한국추진공학회 2007년도 춘계학술대회 논문집 pp.175~178　　2007 KSPE Spring Conference

나노인공위성 추진용 콜로이드 추력기 해석

박건중* · 김호영** · 송성진**

Analysis of Colloid Thrusters for Nano-satellite Propulsion

Kun Joong Park* · Ho-Young Kim** · Seung Jin Song**

ABSTRACT

The mode transition from cone-jet to dripping in colloid thruster operation has been analytically investigated. The transition has been predicted by the dynamic behavior of a liquid drop at the tip of the cone-jet. Conservation laws are applied to determine the upward motion of the drop, and an instability model of electrified jets is used to determine the jet breakup. Finally, for the first time, the analysis enables prediction of the transition in terms of the Weber number and electric Bond number. The predictions are in good agreement with experimental data.

Key Words: Colloid Thrusters(콜로이드 추력기), Nano-satellite(나노인공위성), Cone-jet(콘제트), Stability(안정성)

1. 서 론

The interest in colloid thrusters has been intensively increased in the last decade because of the emphasis on the miniaturization of spacecrafts and satellites. Specifically, colloid thrusters is known to be adequate for future space missions such as precision orbit and attitude control of nano-satellites.

Colloid thrusters are operated by using electrospraying, which refers to electrostatic or electrohydrodynamic spraying of liquids. When a liquid propellant is pumped through a capillary needle under an electric potential on the order of several kV to several tens of kV, the liquid meniscus forms a unique cone shape at the needle tip. Also, a thin jet emits from the cone apex and a spray forms. Taylor (1964) first analyzed the existence of the cone shape and validated his theory with experiments[1]. Thus, the liquid cone is referred to as the Taylor cone.

One major unresolved issue is the stability or transition of the Taylor cone-jet. Fernandez de la Mora & Loscertales (1994) investigated the transition[2], and since then it has been known that the mode change from cone-jet to dripping occurs when the nondimensional flow rate $\eta \equiv \sqrt{\rho K Q / \gamma \varepsilon \varepsilon_0}$, where ρ, K, γ, and ε are the density, electrical conductivity, surface tension coefficient, and dielectric

* 서울대학교 기계항공공학부 박사후연구원
** 서울대학교 기계항공공학부
송성진, E-mail: sjsong@snu.ac.kr

constant of the liquid, ε_0 the permittivity in vacuum, and Q the flow rate, has a value of approximately 1. However, the critical value over which the Taylor cone-jet ensues has been different depending on various parameters, and many researchers have tried to accurately predict the transition[3-6]. Nonetheless, the predictions have been difficult probably because the effects of the applied voltage and electrode geometry have not been considered.

Therefore, the objective of this research is to predict the stability of the Taylor cone-jet, or the transition from the cone-jet mode to the dripping mode, through an analytical investigation.

2. 본　　론

In Fig. 1(a), a drop is attached to a thin thread emerging from the cone. The liquid is assumed to be inviscid. In Fig. 1(b), D is the diameter of the jet. Here D is assumed to be constant along the axial direction. V_0 is the downward velocity of the jet.

2.1 Upward drop motion

In Fig. 1(b), ζ is the displacement of the drop and t the time. If at $t=0$ a drop completely detaches from the jet, the remaining part of the jet begins to recede upward towards the cone and a new drop is formed at $\zeta=0$. The mass of the drop, M, is

$$M=\rho S(\zeta+V_0 t), \tag{1}$$

where S is the cross-section area of the jet. If v is the velocity of the drop, the equation of motion of the drop can be written as

$$\frac{d}{dt}(Mv)=\pi D\gamma-\rho S V_0(v+V_0)-F_e, \tag{2}$$

where F_e represents the electric force downward. Substituting Eq. 2 into Eq. 1, then

$$\frac{d}{dt}\left[(\zeta+V_0 t)\frac{d\zeta}{dt}\right]=V_s^2-V_0\left(\frac{d\zeta}{dt}+V_0\right)-\frac{1}{\rho S}F_e, \tag{3}$$

