하이브리드 광탄성법에 의한 응력확대계수 측정

Determination of Stress Intensity Factor of Cracks by Use of Hybrid Photoelasticity

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1. INTRODUCTION

According to the theories of material strength, where there is a crack in the structure part, an abrupt change in cross section occurs and the stress distribution around the crack will vary correspondingly. Even though the size of crack may be very small, it should be a concern in the design. Due to the complexity of the engineering problems, it is difficult to obtain the stress field around the crack tip directly by theoretical derivation and photoelasticity is a conventional method. But it is a kind of experimental method and can not provide very high-precision results. (1) In this paper, the hybrid method is employed to calculate full-field stress around the crack tip in uni-axially loaded finite width tensile plate and to compare with experimental and FEM results.

2. THEORETICAL FORMULATION

In the absence of body forces and rigid body motion, the stresses under plane and rectilinear isotropy can be written as (2)

$$\sigma_x = 2\,\mathrm{Re}\left[\mu_1^2 \frac{\phi'(\zeta_1)}{\omega_1'(\zeta_1)} + \mu_2^2 \frac{\psi'(\zeta_2)}{\omega_2'(\zeta_2)}\right] \tag{1a}$$

$$\sigma_y = 2\,\mathrm{Re}\left[\frac{\phi'(\zeta_1)}{\omega_1'(\zeta_1)} + \frac{\psi'(\zeta_2)}{\omega_2'(\zeta_2)}\right] \tag{1b}$$

$$\tau_{xy} = -2\,\mathrm{Re}\left[\mu_1 \frac{\phi'(\zeta_1)}{\omega_1'(\zeta_1)} + \mu_2 \frac{\psi'(\zeta_2)}{\omega_2'(\zeta_2)}\right] \tag{1c}$$

where $\phi(\zeta_1) = \sum_{k=-m}^{m} \beta_k \zeta_1^k \quad (k \neq 0)$, $\psi(\zeta_2) = \sum_{k=-m}^{m} (\bar{\beta}_k B \zeta_2^k + \beta_k C \zeta_2^k)$, $\omega'(\zeta_1) = d\omega / d\zeta_1$ $\omega'(\zeta_2) = d\omega / d\zeta_2$. Complex material parameters μ_j (j = 1,2) are the roots of the characteristic $S_{11}\mu^4 + (2S_{12} + S_{66})\mu^2 + S_{22} = 0$ for an isotropic material under plane stress and $S_{ij}(i, j = 1, 2, 6)$ are the elastic compliances. Complex quantities *B* and *C* depend on material properties.

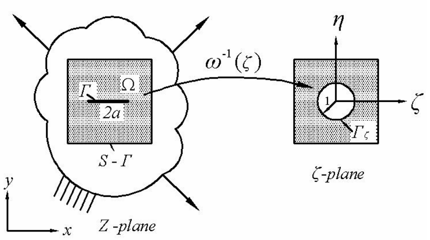


Fig. 1 Conformal mapping of a crack in the physical *z*-plane into a unit circle in the *ζ*-plane.

The inverse of the mapping function ω namely ω^{-1}, maps the geometry of interest from the physical *z*-plane into the *ζ*-plane ($\zeta_l = \xi + \mu_l \eta$). For isotropic materials, the conformal transformation from the unit circle in the ζ_l -plane to the crack in the *z*-plane of length *L* = *a/2* is shown in Fig. 1 and is given by

$$\omega_j = \frac{a}{2}(\cos\alpha + \mu_j \sin\alpha)\left(e^{-i\alpha}\zeta_j + e^{i\alpha}\zeta_j^{-1}\right) \tag{2}$$

where $i = \sqrt{-1}$. The inverse of Eq. (2) is

$$\zeta_j = \frac{e^{i\alpha}\left\{z_j \pm \sqrt{z_j^2 - a^2(\cos\alpha + \mu_j \sin\alpha)^2}\right\}}{a(\cos\alpha + \mu_j \sin\alpha)} \tag{3}$$

The branch of the square root of Eq. (3) is chosen so that $|\zeta_j| \geq 1$ ($j = 1, 2$). Combining Eqs. (1), (2), and (3) gives the following expression for the stress through regions Ω of Fig. 1. In matrix form

$$\{\sigma\} = [V]\{\beta\} \tag{4}$$

where $\{\sigma\} = \{\sigma_x, \sigma_y, \tau_{xy}\}^T$, $\{\beta\}^T = \{b_{-m}, c_{-m}, \cdots, b_m, c_m\}$ and $[V]$ is a rectangular coefficient matrix whose size depends on material properties, positions and the number of terms m of the power series expansions of Eqs. (1).

3. EXPERIMENT AND ANALYSIS

3.1 Photoelasticity experiment

In this experiment, a PSM-1 (3) plate shown as Fig.1 is subjected to the uni-axial tension $\sigma = 3.27 MPa$, the thickness of specimen is 3.175 mm, material fringe constant $f_\sigma = 7005 N/m$, Young's modulus *E*=2482 MPa, Poisson's ratio *ν*= 0.38, the width of crack is 0.5 mm.

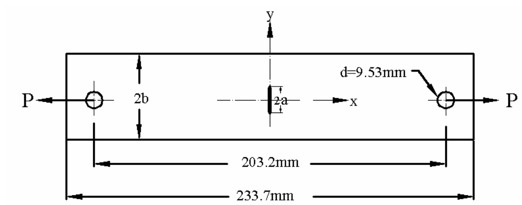


Fig. 2 Finite-width uni-axially loaded tensile plate containing a central crack.

