Fuzzy (r, s)-semi-preopen sets and fuzzy (r, s)-semi-precontinuous maps

퍼지 (r, s)-semi-preopen 집합과 퍼지 (r, s)-semi-precontinuous 함수

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Abstract

In this paper, we introduce the concepts of fuzzy (r, s)-semi-preopen sets and fuzzy (r, s)-semi-precontinuous maps on intuitionistic fuzzy topological spaces in Sostak's sense. The relations among fuzzy (r, s)-semicontinuous, fuzzy (r, s)-precontinuous, and fuzzy (r, s)-semi-precontinuous maps are discussed. The concepts of fuzzy (r, s)-semi-preinterior, fuzzy (r, s)-semi-preclosure, fuzzy (r, s)-semi-preneighborhood, and fuzzy (r, s)-quasi-semi-preneighborhood are given. Using these concepts, the characterization for the fuzzy (r, s)-semi-precontinuous map is obtained. Also, we introduce the notions of fuzzy (r, s)-semi-preopen and fuzzy (r, s)-semi-preclosed maps on intuitionistic fuzzy topological spaces in Sostak's sense, and then we investigate some of their characteristic properties.

Keywords : fuzzy (r, s)-semi-preopen set, fuzzy (r, s)-semi-precontinuous map, fuzzy (r, s)-semi-preopen map, fuzzy (r, s)-semi-preclosed map

1. Introduction

The concept of fuzzy set was introduced by Zadeh [17]. Chang [2] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Sostak [15], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay and his colleagues [3], and by Ramadan [14].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Coker and his colleagues [4, 7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Coker and Demirci [6] defined intuitionistic fuzzy topological spaces in Sostak's sense as a generalization of smooth topological spaces and intuitionistic fuzzy topological spaces. Thakur and Singh [16] introduced the concepts of fuzzy semi-preopen sets and fuzzy semi-precontinuous maps on Chang's fuzzy topological spaces and Jun and Song [9] considered this concepts on intuitioinistic fuzzy topological spaces.

In this paper, we introduce the concepts of fuzzy (r, s)-semi-preopen sets and fuzzy (r, s)-semi-precontinuous maps on intuitionistic fuzzy topological spaces in Sostak's sense. The relations among fuzzy (r, s)-semicontinuous, fuzzy (r, s)-precontinuous, and fuzzy (r, s)-semi-precontinuous maps are discussed. The concepts of fuzzy (r, s)-semi-preinterior, fuzzy (r, s)-semi-preclosure, fuzzy (r, s)-semi-preneighborhood, and fuzzy (r, s)-quasi-semi-preneighborhood are given. Using these concepts, the characterization for the fuzzy (r, s)-semi-precontinuous map is obtained. Also, we introduce the notions of fuzzy (r, s)-semi-preopen and fuzzy (r, s)-semi-preclosed maps on intuitionistic fuzzy topological spaces in Sostak's sense, and then we investigate some of their characteristic properties.

2. Preliminaries

Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of non-membership, respectively and $\mu_A + \gamma_A \leq 1$. Obviously every fuzzy set μ in X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1} - \mu)$.

Definition 2.1. ([1]) Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy sets in X. Then

(1) $A \subseteq B$ iff $\mu_A \leq \mu_B$ and $\gamma_A \geq \gamma_B$.

(2) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.

(3) $A^c = (\gamma_A, \mu_A)$.

(4) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.

(5) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.

(6) $\underline{0} = (\tilde{0}, \tilde{1})$ and $\underline{1} = (\tilde{1}, \tilde{0})$.

Let $I(X)$ be a family of all intuitionistic fuzzy sets in X and let $I \otimes I$ be the set of the pair (r, s) such that $r, s \in I$ and $r + s \leq 1$.

Definition 2.2. ([6]) Let X be a nonempty set. An *intuitionistic fuzzy topology in Sostak's sense*(SoIFT for short) $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2)$ on X is a map $\mathcal{T} : I(X) \to I \otimes I$ which satisfies the following properties :

(1) $\mathcal{T}_1(\underline{0}) = \mathcal{T}_1(\underline{1}) = 1$ and $\mathcal{T}_2(\underline{0}) = \mathcal{T}_2(\underline{1}) = 0$.

