저 복잡도 LPDC 부호화기

Low Complexity LDPC Encoder

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Abstract - In this paper, we will introduce an encoding algorithm of LDPC Codes in Direct-Sequence UWB systems. We evaluate the performance of the coded systems in an AWGN channel. This new algorithm is based on the Jacket matrics. Mathematically let $A = (a_{kl})$ be a matrix, if $A^{-1} = (a_{kl}^{-1})^T$, then the matrix A is a Jacket matrix. If the Jacket matrices if Low density, the inverse matrices is also Low density which is very important to the introduced encoding algorithm.

Key Words :LDPC Encoder, Jacket Matrix.UWB.

1. Introduction

Ultra Wideband UWB is a wireless technology for transmitting digital data at very high rate over a wide spectrum of frequency bands using very low power [1]. The specifications for UWB are oriented for wireless personal area network WPAN communications.

In most wireless systems, forward error correction (FEC) techniques are employed to correct the transmission errors occurring in channels. Low-Density Parity-Check (LDPC) codes were introduced by Gallager 1963 [2] and re-discovered in 1996 by MacKay and Neal [3]. Because of their capacity-achieving performance and the existence of effective decoding schemes have recently received a lot of interest for reliable high speed communication applications such as future telecommunication standards and are already part of new DVB-S2 standard [4]. Based on the Low density property, the complexity of the LDPC decoder is very low. However, the encoding problem becomes an obstacle for high-speed applications because the complexity of encoding is quadratic in the block length. In this paper we will introduce an new encoding algorithm of LDPC Codes based on the Jacket Matrices. In our design, both of the generate matrix G and The parity check matrix H are low density, so both of the encoder and decoder will have low complexity.

The paper is structured as follows. In Section two, we will introduce the preliminary of this paper. The definition of Jacket matrices will be introduced in section three. In section four, a new encoding algorithm is proposed and the complexity of the proposed algorithm is compared to the complexity of Richardson encoding algorithm [5].

2. Preliminary

Let C be a low-density parity check matrix (LDPC) code with code length N and dimension K. We write P=N-K to represent the number of parity check symbols of C. Let H be a parity check matrix of the code C, and consider to represent H as $H = [H_1 | H_2]$ where H1 and H2 are P by K and P by P submatrices, respectively. Assume that H2 is nonsingular. This assumption is equivalent to assuming that C is systematic and the first K symbols of a codeword of are information symbols. For two vectors s and p, their concatenation sgp is a correct codeword of C if and only if $H(sgp) = 0$. Therefore, the encoding can be regarded as a procedure to find the vector p which satisfies $H_1 S^T = H_2 P^T$. According to this observation, we can consider the following two-step encoding procedure.

Step 1: Compute $u^T = H_1 s^T$.

Step 2: Solve $H_2 p^T = u^T$ mod 2 with respect to p.

If C is an LDPC code, then H, H1 and H2 are all

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sparse (low-density). Therefore, by using the algorithm for the sparse matrix multiplication, the computation in the step 1 above is possible in a linear order (precisely, the complexity is proportional to the number of nonzero components in H1). On the other hand, the complexity needed to execute the step 2 is beyond linear order in general. Because usually H_2^{-1} is not low density. So the computing $p^T = H_2^{-1}u^T$ is quadratic in the block length of H_2^{-1}. In this study, we consider to use Jacket matrix for solving this problem efficiently.

3. Jacket Matrices

Let a square matrix $[J]_{m\times m} = [J_{ij}]_{m\times m}$. If its inverse matrix is obtained simply by an element-wise inverse, i.e., like $[J]^{-1}_{m\times m} = \frac{1}{C}[1/J_{ij}]^T{}_{m\times m}$, for $1 \le i, j \le m$, where C is a nonzero constant, then we call matrix $[J]_{N\times N}$ a Jacket matrix, such as

$$[J]_m = \begin{bmatrix} j_{0,0} & j_{0,1} & \cdots & j_{0,m-1} \\ j_{1,0} & j_{1,1} & \cdots & j_{1,m-1} \\ \vdots & \vdots & & \vdots \\ j_{m-1,0} & j_{m-1,1} & \cdots & j_{m-1,m-1} \end{bmatrix}, \qquad (1)$$

and its inverse is

$$[J]_m^{-1} = \frac{1}{C}\begin{bmatrix} 1/j_{0,0} & 1/j_{0,1} & \cdots & 1/j_{0,m-1} \\ 1/j_{1,0} & 1/j_{1,1} & \cdots & 1/j_{1,m-1} \\ \vdots & \vdots & & \vdots \\ 1/j_{m-1,0} & 1/j_{m-1,1} & \cdots & 1/j_{m-1,m-1} \end{bmatrix}^T, \qquad (2)$$

