = K^{-1}B^T P^T s + \dot{\alpha}_1$. From Properties 1, 2 and 3, term Δ_1 can be bounded as

$$\begin{aligned}\|\Delta_1\| &= \xi_1\|s\| + \Big(\left\|\tfrac{\partial\alpha_1}{\partial q_1}\right\|\|\dot{q}_1\| + \left\|\tfrac{\partial\alpha_1}{\partial q_{1d}}\right\|\|\dot{q}_{1d}\| + \left\|\tfrac{\partial\alpha_1}{\partial\dot{q}_{1d}}\right\|\|\ddot{q}_{1d}\| \\ &\quad + \left\|\tfrac{\partial\alpha_1}{\partial\ddot{q}_{1d}}\right\|\|q_{1d}^{(3)}\| + \left\|\tfrac{\partial\alpha_1}{\partial\dot{q}_1}\right\|\|\ddot{q}_1\|\Big)\end{aligned} \tag{30}$$

where ξ_1 is a positive constant, and

$$\begin{aligned}\|\ddot{q}_1\| &= \left\|M^{-1}(q_1)\left[-C(q_1,\dot{q}_1)\dot{q}_1 - G(q_1) + K(x_1 - q_1)\right]\right\| \\ &= \beta_0 + \beta_1\|\dot{q}_1\|^2 + \beta_2\|x_1 - q_1\|,\end{aligned} \tag{31}$$

with β_0, β_1, β_2, all positive constants satisfying

$$\beta_0 \ge \alpha_I\alpha_G > 0,\ \ \beta_1 \ge \alpha_I\alpha_C > 0,\ \beta_2 \ge \alpha_I\alpha_K > 0. \tag{32}$$

Note that Property 1, 2 and 3 are used in deriving the inequality in (32). Equation (30) is can be represented the upper bounded function as

$$\begin{aligned}\|\Delta_1\| &\le \xi_1\|s\| + \xi_2\|\dot{q}_1\|^2 + \xi_3\|x_1 - q_1\| + \xi_4 \\ &\le \xi_5\|\dot{e}\| + \xi_6\|e\| + \bar{\rho}_1(x_1, q_1, \dot{q}_1)\end{aligned} \quad (33)$$

where

$\xi_2\|\dot{q}_1\|^2 = \left\|\frac{\partial\alpha_1}{\partial q_1}\right\|\|\dot{q}_1\| + \left\|\frac{\partial\alpha_1}{\partial\dot{q}_1}\right\|\beta_1\|\dot{q}_1\|^2$,

$\xi_3\|x_1 - q_1\| = \left\|\frac{\partial\alpha_1}{\partial\dot{q}_1}\right\|\beta_2\|x_1 - q_1\|$,

$\xi_4 = \left\|\frac{\partial\alpha_1}{\partial q_{1d}}\right\|\|\dot{q}_{1d}\| + \left\|\frac{\partial\alpha_1}{\partial\dot{q}_{1d}}\right\|\|\ddot{q}_{1d}\| + \left\|\frac{\partial\alpha_1}{\partial\ddot{q}_{1d}}\right\|\|q_{1d}^{(3)}\| + \left\|\frac{\partial\alpha_1}{\partial\dot{q}_1}\right\|\beta_0$

and ξ_i for $i = 2,...,6$ is a positive constant. Therefore, (29) is represented by

$$\dot{V}_2 \le \gamma^2\|w\|^2 - \|z\|^2 + 2e_1^T K e_2 + 2e_1^T K(\Delta_1 + \alpha_2) \quad (34)$$

To satisfy the Lyapunov stability, robust control input of

$$\alpha_2 = -\Lambda_2 e_1 - k_{d2}e_1\dot{e}^2 - k_{p2}e_1 e^2 - e_1\bar{\rho}_1\bar{K}/(\|e_1\| + \varepsilon)\underline{K}. \quad (35)$$

is designed, based on the bound of the stiffness matrix as $\underline{K}I \le K \le \bar{K}I$, where ε is small value and k_{p2}, k_{d2} are positive constants matrices. To make the control input differentiable, we need to make the second modification that redefines the control to be

$$\alpha_2 = -\Lambda_2 e_1 - k_{d2}e_1\dot{e}^2 - k_{p2}e_1 e^2 - e_1\|e_1\|^2\bar{\rho}_1\bar{K}/(\|e_1\|^3 + \varepsilon)\underline{K}$$

