A Fast Converging Pulse Coupling Oscillator Synchronicity Model

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Abstract

The Pulse Coupling Oscillator (PCO) is a synchronicity model inspired by nature. However, the PCO model has some limitations. The Fast PCO model is proposed in this paper. It addresses the problem of the phase swing actions in the original PCO model. Benefits are the fast synchronicity speed and associated energy saving.

1. Introduction of PCO

Today's computing devices are equipped with a hardware oscillator assisted clock. The PCO model [1], [2], [3], [4] regards every distributed clock as an oscillator with the same fixed frequency but different initiate phase. "Pulse coupling" means using a fired pulse as signal to transmit the oscillating information. The object is to let these oscillators act with the same phase, same frequency, so-called getting synchronized. $f(\phi)$ is the oscillator's oscillator function, ϕ represents the oscillator phase. Phase ϕ and state $\varphi = f(\phi)$ both are defined on [0, 1]. The function cycle period is T. The f should be monotonically increasing and concave down (that is: $f' > 0$ and $f'' < 0$; $f(0) = 0$, $f(1) = 1$).

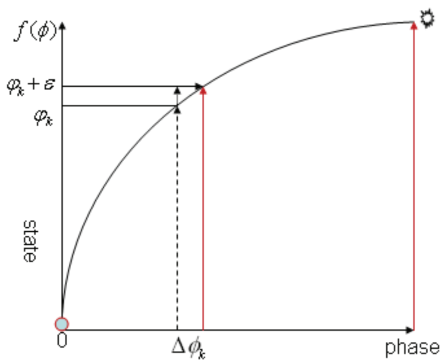

Fig.1. Oscillator jump action

When there is only one oscillator, the state will follow the function curve from 0 to 1 at a regular rate: $d\phi / dt = 1/T$. When it arrives at 1, it fires and emits a pulse, then resets the phase to 0 and runs again. However, if during the walking path, at time *t*, it receives a pulse from the other oscillator, the state will jump an amount ε (Figure 1). The corresponding phase becomes $MIN(f^{-1}(f(\phi_t) + \varepsilon, 1)$. And if the state jumps beyond 1, we set it as 1, and thinking it gets synchronized with the firing one. In such a way, oscillators interact with each other and adjust their phases to an agreed one.

2. Converging direction determinant formula

The discussion in [2] is extended and enhanced in this section to more clearly describe the dynamic nature of the converging process and get the converging direction determinant formula. Here we give a graphic explanation on a pair nodes' converging procedure (Figure 2)

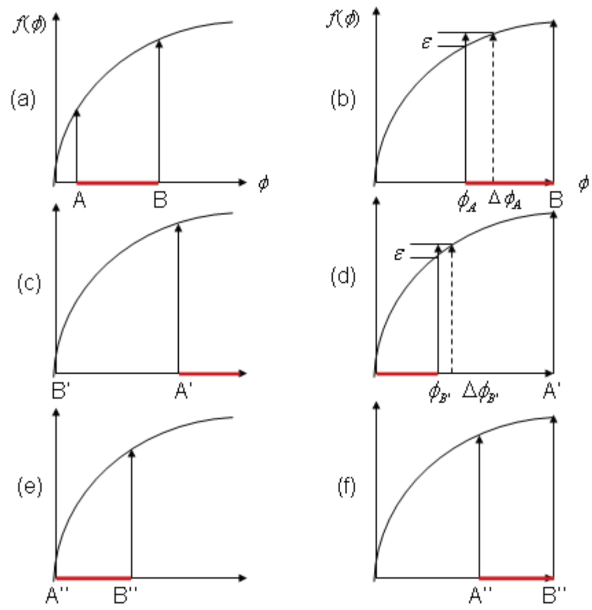

Fig. 2 A pair nodes' phase converging procedure in one cycle.