3.2 FEM analysis

In this paper, a common FEM software ABAQUS (4) is used to discretize the specimen into the CPS4R element, a kind of bilinear plane stress quadrilateral element. The finite element model is shown as Fig. 4.

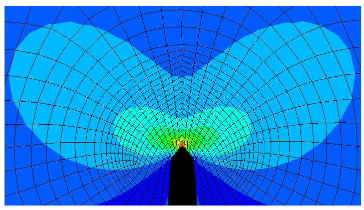
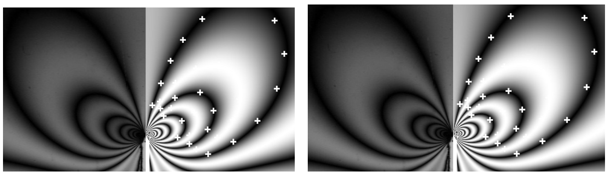


Fig. 3 The stress distribution in the vicinity of crack tip

Fig. 3 illustrates that the region with same color is subjected to the same level stress in this region. The stress varies with the difference of colors. Stress concentration can be intuitively observed in the vicinity of crack tip from Fig. 3.

3.3 Hybrid method analysis

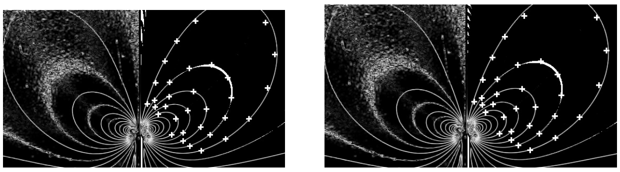
For conveniently determining the fringe orders of given points, all of them shown as the cross marks (+) in Fig .4 are obtained in the fringe loops whose fringe orders are easy to determine, such as fringe order are 1.5,1.75, 2, 2.25, 2.5 and so on. The coordinates and isochromatic fringe orders can be obtained from the photoelasticity images and used as input data of complex variable formulations to get the stress field distribution.



$m = 3$ $m = 5$

Fig. 4 Two times multiplied actual fringe pattern (left half) and reconstructed (right half) fringes for different number of terms (m) of series type stress function.

In order to accurately compare calculated fringes with experimental ones, both actual and regenerated photoelastic fringe patterns are two times multiplied and sharpened shown as Fig. 4 and Fig. 5 by digital image processing. From the figures, we can see that the regenerated fringes of m=3,5 by hybrid method are quite comparable to actual fringes [5,6].



$m = 3$ $m = 5$

Fig. 5 Fringe-sharpened lines extracted from two times multiplied actual fringe pattern (left half) and reconstructed (right half) fringes for different number of terms (m) of series type stress function.

The stress intensity factor K_I is obtained and normalized by hybrid method, FEM and theoretical formulation and shown as Fig. 6. Comparing the results with each other, we can see the error of hybrid method presented in this paper is less than 5%.

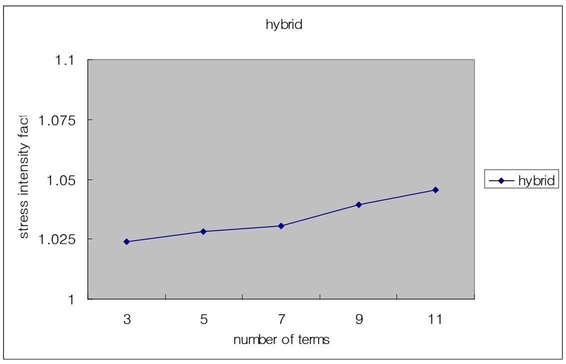


Fig. 6 The stress intensify factor varies with different number of terms (m)

4. DISCUSSION AND CONCLUSION

In this study, we use only isochromatic data with their respective coordinates. By this information we can easily obtain isochromatic fringe order and stress intensity factor of Mode I at the geometric discontinuity. Excellent results were obtained at m =3,5,7,9,11 with less than 2 per cent difference on isochromatic data and less than 5% difference on stress intensity factor from the results obtained from FEM. We can see that the technique is very effective and reliable.

Here we utilized FEM to obtain the isochromatic data and coordinate information of given points. This study will be extended to the consideration of other types of geometric orientation like circular hole, elliptical hole and/or edge crack. The use of hybrid method has a potential future and the results attained in this study can be used for bench mark test in theoretical simulation and experiment.

REFERENCE

1. Savin, G. N., "Stress Concentration Around Holes," Pergamon Press, New York, USA (1961).
2. Gerhardt, G. D., "A Hybrid/Finite Element Approach for Stress Analysis of Notched Anisotropic Materials", ASME Journal of Applied Mechanics, Vol. 51, pp. 804-810 (1984).
3. "Photoelastic Division", Measurement Group, Inc., Raleigh, NC 27611, USA.
4. ABAQUS Analysis User's Manual, ABAQUS Inc., Providence, RI 02909, USA. (2003).
5. Baek, T. H., "Digital Image Processing Technique for Photoelastic Isochromatic Fringe Sharpening", Journal of the Korean Society for Precision Engineering, Vol. 10, No. 3, pp. 220-230 (1993).
6. Baek, T. H. and Burger, C. P., "Accuracy Improvement Technique for Measuring Stress Intensity Factor in Photoelastic Experiment", KSME International Journal, Vol. 5, No. 1, pp. 22-27 (1991).