The designed control input considered about the stiffness uncertainty. If the stiffness of system is changing, the designed control input guarantees the negative function in the bound range. Therefore, (34) is rewritten such as

$$\begin{aligned}\dot{V}_2 \le{}& \gamma^2\|w\|^2 - \|z\|^2 - 2e_1^T K\Lambda_2 e_1 + 2e_1^T K e_2 \\ &+ 2e_1^T K\left(\bar{\rho}_1 - e_1\|e_1\|^2\bar{\rho}_1\bar{K}/(\|e_1\|^3 + \varepsilon)\underline{K}\right)\end{aligned} \quad (37)$$

where Λ_2 is positive diagonal constant gain matrix. The designed control input minimizes the magnitude of the control in the worst case, i.e., when the uncertainty is its maximum in size. Therefore, if $e_2 = 0$ then

$\dot{V}_2 \le \gamma^2\|w\|^2 - \|z\|^2$,

and with $w = 0$ becomes asymptotically stable.

3.4 Actual Robust Control

Since the actual control input is related to u in (5), differentiating e_2 with respect to time is needed to derive a control law for u. Thus,

$$\dot{e}_2 = \dot{x}_2 - \dot{\alpha}_2 = J^{-1}u - J^{-1}K(x_1 - q_1) - \dot{\alpha}_2 \quad (38)$$

For a stable control law, a Lyapunov function of $V_3 = V_2 + \frac{1}{2}e_2^T KJe_2$ is defined. And, differentiating it with respect to time yields

$$\begin{aligned}\dot{V}_3 ={}& \gamma^2\|w\|^2 - \|z\|^2 + e_2^T K\left(u + e_1 - \hat{K}(x_1 - q_1) - \Delta_2\right) \\ &- e_1^T K\Lambda_2 e_1 + e_1^T K\left(\Delta_1 - e_1\|e_1\|^2\bar{\rho}_1\underline{K}/\bar{K}(\|e_1\|^3 + \varepsilon)\right).\end{aligned} \quad (39)$$

Term Δ_2 denotes uncertainty which has properties of stiffness and derivation values of the designed control input α_2.

$$\begin{aligned}&\Delta_2 \triangleq \Delta K(x_1 - q_1) + J\dot{\alpha}_2 \\ &\|\Delta_2\| \le \xi_7\|x_1 - q_1\| + \xi_8\left\|\hat{\dot{\alpha}}_2(e_1, e_2, \dot{q}_2)\right\| \\ &\qquad \le \xi_7\|x_1 - q_1\| + \xi_9\|\dot{q}_2\| + \xi_{10}\|e_1\| + \xi_{11}\|e_2\| + \xi_{12} \\ &\qquad \le \xi_{10}\|e_1\| + \xi_{11}\|e_2\| + \bar{\rho}_2(x_1, q_1, \dot{q}_2)\end{aligned} \quad (40)$$

where ξ_i for $i = 7,...,12$ is a positive constant, ΔK is the stiffness distinction. Equation (39) is represented as

$$\dot{V}_3 \le \gamma^2\|w\|^2 - \|z\|^2 + e_2^T K\left(u + e_1 + \hat{K}(q_1 - x_1) - \Delta_2\right). \quad (41)$$

Thus, for a control law of

$$\begin{aligned}u_d ={}& -e_1 - \Lambda_3 e_2 - k_{p3}e_2 e_1^2 - k_{d3}e_2 e_2^2 - \hat{K}(q_1 - x_1) \\ &- \bar{\rho}_2\bar{K}\mu/(\underline{K}\mu + \varepsilon\varphi)\end{aligned} \quad (42)$$

where Λ_3 is a diagonal matrix with positive elements, $\mu(q,t) \triangleq 2e_2\underline{K}\bar{\rho}_2$, φ is small value, k_{p3}, k_{d3} are positive constant matrices, then it follow

$$\dot{V}_3 \le \gamma^2\|w\|^2 - \|z\|^2 + e_2^T\hat{K}J\left(\bar{\rho}_2 - \mu\bar{\rho}_2\bar{K}/(\mu + \varepsilon\varphi)\underline{K}\right). \quad (43)$$