(2) $\mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$ and $\mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$.

(3) $\mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i)$ and $\mathcal{T}_2(\bigcup A_i) \leq \bigvee \mathcal{T}_2(A_i)$.

The $(X, \mathcal{T}) = (X, \mathcal{T}_1, \mathcal{T}_2)$ is said to be an *intuitionistic fuzzy topological space in Sostak's sense*(SoIFTS for short). Also, we call $\mathcal{T}_1(A)$ a *gradation of openness* of A and $\mathcal{T}_2(A)$ a *gradation of nonopenness* of A.

Definition 2.3. ([5, 8]) Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS and $(r, s) \in I \otimes I$. Then

(1) an intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X is said to be *quasi-coincident* with the intuitionistic fuzzy set A in X, denoted by $x_{(\alpha,\beta)} qA$, if and only if $\mu_A(x) > \beta$ or $\gamma_A(x) < \alpha$.

(2) two intuitionistic fuzzy sets A and B in X are said to be *quasi-coincident*, denoted by AqB, if and only if there exists an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$.

The word 'not quasi-coincident' will be abbreviated as $\tilde{q}$.

Theorem 2.4. ([13]) Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS and $(r, s) \in I \otimes I$.

(1) If $\{A_i\}$ is a family of fuzzy (r, s)-preopen sets in X, then $\bigcup A_i$ is fuzzy (r, s)-preopen.

(2) If $\{A_i\}$ is a family of fuzzy (r, s)-preclosed sets in X, then $\bigcap A_i$ is fuzzy (r, s)-preclosed.

3. Fuzzy (r, s)-semi-preopen sets and fuzzy (r, s)-semi-precontinuous maps

Now, we define the notions of fuzzy (r, s)-semi-preopen sets and fuzzy (r, s)-semi-precontinuous maps on intuitionistic fuzzy topological spaces in Sostak's sense, and then we investigate some of their properties.

Theorem 3.1. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is a fuzzy (r, s)-preopen set in X if and only if there is a fuzzy (r, s)-open set B in X such that $A \subseteq B \subseteq \mathrm{cl}(A, r, s)$.

Definition 3.2. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is called

(1) a *fuzzy (r, s)-semi-preopen* set if there is a fuzzy (r, s)-preopen set B in X such that $B \subseteq A \subseteq \mathrm{cl}(B, r, s)$.

(2) a *fuzzy (r, s)-semi-preclosed* set if there is a fuzzy (r, s)-preclosed set B in X such that $\mathrm{int}(B, r, s) \subseteq A \subseteq B$.

Theorem 3.3. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then the following statements are equivalent :

(1) A is a fuzzy (r, s)-semi-preopen set.

(2) A^c is a fuzzy (r, s)-semi-preclosed set.

Theorem 3.4. Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then the following statements are true :

(1) For each fuzzy (r, s)-semi-preopen set B in X, $B \subseteq A \subseteq \mathrm{cl}(B, r, s)$ implies that A is fuzzy (r, s)-semi-preopen in X.

(2) For each fuzzy (r, s)-semi-preclosed set B in X, $\mathrm{int}(B, r, s) \subseteq A \subseteq B$ implies that A is fuzzy (r, s)-semi-preclosed in X.

Theorem 3.5. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS and $(r, s) \in I \otimes I$.

(1) If $\{A_i\}$ is a family of fuzzy (r,s)-semi-preopen sets in X, then $\bigcup A_i$ is fuzzy (r,s)-semi-preopen.

(2) If $\{A_i\}$ is a family of fuzzy (r,s)-semi-preclosed sets in X, then $\bigcap A_i$ is fuzzy (r,s)-semi-preclosed.

The following example shows that the intersection(resp. union) of two intuitionistic fuzzy (r,s)-semi-preopen(resp. fuzzy (r,s)-semi-preclosed) sets need not be a fuzzy (r,s)-semi-preopen(resp. fuzzy (r,s)-semi-preclosed) set for each $(r,s)\in I\otimes I$.