4. Proposed Algorithm

Because the inverse of the Jacket matrices is the element wise inverse and transpose, so the density of $[J]^{-1}_{m\times m} = \frac{1}{C}[1/J_{ij}]^T{}_{m\times m}$ is equal to the density of $[J]_{m\times m} = [J_{ij}]_{m\times m}$, low density. So if we use Jacket matrices in H2, the inverse matrices $H_2^{-1} = J^{-1}$ is low density and the complexity of $p^T = H_2^{-1}u^T$ is possible in a linear order.

Now we will discuss the complexity of encoding algorithms by means of the number of operations. Let $|M|$ denote the number of nonzero components in a matrix M, and let x and y be vectors with an appropriate length. We define The number of operations necessary for computing M*x is $|M|$, Thenumber of operations necessary for computing $T^{-1}x$ is $|T|$ where T is a triangular matrix. The number of operations necessary for computing x+y is the length of x(y).

Assumption 1: A dense matrix has almost equal number of zeros and ones.

Assumption 2: Nonzero components in H distribute "uniformly" in the matrix H except the triangular zero-part of H.

Let d be the ratio of nonzero components in H, and consider that a submatrix of which has m components contains $m \cdot d$ nonzero components.

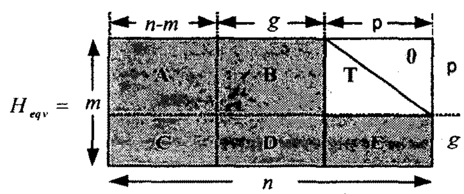
We assume $H_2 = J$ is a Jacket Matrix. J is composed of permutation matrix and J^{-1} is the element inverse and transpose of J. Element inverse of permutation matrix is still a permutation. So J and J^{-1} has the same density d.

(1) complexity of our design.

$$|H_1| + |J^{-1}|$$

Complexity $= |H_1| + p^2 \cdot d$

(2) Complexity of Richardson Encoding algorithm [5]



In the encoding method [5], first we need to make the element of the up right triangle of the matrix to be all zero by row and column permutation. So, obviously the density of H2 (B,T,D,E) will be

$$\rho = \frac{d\,gp^2}{p^2 - \frac{(p-g)^2}{2}} \qquad (3)$$

Next, we will calculate the complexity of the encoding algorithm [5] for cooperation Let us assume s is the message

$$\begin{bmatrix} I & 0 \\ ET^{-1} & I \end{bmatrix} \cdot \begin{bmatrix} A & B & T \\ C & D & E \end{bmatrix} = \begin{bmatrix} A & B & T \\ -ET^{-1}A+C & -ET^{-1}B+D & 0 \end{bmatrix}$$

Table.1 The operation and Complexity

Operation	Comment	Complexity
As^T	Multiplication by sparse matrix	$O(n)$
Bp_1^T	Multiplication by sparse matrix	$O(n)$
$[As^T]+[Bp_1^T]$	Addition	$O(n)$
$-T^{-1}[As^T+Bp_1^T]$	$-T^{-1}[As^T+Bp_1^T]=y^T$	$O(n)$

Operation	Comment	Complexity
As^T	Multiplication by sparse matrix	$O(n)$
$T^{-1}[As^T]$	$T^{-1}[As^T]=y^T \Leftrightarrow [As^T]=Ty^T$	$O(n)$
$-E[T^{-1}As^T]$	Multiplication by sparse matrix	$O(n)$
Cs^T	Multiplication by sparse matrix	$O(n)$
$[-ET^{-1}As^T]+[Cs^T]$	Addition	$O(n)$
$-\delta^{-1}[-ET^{-1}As^T+Cs^T]$	Multiplication by dense g g matrix	$O(g^2)$

$$As^T + Bp_1^T + Tp_2^T = 0 \qquad (4)$$

$$(-ET^{-1}A + C)s^T + (-ET^{-1}B + D)p_1^T = 0 \qquad (5)$$

We get the code word $[s, p_1, p_2]$ the table below show the operation and complexity of the encoding algorithm.