The saturation type control input designed for the stiffness uncertainty in acceleration level of motor side which is relation to the link side. If u_d is designed from the robust nonlinear method, the Lyapunov function satisfied $\dot{V}_3 \le \gamma^2\|w\|^2 - \|z\|^2$. The control law in (42) guarantees that the closed-loop system is dissipative at the equilibrium point, i.e., $s = e_1 = e_2 = 0$. Therefore, the overall system is said to be L_2-gain less than or equal to 1 and guarantees the asymptotically stable with $w = 0$.

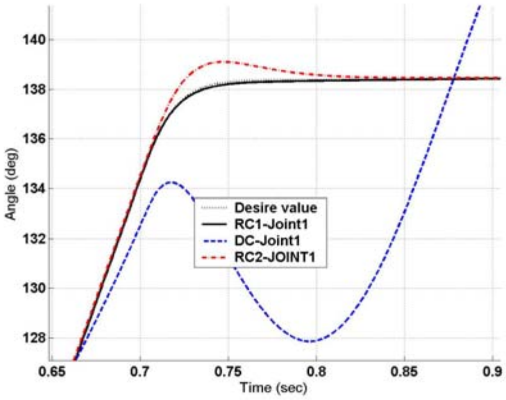
4. SIMULATION

The performance robustness of proposed controller is verified through simulations against continuous parameter variations, random parameter changes, and an external impulse disturbance. A 2-DOF flexible joint robot manipulator is assumed to have uniform links. Table I shows the nominal values of its physical parameters.

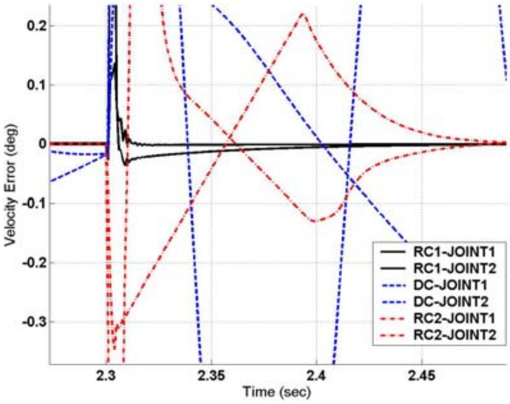
Table 1 Parameters of the manipulator in simulation

symbol	m_1	m_2	l_1	l_2	k_1	k_2
value	6	4	0.30	0.30	1,500	1,200
unit	kg	kg	m	m	Nm/rad	Nm/rad

The performance of the proposed controller is compared with that of the PID controller with the dynamic feed-forward compensation, and that of the robust nonlinear H-infinity controller based on the back-stepping method in [13]. Initially, the links of the manipulator are stretched horizontally. The desired motion of the first link is to turn 180° in the CCW direction, and that of the second link is to turn 90° in the CCW direction relative to the first link. First, the effect of a constant uncertainty is measured. It is assumed that the actual values of the inertias and the joint stiffness are 20% higher than the measured. The resulting motions of the manipulator with various controllers are shown in Fig. 1(a)



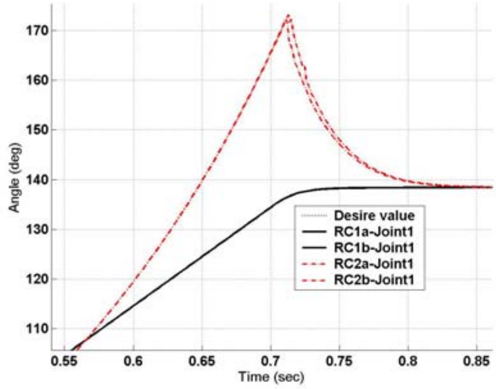
(a)



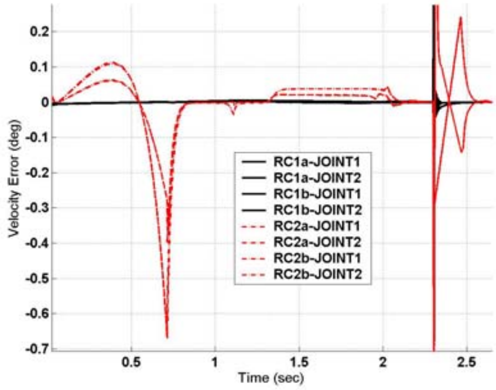
(b)