Example 3.6. Let $X=\{x,y\}$ and let A_1 and A_2 be intuitionistic fuzzy sets in X defined as

$$A_1(x)=(0.1,0.7),\quad A_1(y)=(0.4,0.3);$$

and

$$A_2(x)=(0.8,0.1),\quad A_2(y)=(0.2,0.4).$$

Define $\mathcal{T}:I(X)\to I\otimes I$ by

$$\mathcal{T}(A)=(\mathcal{T}_1(A),\mathcal{T}_2(A))=\begin{cases}(1,0) & \text{if } A=\underline{0},\underline{1},\\ (\frac{1}{2},\frac{1}{3}) & \text{if } A=A_1,\\ (0,1) & \text{otherwise.}\end{cases}$$

Then clearly $\mathcal{T}$ is a SoIFT on X. The intuitionistic fuzzy sets A_1 and A_2 are fuzzy $(\frac{1}{2},\frac{1}{3})$-semi-preopen. But $A_1\cap A_2$ is not fuzzy $(\frac{1}{2},\frac{1}{3})$-semi-preopen.

Definition 3.7. Let $(X,\mathcal{T}_1,\mathcal{T}_2)$ be a SoIFTS. For each $(r,s)\in I\otimes I$ and for each $A\in I(X)$, the *fuzzy (r,s)-semi-preinterior* is defined by

$$\text{spint}(A,r,s)=\bigcup\{B\in I(X)\mid B\subseteq A,\ B \text{ is fuzzy } (r,s)\text{-semi-preopen}\}$$

and the *fuzzy (r,s)-semi-preclosure* is defined by

$$\text{spcl}(A,r,s)=\bigcap\{B\in I(X)\mid A\subseteq B,\ B \text{ is fuzzy } (r,s)\text{-semi-preclosed}\}.$$

Obviously $\text{spcl}(A,r,s)$ is the smallest fuzzy (r,s)-semi-preclosed set which contains A and $\text{spint}(A,r,s)$ is the greatest fuzzy (r,s)-semi-preopen set which is contained in A. Also, $\text{spcl}(A,r,s)=A$ for any fuzzy (r,s)-semi-preclosed set A and $\text{spint}(A,r,s)=A$ for any fuzzy (r,s)-semi-preopen set A. Moreover, we have

$$\begin{aligned}\text{int}(A,r,s) &\subseteq \text{pint}(A,r,s)\subseteq\text{spint}(A,r,s)\subseteq A\\ &\subseteq \text{spcl}(A,r,s)\subseteq\text{pcl}(A,r,s)\subseteq\text{cl}(A,r,s).\end{aligned}$$

Also, we have the following results :
(1) $\text{spcl}(\underline{0},r,s)=\underline{0}$, $\text{spcl}(\underline{1},r,s)=\underline{1}$.
(2) $\text{spcl}(A,r,s)\supseteq A$.
(3) $\text{spcl}(A\cup B,r,s)\supseteq\text{spcl}(A,r,s)\cup\text{spcl}(B,r,s)$.
(4) $\text{spcl}(\text{spcl}(A,r,s),r,s)=\text{spcl}(A,r,s)$.
(5) $\text{spint}(\underline{0},r,s)=\underline{0}$, $\text{spint}(\underline{1},r,s)=\underline{1}$.
(6) $\text{spint}(A,r,s)\subseteq A$.
(7) $\text{spint}(A\cap B,r,s)\subseteq\text{spint}(A,r,s)\cap\text{spint}(B,r,s)$.
(8) $\text{spint}(\text{spint}(A,r,s),r,s)=\text{spint}(A,r,s)$.

Definition 3.8. Let A be an intuitionistic fuzzy set and $x_{(\alpha,\beta)}$ an intuitionistic fuzzy point in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$ and $(r,s)\in I\otimes I$. Then A is called

(1) a *fuzzy (r,s)-semi-preneighborhood* of $x_{(\alpha,\beta)}$ if there is a fuzzy (r,s)-semi-preopen set B in X such that $x_{(\alpha,\beta)}\in B\subseteq A$.

(2) a *fuzzy (r,s)-quasi-semi-preneighborhood* of $x_{(\alpha,\beta)}$ if there is a fuzzy (r,s)-semi-preopen set B in X such that $x_{(\alpha,\beta)}\text{q}B\subseteq A$.