where $V_s\equiv\sqrt{4\gamma/\rho D}$. Here the scaling of D, presented by Ganan-Calvo (1997), is

$$D=2\pi^{-2/3}\left(\frac{\rho\varepsilon_0}{\gamma K}\right)^{1/6}Q^{1/2}f_b, \tag{4}$$

where f_b is a scaling constant[7]. In addition, F_e is predicted as

$$F_e=\pi D(\zeta+V_0 t)\sigma_s E_0, \tag{5}$$

where σ_s is the surface charge density of the jet and E_0 the axial electric field acting on the drop. Then, the substitution of Eq. 5 into Eq. 3 yields an analytic solution for ζ:

$$\zeta=-\frac{g_e}{6}t^2+(V_s-V_0)t, \tag{6}$$

where "acceleration of electric force" $g_e\equiv 4\sigma_s E_0/\rho D$, termed analogous to that of gravity. Therefore, the maximum displacement of the drop,

$$\zeta_{\max}=\frac{3(V_s-V_0)^2}{2g_e}. \tag{7}$$

2.2 Jet breakup

Once the drop reaches $\zeta_{\max}$, the drop begins to move downward and detach from the jet. This jet breakup is called necking. From Basset (1894) and Taylor (1969)[8,9], the necking time τ_n can be written as

$$\tau_n = C_n\left(\frac{\rho D^3}{8\gamma}\right)^{1/2}, \qquad (8)$$

where C_n depends on the applied voltage Φ_0.

If the drop moves downward a distance l_n during the jet breakup, then

$$l_n \approx V_0 \tau_n, \qquad (9)$$

where l_n is called the necking distance.

2.3 Transition from cone-jet to dripping

If $l_n < \zeta_{\max}$, the necking point moves upward, and the cone-jet mode transitions to the dripping mode. On the other hand, if $l_n > \zeta_{\max}$, the necking point moves downward so that the jet is extended and maintained. Consequently, the transition condition is $l_n = \zeta_{\max}$, and it becomes

$$\frac{V_0}{V_s} = 1 + \frac{g_e \tau_n}{3V_s} - \left[\left(1 + \frac{g_e \tau_n}{3V_s}\right)^2 - 1\right]^{1/2}. \qquad (10)$$

Here V_0/V_s is replaced with the Weber number defined as $We \equiv \rho V_0^2 D/\gamma$, the ratio of inertia to surface tension. And the electric Bond number Bo_E is newly derived as

$$Bo_E \equiv \left(\frac{\varepsilon_0}{\gamma}\right)^{1/2} \frac{\Phi_0}{r_c^{1/2}\ln(4z_0/r_c)}, \qquad (11)$$

where r_c is the radius of the needle. Therefore, the transition condition, in terms of We and Bo_E, is

$$Bo_E = \frac{3}{2}\frac{1}{C_n C_a}\left[\frac{1}{4}\sqrt{We} + \frac{1}{\sqrt{We}} - 1\right]. \qquad (12)$$

Also, the relationship between We and η is newly given by

$$We = \frac{2\sqrt{\varepsilon}}{f_b^3}\eta. \qquad (13)$$

Thus, the transition condition, in terms of η and Bo_E, is

$$Bo_E = \frac{3}{2}\frac{1}{C_n C_a}\left[\frac{1}{4}\frac{\sqrt{2}\varepsilon^{1/4}}{f_b^{3/2}}\sqrt{\eta} + \frac{f_b^{3/2}}{\sqrt{2}\varepsilon^{1/4}}\frac{1}{\sqrt{\eta}} - 1\right]. \qquad (14)$$

3. 결과 및 토의

Figure 2 shows a comparison of the predicted and measured transition conditions[10]. The agreement is excellent. Here, if we divide the transition condition into two parts based on the critical point, with increasing η, Bo_E at the transition decreases in the left but increases in the right. This "V" shape forms because, in Eq. 14, the term involving $1/\sqrt{\eta}$ dominates the left and the term involving $\sqrt{\eta}$ the right.

