Taylor Series Discretization Method for Input-Delay Nonlinear Systems

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Abstract - Anew discretization method for the input-driven nonlinear continuous-time system with time delay is proposed. It is based on the combination of Taylor series expansion and first-order hold assumption. The mathematical structure of the new discretization scheme is explored. The performance of the proposed discretization procedure is evaluated by case studies. The results demonstrate that the proposed discretization scheme can assure the system requirements even though under a large sampling period. A comparison between first order hold and zero-order hold is simulated also.

Key Words : Nonlinear System; First Order Hold; Taylor Series; Time Delay; Time Discretization.

1. Introduction

In many physical, industrial and engineering systems, delays occur due to the finite capabilities of information processing and data transmission among various parts of the system. Typical examples of time-delay systems are communication networks, chemical processes, teleoperation systems, biosystems, underwater vehicles and so on. The presence of delays makes system analysis and control much more complicated [1][2]. Thus, new control system design methods that can solve a system with time delays are necessary [3][4].

The proposed discretization scheme is based on the Taylor-Lie series and uses a similar mathematical framework previously developed for delay-free nonlinear systems [5][6][7][8]. Many traditional approaches require a "small" time step in order to be deemed accurate, and this may not be the case in control applications where large sampling periods are inevitably introduced due to physical and technical limitations. In these large sampling period systems, Taylor series method was used to improved the performance of the controller [9]. However, in the previous paper zero-order hold (ZOH) assumption was used in the discretization method. The performance of ZOH assumption is seriously depended on the input signal and the sampling time should be short enough for a certain control precision.

A high-order method is a method that provides extra digits of accuracy with only a modest increase in computational cost [10][11]. Except the square wave and unit step input signals, ZOH assumption will no longer keep the good performance of control. Therefore, first-order hold (FOH) assumption is introduced in this paper to enhance the performance.

The paper is organized as follows,Section 2 contains some mathematical preliminaries, Sec. 3 shows the proposed method, Section 4 case studies is given, and Sec. 5 provides a few concluding remarks drawn from this study.

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2. First Order Hold for Input-Delay System

In the present study single-input nonlinear continuous-time control systems are considered with a state-space representation of the form:

$$\frac{dx(t)}{dt} = f(x(t)) + g(x(t))u(t-D) \tag{1}$$

where $x \in X \subset R^n$ is the vector of the states and an open and connected set, $u \in R$ is the input variable and D is the system's constant time-delay (dead-time) that directly affects the input. It is assumed that $f(x)$, $g(x)$ are real analytic vector fields on X.

An equidistant grid on the time axis with mesh $T = t_{k+1} - t_k > 0$ is considered, where $[t_k, t_{k+1}] = [kT, (k+1)T)$ is the sampling interval, T is the sampling period. It is also assumed that system (1) is driven by an input that is piecewise linear over the sampling interval, i.e. the first-order hold (FOH) assumption holds true.

For FOH, while $D = 0$ and $kT \le t < kT + T$,

$$\begin{aligned} u(t) &= u(kT) + \frac{u(kT) - u[(k-1)T]}{T}(t - KT) \\ &\equiv u(k) + \frac{u(k) - u(k-1)}{T}(t - kT) \end{aligned} \tag{2}$$

Furthermore, let,

$$D = qT + \gamma \tag{3}$$

where $q \in \{0,1,2,...\}$ and $0 < \gamma \le T$. Equivalently, the time-delay $D$is customarily represented as an integer multiple of the sampling period plus a fractional part of T [5][6]. Under the FOH assumption and the above notation, it is rather straightforward to verify that the "delayed"input variable attains the following values with expressions within the sampling interval, while $D \neq 0$.

Introducing functions $u_1(t)$ and $u_2(t)$, we get a compact expression, as follows,

$$u(t-D)=\begin{cases} u_1(t) & t\in[kT,kT+\gamma) \\ u_2(t) & t\in[kT+\gamma,kT+T) \end{cases} \quad (4)$$

where,

$$u_1(t)=u(k-q-1)+\frac{u(k-q-1)-u(k-q-2)}{T}[t-kT+(T-\gamma)] \quad (5)$$

$$u_2(t)=u(k-q)+\frac{u(k-q)-u(k-q-1)}{T}[t-kT-\gamma] \quad (6)$$

3. Taylor Series Based Discretization Method

It is now feasible to extend the aforementioned Taylor discretization method to nonlinear continuous-time systems with a constant time-delay ($D\neq 0$) in the input.