Let $|\overline{AB}|$, the red (dark) lines in figure 2 represent the phase distance vector from *A* to *B*. When oscillator *B* is fired, A jumps (figure 2 (b), (c)), *AB* distance changes to:

$$|\overrightarrow{A'B'}| = |\overrightarrow{AB}| - \Delta\phi_A \qquad (1)$$

Next after oscillator B firing, A jumps (Figure 2(d), (e)), the distance becomes:

$$|\overrightarrow{A''B''}| = |\overrightarrow{A'B'}| + \Delta\phi_{B'} = |\overrightarrow{AB}| - \Delta\phi_A + \Delta\phi_{B'} \qquad (2)$$

Using the definition of inverse function definition (Figure 1):

$$\Delta\phi_k = f^{-1}(\varphi_k + \varepsilon) - f^{-1}(\varphi_k) \qquad (3)$$

For $f'' < 0$ and $f' > 0$,

$$\Delta\phi' = \frac{1}{f'(\varphi+\varepsilon)} - \frac{1}{f'(\varphi)} = \frac{f'(\varphi) - f'(\varphi+\varepsilon)}{f'(\varphi)f'(\varphi+\varepsilon)} > 0 \qquad (4)$$

$\Delta\phi$ is monotonically increasing.

$$\phi_{B'} = 1 - \phi_{A'} = 1 - f^{-1}(f(\phi_A) + \varepsilon)$$

$$\phi_A - \phi_{B'} = \phi_A + f^{-1}(f(\phi_A) + \varepsilon) - 1 \qquad (5)$$

So when ϕ_B is at critical fire point ($\phi_B = 1$), if

$\phi_A + f^{-1}(f(\phi_A) + \varepsilon) > 1$, $|\overrightarrow{A''B''}| < |\overrightarrow{AB}|$,

$\Delta\phi_A > \Delta\phi_{B'}$, AB will converge to 0

$\phi_A + f^{-1}(f(\phi_A) + \varepsilon) < 1$, $|\overrightarrow{A''B''}| > |\overrightarrow{AB}|$,

$\Delta\phi_A < \Delta\phi_{B'}$, AB will converge to 1

3 Fast converging PCO

From the pair oscillators' converging procedure, we notice that the "phase difference" swings: $|\overrightarrow{AB}| > |\overrightarrow{A'B'}|$ *and* $|\overrightarrow{A'B'}| < |\overrightarrow{A''B''}|$ The distance decreases and increases again in one cycle. In Figure 3 we draw out the phase difference changing procedure of a pair of oscillators' convergence, the blue (upper) line describes the swing action. The unnecessary swing between two directions causes converging time to be longer and to consume more energy.

Based on our determinant formula, if we know the converging direction in advance by calculating the value of $\phi_A + f^{-1}(f(\phi_A) + \varepsilon)$, then we can avoid the unnecessary actions. To be concrete, when an oscillator hears one pulse, it calculates current $\phi_A + f^{-1}(f(\phi_A) + \varepsilon)$ value and checks that the coming jump is to its final converging direction or not. If it is, then jump the oscillator, otherwise, ignore it. Thus the reverse jump is replaced by just staying its original place, and the up jump fold parts in the blue (upper) line become the level parts in the red (lower) line as shown in Figure 3.

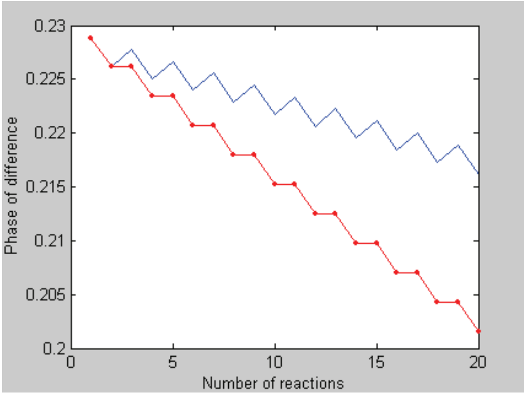

Fig. 3 Swing action of the phase difference

4 Conclusions

The FPCO saves almost half of converging time in the two synchrony group stage. Furthermore, the converging cycle is reduced by half; in every cycle, half of the jump actions are ignored. The total energy consumption on synchrony stage will be definitely saved.

Acknowledgment

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reference

[1] C. Peskin, *Mathematical Aspects of Heart Physiology*, New York University, New York, 1975.
[2] R. Mirollo and S. Strogatz, "Synchronization of pulse-coupled biological oscillators," *SIAM.*, vol. 50, no. 6, p. 1645C1662, 1990.
[3] G. W. Allen, G. Tewari, and A. Patel, "Firefly inspired sensor network synchronicity with realistic radio effects," in *SenSys05*, San Diego, California, USA., November 2005.
[4] Y. Hong and A. Scaglione, "Time synchronization with pulse coupled oscillators for uwb wireless ad hoc networks," in *In IEEE Conf. on Ultra Wideband Systems and Technologie s*, Nov 2003.