In Fig. 2, at a certain point in the cone-jet mode, if Q is decreased at a fixed Φ_0, the mode transition occurs. Q corresponding to this transition is called the minimum flow rate $Q_{\min}$, and is clearly dependent on Φ_0. In our analysis, the mode transition to dripping is predicted in terms of Φ_0 as shown in Fig. 2.

4. 결 론

The transition from the Taylor cone-jet mode to the dripping mode has been analyzed, and, for the first time, the transition has been accurately predicted. Therefore, the mechanism behind this transition is concluded to be based on the dynamic behaviour of the drop at the tip of the cone-jet.

참고문헌

1. Taylor, G. I., "Disintegration of Water Drops in an Electric Field," Proc. R. Soc. London, Ser. A, Vol. 280, 1964, pp. 383-397.
2. Fernandez de la Mora, J. and Loscertales, I. G., "The Current Emitted by Highly Conducting Taylor Cones," J. Fluid Mech., Vol.260, 1994, pp. 155-184.
3. Ganan-Calvo, A. M., "The Size and Charge of Droplets in the Electrospraying of Polar Liquids in Cone-jet Mode, and the Minimum Droplet Size," J. Aerosol Sci., Vol. 25, 1994, pp. S309-S310.
4. Chen, D.-R., Pui, D. Y. H. and Kaufman, S. L., "Electrospraying of Conducting Liquids for Monodisperse Aerosol Generation in the 4 nm to 1.8 μm Diameter Range," J. Aerosol Sci., Vol. 26, 1995, pp. 963-977.
5. Chen, D.-R. and Pui, D. Y. H., "Experimental Investigation of Scaling Laws for Electrospraying: Dielectric Constant Effect," Aerosol Sci. Technol., Vol. 27, 1997, pp. 367-380.
6. Ganan-Calvo, A. M., Davila, J. and Barrero, A., "Current and Droplet Size in the Electrospraying of Liquids. Scaling Laws," J. Aerosol Sci., Vol. 28, 1997, pp. 249-275.
7. Ganan-Calvo, A. M., "Cone-jet Analytical Extension of Taylor's Electrostatic Solution and the Asymptotic Universal Scaling Laws in Electrospraying," Phys. Rev. Lett., Vol. 79, 1997, pp. 217-220.
8. Basset, A. B., "Waves and Jets in a Viscous Liquid," Am. J. Math., Vol. 16, 1894, pp. 93-110.
9. Taylor, G. I., "Electrically Driven Jets," Proc. R. Soc. London, Ser. A, Vol. 313, 1969, pp. 453-475.
10. Tang, K. and Gomez, A., "Generation by Electrospray of Monodisperse Water Droplets for Targeted Drug Delivery by Inhalation," J. Aerosol Sci., Vol. 25, 1994, pp. 1237-1249.

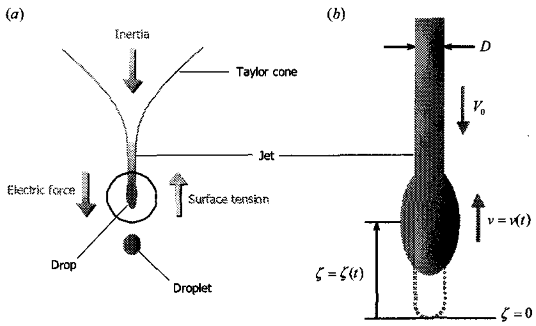

Fig. 1 A Drop at the Tip of the Taylor Cone-jet. (a) Three xternal Forces; (b) Upward Drop Motion

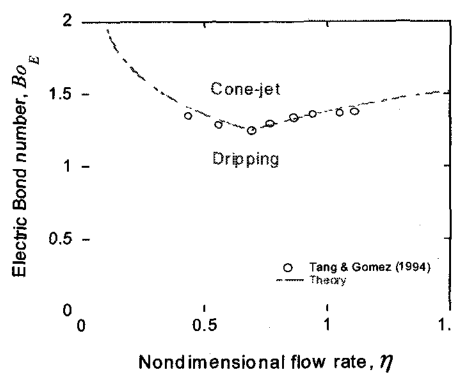

Fig. 2 Predicted and Measured Transition Conditions