Fig. 1 Angular velocity under uncertainty and impulse disturbance at t=2.3s: (a) link velocity (b) error in link velocity (RC1: the proposed controller, solid line, RC2: the other robust controller, dash & dot line, DC: the model based dynamic controller, dash line)

The model-based dynamic controller, with feed-forward compensation, becomes unstable. The robust H-infinity controller of [13] perform better than the model-based dynamic controller. However, its tracking error is significant. On the other hand, the proposed controller exhibits very close tracking to the desired trajectory. The advantage of the proposed controller over the others is more apparent when convergence to the desired velocity is evaluated after an impulse disturbance at t=2.3s, as shown in Fig. 1(b).



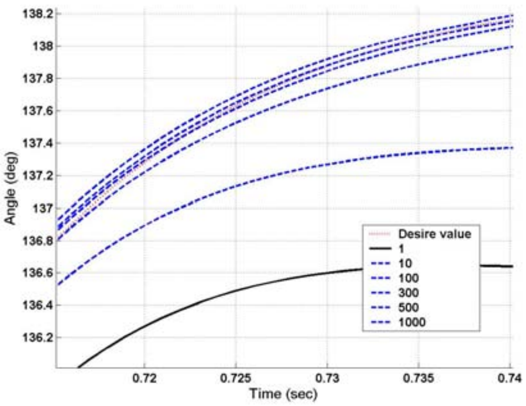
(a)



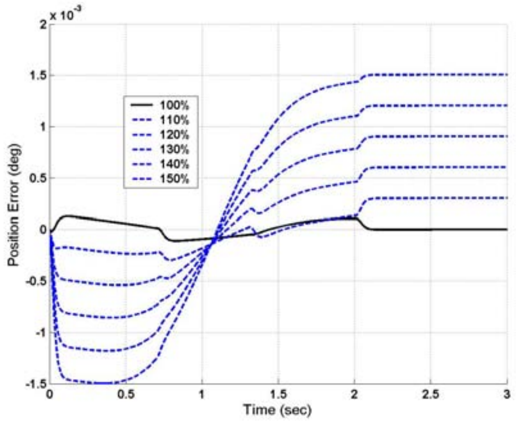
(b)

Fig. 2 Angular velocity (a) and velocity error (b) under the continuous parameter variations with initial uncertainty (a: the sine function inertia, b: the random function inertia)

Figure 2 shows the robust performance and stability in case of the sinusoidal fluctuations of the parameters, whose peaks are about 30% of their nominal values. The tracking performance of the proposed controller is compared with that of the H-infinity controller used in [13]. Note that in Fig. 2(a) the transient response of the H-infinity controller of [13] is unsatisfactory: its maximum overshoot is almost 100%. Figure 2(b) show that the effect of fluctuating parameter values has much less effects on the proposed controller. Again, it also shows that the effect of an impulse disturbance quickly dies for the proposed controller, while it is persistent for the H-infinity controller of [13].



(a)



(b)

Fig. 3 Angular velocity (a) and position error (b) in Joint_1 under the gain change and the parameter uncertainty

Fig. 3(a) shows the performance relation according to lambda gain. Fig. 3(b) shows the position error bound in parameter uncertainties which are Mass, Inertia and Stiffness at the same time.

5. CONCLUSION

We proposed a robust back-stepping control method for robot manipulators with flexible joints which uses the nonlinear H-infinity and the saturation control method. In each design step, the robust input is designed against the parameter uncertainty by satisfying the L_2 -gain property. The acceleration signal is defined from the relations in the link dynamic equation. The desire jerk is assumed to be constant by defining the bounded function. Thus, the proposed controller does not need to measure the acceleration for the feedback and to design the jerk information as compared with the previous researches. From the simulation result, the polynomial-type PD input has directly performance relation with robust gain. Also, the proposed control method has more performance robustness, stability, and convergence than many existing robust controllers under the parameter uncertainty, variations and impulse disturbances.

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