Theorem 3.9. Let A be an intuitionistic fuzzy set in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$ and $(r,s)\in I\otimes I$. Then A is fuzzy (r,s)-semi-preopen if and only if for each intuitionistic fuzzy point $x_{(\alpha,\beta)}\in A$, A is a fuzzy (r,s)-semi-preneighborhood of $x_{(\alpha,\beta)}$.

Theorem 3.10. Let A be an intuitionistic fuzzy set in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$ and $(r,s)\in I\otimes I$. Then an intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is contained in $\text{spcl}(A,r,s)$ if and only if every fuzzy (r,s)-quasi-semi-preneighborhood of $x_{(\alpha,\beta)}$ is quasi-coincident with A.

Definition 3.11. Let $f:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\mathcal{U}_1,\mathcal{U}_2)$ be a map from a SoIFTS X to a SoIFTS Y and $(r,s)\in I\otimes I$. Then f is called a *fuzzy (r,s)-semi-precontinuous* map if $f^{-1}(B)$ is a fuzzy (r,s)-semi-preopen set in X for each fuzzy (r,s)-open set B in Y.

Theorem 3.12. Let $f:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\mathcal{U}_1,\mathcal{U}_2)$ be a map from a SoIFTS X to a SoIFTS Y and $(r,s)\in I\otimes I$. Then the following statements are equivalent :

(1) f is fuzzy (r,s)-semi-precontinuous.

(2) For each fuzzy (r,s)-closed set B in Y, $f^{-1}(B)$ is a fuzzy (r,s)-semi-preclosed set in X.

(3) For every intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and every fuzzy (r,s)-open set B in Y such that $f(x_{(\alpha,\beta)})\in B$, there is a fuzzy (r,s)-semi-preopen set A in X such that $x_{(\alpha,\beta)}\in A$ and $f(A)\subseteq B$.

(4) For every intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and every fuzzy (r,s)-neighborhood B of $f(x_{(\alpha,\beta)})$, $f^{-1}(B)$ is a fuzzy (r,s)-semi-preneighborhood of $x_{(\alpha,\beta)}$.

(5) For every intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and every fuzzy (r,s)-neighborhood B of $f(x_{(\alpha,\beta)})$, there is a fuzzy (r,s)-semi-preneighborhood A of $x_{(\alpha,\beta)}$ such that $f(A) \subseteq B$.

(6) For every intuitionistic fuzzy set B in Y, $\text{spcl}(f^{-1}(B),r,s) \subseteq f^{-1}(\text{cl}(B,r,s))$.

(7) For every intuitionistic fuzzy set A in X, $f(\text{spcl}(A,r,s)) \subseteq \text{cl}(f(A),r,s)$.

(8) For every intuitionistic fuzzy set B in Y, $f^{-1}(\text{int}(B,r,s)) \subseteq \text{spint}(f^{-1}(B),r,s)$.

4. Fuzzy (r,s)-semi-preopen and fuzzy (r,s)-semi-preclosed maps

We define the notions of fuzzy (r,s)-semi-preopen and fuzzy (r,s)-semi-preclosed maps, and then we investigate some of their properties.

Definition 4.1. Let $f:(X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a map from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is called

(1) a *fuzzy (r,s)-semi-preopen* map if $f(A)$ is a fuzzy (r,s)-semi-preopen set in Y for each fuzzy (r,s)-open set A in X,

(2) a *fuzzy (r,s)-semi-preclosed* map if $f(A)$ is a fuzzy (r,s)-semi-preclosed set in Y for each fuzzy (r,s)-closed set in X.

Theorem 4.2. Let $f:(X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a map from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is a fuzzy (r,s)-semi-preopen map if and only if $f(\text{int}(A,r,s)) \subseteq \text{spint}(f(A),r,s)$ for each intuitionistic fuzzy set A in X.

Theorem 4.3. Let $f:(X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a map from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is a fuzzy (r,s)-semi-preclosed map if and only if $\text{spcl}(f(A),r,s) \subseteq f(\text{cl}(A,r,s))$ for each intuitionistic fuzzy set A in X.

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