3.1 Linear Control Systems With Time-Delay

In order to motivate the development of the proposed discretization procedure and draw the appropriate analogies from the field of linear systems, let us first begin the exposition of the paper's main results by briefly reviewing the ones available in the case of linear systems [12, 13],

$$\frac{dx(t)}{dt}=Ax(t)+bu(t-D) \quad (7)$$

where A, b are constant matrices of appropriate dimensions. It is know that for any time interval $I=[t_i,t_f)$, the following formula holds true,

$$x(t_f)=e^{A(t_f-t_i)}x(t_i)+\int_{t_i}^{t_f}e^{A(t_f-\tau)}bu(\tau)d\tau \quad (8)$$

As shown in (4), under FOH assumption, the input variable expressions are different within the two subintervals $[kT,kT+\gamma)$ and $[kT+\gamma,kT+T)$. Successively applying formula (8), we readily obtain,

$$x(kT+\gamma)=e^{A\gamma}x(kT)+\int_{kT}^{kT+\gamma}e^{A(kT+\gamma-\tau)}bu_1(\tau)d\tau \quad (9)$$

and

$$x(kT+T)=e^{A(T-\gamma)}x(kT+\gamma)+\int_{kT+\gamma}^{kT+T}e^{A(kT+T-\tau)}bu_2(\tau)d\tau \quad (10)$$

In light of equation (9) and (10) yields,

$$\begin{aligned} x(kT+T)&=e^{A(T-\gamma)}e^{A\gamma}x(kT)+\int_{kT+\gamma}^{kT+T}e^{A(kT+T-\tau)}bu_2(\tau)d\tau \\ &+e^{A(T-\gamma)}\int_{kT}^{kT+\gamma}e^{A(kT+\gamma-\tau)}bu_1(\tau)d\tau \\ &=e^{AT}x(kT)+\int_0^{T-\gamma}e^{A(T-\gamma-\tau)}bu_2(kT+\gamma+\tau)d\tau \\ &+\int_{T-\gamma}^{T}e^{A(2T-\gamma-\tau)}bu_1((k-1)T+\gamma+\tau)d\tau \end{aligned} \quad (11)$$

Notice, that the value of the state vector at $(k+1)T$ is defined by the states evaluated at kT and the two subinterval expressions, which can be obtained by the time-delay D and equation (4).

3.2 Nonlinear Control Systems With Time-Delay

Motivated by the linear approach described in section 3.1, a similar line of thinking is adopted for the nonlinear case as well. Indeed, by applying the Taylor series discretization method for nonlinear systems presented before to the $[kT,kT+\gamma)$ subinterval one immediately obtains the state vector evaluated at $kT+\gamma$,

$$x(kT+\gamma)=\Phi_\gamma(x(kT),u_1(kT)) \quad (12)$$

where the map Φ_γ can be derived through a direct application of formula (13)and the subsequent calculation of the corresponding Taylor coefficients can be realized through the recursive formulas (14).

$$\Phi_T(x(k),u(k))=x(k)+\sum_{l=1}^{\infty}A^{[l]}(x(k),u(k))\frac{T^l}{l!} \quad (13)$$

where, $A^{[l]}(x,u)$ are determined recursively by,

$$A^{[1]}(x,u)=f(x)+ug(x)$$

$$A^{[l+1]}(x,u)=\frac{\partial A^{[l]}(x,u)}{\partial x}(f(x)+ug(x)) \quad (14)$$

$x(kT)$ and $u_1(kT)$ are the instantaneous state vector and input value respectively at time kT. Furthermore, it can be derived from (4) that,

$$u_1(kT)=u(k-q-1)+\frac{u(k-q-1)-u(k-q-2)}{T}(T-\gamma) \quad (15)$$

Similarly,

$$x(kT+T)=\Phi_{T-\gamma}(x(kT+\gamma),u_2(kT+\gamma)) \quad (16)$$

and,

$$u_2(kT+\gamma)=u(k-q) \quad (17)$$

Based on (14), the above equation (12) and (16) can be rewritten as follows,

$$x(kT+\gamma)=x(kT)+\sum_{l=1}^{\infty}A^{[l]}(x(kT),u_1(kT))\frac{\gamma^l}{l!} \quad (18)$$

$$x(kT+T)=x(kT+\gamma)+\sum_{l=1}^{\infty}A^{[l]}(x(kT+\gamma),u_2(kT+\gamma))\frac{(T-\gamma)^l}{l!} \quad (19)$$

And furthermore, according to (13), the approximate sampled-data representation (ASDR) of equation (18) and (19) are resulted from a truncation of the Taylor series order N, as shown below,

$$\begin{aligned} x(kT+\gamma)&=\Phi_\gamma^N(x(kT),u_1(kT)) \\ &=x(kT)+\sum_{l=1}^{N}A^{[l]}(x(kT),u_1(kT))\frac{\gamma^l}{l!} \end{aligned} \quad (20)$$

$$\begin{aligned} x(kT+T)&=\Phi_{T-\gamma}^N(x(kT+\gamma),u_2(kT+\gamma))=x(kT+\gamma) \\ &+\sum_{l=1}^{N}A^{[l]}(x(kT+\gamma),u_2(kT+\gamma))\frac{(T-\gamma)^l}{l!} \end{aligned} \quad (21)$$

It should be emphasized, that the functional representation of the $A^{[l]}$ coefficients of the map $\Phi_{T-\gamma}$ remains exactly the same subpart as for the subinterval $[kT,kT+\gamma)$, and it is only need to reuse the same part with the aid of a symbolic software package such as MAPLE.