The approximate number of operations is shown in the Table.2 below. ϕ^{-1} is a $g \times g$ matrix and not low density, so density of ϕ^{-1} is 1/2.So the number of operations is

$$|A| + |T| + |E| + |C| + g + |\phi^{-1}| + |B| + p - g + |T|$$
$$= |H_1| + (p-g)^2 \rho / 2 + (p-g) g \rho + g + g^2/2$$
$$+ g(p-g)\rho + p - g + (p-g)^2 g/2$$
$$= |H_1| + (p-g)^2 \rho + 2(p-g) g \rho + g^2/2 + p$$
$$= |H_1| + (p-g)(p+g)\rho + g^2/2 + p$$
$$= |H_1| + p^2\rho - g^2\rho + g^2/2 + p$$
$$= |H_1| + p^2 g \frac{d\, g p^2}{\left(p^2 - \frac{(p-g)^2}{2}\right) - g^2 g \left(p^2 - \frac{(p-g)^2}{2}\right)}$$
$$+ g^2/2 + p$$
$$= |H_1| + \frac{2dp^4 - 2dp^2g^2}{2p^2 - (p-g)^2} + g^2/2 + p$$
$$= |H_1| + \frac{2dp^4 - 2dp^2g^2}{p^2 + 2pg - g^2} + g^2/2 + p$$

Since $p ?\ g$

$$\approx |H_1| + \frac{2dp^4 - 2dp^2g^2}{p^2} + g^2/2 + p$$
$$= |H_1| + 2dp^2 - 2dg^2 + g^2/2 + p$$
$$\approx |H_1| + 2dp^2 + p - 2dg^2 \qquad (6)$$

Obviously the complexity of our design is lower than the encoding algorithm [5], and the BER performance of our designed parity check matrices in a four users UWB communication systems is show in figure.1.

Table.2 The Approximate number of operations

Operation	# of Operations	Approximate of # of Operations
As^T	$\|A\|$	$p \cdot (n-m) \cdot \rho$
$T^{-1}[As^T]$	$\|T\|$	$\|T\| = (p-g)^2 \rho / 2$
$-E[T^{-1}As^T]$	$\|E\|$	$\|E\| = (p-g) g \rho$
Cs^T	$\|C\|$	$g \cdot (n-m) \cdot \rho$
$[-ET^{-1}As^T] + [Cs^T]$	g	g
$-\phi^{-1}[-ET^{-1}As^T]$	$\|\phi^{-1}\|$	$\|\phi^{-1}\| = g^2/2$
Bp_1^i	$\|B\|$	$\|B\| = g(p-g)\rho$
$\|As^T\| + \|Bp_1^T\|$	p-g	p-g
$T^{-1}\|As^T + Bp_1^T\|$	$\|T\|$	$\|T\| = (p-g)^2 \rho / 2$

Conclusion

We proposed an new encoding algorithm for LDPC Codes for UWB communication systems. The algorithm is based on the Jacket Matrices. Byusing the Jacket matrices in the parity check matrix of LDPC Codes. The complexity of encoding has been reduced greatly. We compared the complexity of our design to the Richardson's algorithm [4]. The proposed algorithm consumes smaller number of operations than Richardson's algorithm.

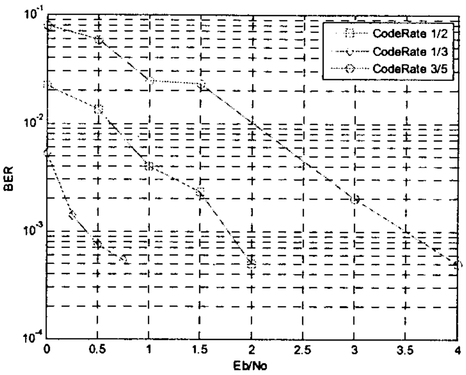


Fig.1 BER performance of our design

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