For the consecutive subintervals, combing equations (12) and (16), the desired sampled-data representation of the original system (1) is obtained,

$$\begin{aligned} x(kT+T)&=\Phi_T^D(x(kT),u_1(kT),u_2(kT+\gamma)) \\ &=\Phi_{T-\gamma}(\Phi_\gamma(x(kT),u_1(kT)),u_2(kT+\gamma)) \end{aligned} \quad (22)$$

Notice, that a finite series truncation order N for the above series would naturally produce an ASDR,

$$x(kT+T)=\Phi_T^{N,D}(x(kT),u_1(kT),u_2(kT+\gamma)) \quad (23)$$

4. Case Studies

Different sampling periods and different input signals are introducedin the simulation under two different input signals. The partial derivative terms involved in the Taylor series expansion are determined recursively by Maple. At the same time, the Matlab ODE solver is used to obtain the exact solution.

A simple chemical process system isconsidered which is exactly the same system in [7][8]. The system can be described as follow,

$$\frac{dx}{dt} = f(x) + g(x)u$$
$$= -(1+2a)x + au - ux - ax^2 . \qquad (24)$$

In the simulation, $a = 1$ is used. The initial system state is assumed that $x(0) = 0$.

Within the sampling interval, the solution of (24) is obtained using uniformly convergent Taylor series. According to the methodology described in earlier sections, the sampled-data representation of the system is shown as (20) and (21).

In this system, $f(x) = -(1+2a)x - ax^2$,

$$g(x) = (a - x) \qquad (25)$$

So that, the partial derivative terms $A^{[l]}(x,u)$ are determined recursively by (14).

4.1 A Slope Input

The following slopeinput is applied to the system,

$$u(t-D) = 0.9(t-D) \qquad (26)$$

We select the truncated order $N = 3$ for Taylor series method for a sampling $T = 0.01$ and the input time-delay $D = 0.005$.

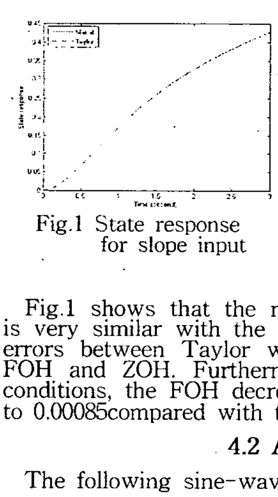

Fig.1 State response for slope input

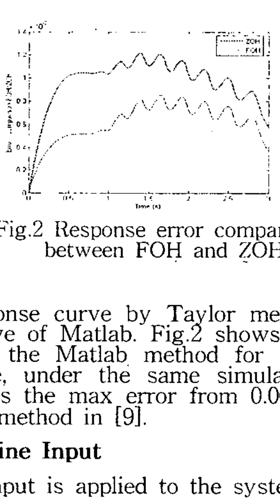

Fig.2 Response error comparison between FOH and ZOH.

Fig.1 shows that the response curve by Taylor method is very similar with the curve of Matlab. Fig.2 shows the errors between Taylor with the Matlab method for both FOH and ZOH. Furthermore, under the same simulating conditions, the FOH decreases the max error from 0.00122 to 0.00085compared with the method in [9].

4.2 A Sine Input

The following sine-wave input is applied to the system,

$$u(t-D) = 0.9\sin(2\pi(t-D)) \qquad (27)$$

We select the truncated order $N = 3$ for Taylor series method for a sampling $T = 0.01$ and the input time-delay $D = 0.015$.

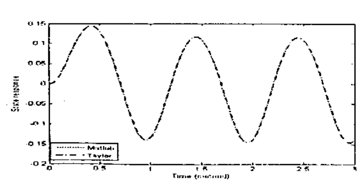

Fig.3 State responsefor sine input

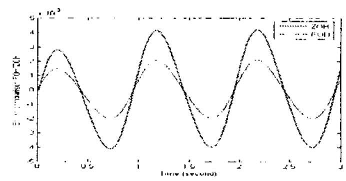

Fig.2 Response error comparison between FOH and ZOH.

Fig.1 shows that the accuracy of the proposed Taylor method is enough. In this fig.2, the FOH combined with Taylor series decreases the max errors from 0.0042 to 0.0021 compared with ZOH in [9].

5. Conclusions

A first-order hold assumption for Taylor series descretizationmethod is proposed for nonlinear control system with input time-delay.

The performance of the proposed time-discretization procedure is evaluated using case studies with two different input signals. In these cases, even when the sampling time is large with input time-delay, Taylor series combined with FOH can reach the accuracy requirement of the systems.

At the same time, the results show that FOH is much better than ZOH method for the two input signals. Further comparison for the FOH and ZOH will be the subject of future publications.

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