Worst-case optimal feedback control policy for a remote electrical drive system with time-delay

Worst-case optimal feedback control policy for a remote electrical drive system with time-delay

고유, 장정, 이창구, 정길도
GaoYu ,Zheng Zhang, Chang Goo Lee, Kil To Chong

Abstract - This paper considers an optimal control problem for a remote control to an electrical drive system with a DC motor. Since it is a linear control system with time-delay subject to unknown but bounded disturbance, we construct a worst-case feedback control policy. This policy can guarantee that, for all admissible uncertain disturbances, the real system state should be in a prescribed neighborhood of a desired value, and the cost functional takes the best guarantee value.

The worst-case feedback control policy is allowed to be corrected at one correction point between the initial to the final time, which is equivalent to solving a 1-level min-max problem. Since the min-max problem at the stage does not yield a simple analytical solution, we consider an approximate control policy, which is equivalent and can be solved explicitly in the numerical experiments.

Key Words : remote control, Electrical Drive System, worst-case feedback, approximate, time-delay

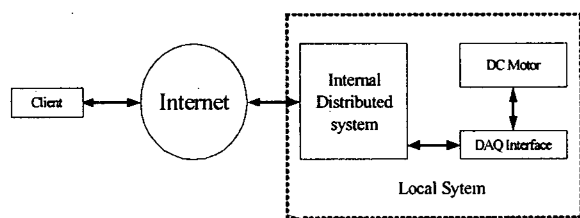
1. Introduction

Optimal control deals with the problem of finding a control law for a given system such that a certain optimality criterion is achieved. The optimal control problem for linear systems with delays is still open, depending on the delay type, specific system equations, criterion, etc. [1] ,[2].

This paper concentrates on the solution of the optimal control problem for an Internet-based remote control to a DC motor. The time-delay is caused by the internet traffic and policy construction. And in the drive system model, the load torque is uncertain but can be bounded in the practice. So we use a worst-case optimal control policy to solve the dynamic system with uncertainties, instead of the classical optimal control method [3].

2. System structure and Model of the DC motor

In previous years, the Internet provides great potential for the high-level control of process plants. It enabled engineer, who is situated in geographically diverse locations, to monitor and adjust. Figure.1 illustrates the general structure of the Internet-base remote control system [6].



Internet time-delay is an essential issue which must be considered in the design of the Internet-based control system. We assume that the total time-delay of a control action per cycle is h and it can be considered as a kind of input time-delay.

Now, we consider the mathematical model of the DC motor, which is described by two linear differential equations:

$$U = R_a i + L_a \frac{di}{dt} + C_e \omega \quad (1)$$
$$C_m = J_r \frac{d\omega}{dt} + \rho\omega + m$$

Where U is the voltage applied to the rotor circuit, i is the current, ω is the rotation speed, m is the resistant torque reduced to the motor shaft, R_a and L_a are the resistance and the inductance of the circuit respectively, J_r the inertia moment referred to the motor shaft, C_e , C_m , are the constants of the motor and ρ is the coefficient of viscous friction.

Add the time-delay of input and above equations can be written in the form of the state equation

$$\dot{z}(t) = Az(t) + bu(t-h) + gw(t) \quad (2)$$

where

저자 소개
* 고유 : 全北大學 電子情報工學科 碩士課程
** 장정 : 全北大學 電子情報學科 博士
*** 이창구 : 全北大學 電子情報工學科 教授・工博
**** 정길도 : 全北大學 電子情報工學科 教授・工博

$$\dot{z}(t)=\begin{bmatrix}\omega(t)\\ i(t)\end{bmatrix}, u(t)=U(t), w(t)=m(t), \quad (3)$$

are the state vector, control variable and the disturbance vector, respectively. The matrices in (2) are

$$A=\begin{bmatrix}-\rho/J_r & C_m/J_r\\ -C_r/L_a & -R_a/L_a\end{bmatrix}, b=\begin{bmatrix}0\\ 1/L_a\end{bmatrix}, g=\begin{bmatrix}-1/J_r\\ 0\end{bmatrix}. \quad (4)$$

3.The worst-case optimal control policy

Now we have got the differential equation form

$$\begin{aligned}&\dot{z}(t)=Az(t)+bu(t-h)+gw(t),\\ &z(0)=z_0; u(0)=u_0*(t), t\in[-h,0],\\ &rank(b,Ab,\dots,A^n b)=n, rank=(g,Ag,\dots,A^n g)=n,\end{aligned} \quad (5)$$

the initial system state $z(0)$ and initial control $u_0*(t), t\in[-h,0]$ are given, $w(t)$ is an unknown in an advance disturbance from a bounded set, which will be defined later.

Follow the closed-loop worst-case formulation, we construct the control policy

$$\pi=\{u_1(\cdot|z_0,u_0(\cdot)), u_2(\cdot|z_1,u_1(\cdot))\} \quad (6)$$

at each control interval $T_1=[t_0,t_1]=[0,h]$; $T_2=[t_1,\overline{t_2}]$; note that $\overline{t_2}=t_*-h$.

The trajectory $z(t)$ should satisfies terminal condition

$$\| z(t_*|\pi,w(\cdot))-x_*\|_2^2\le \delta_*^2 \quad (7)$$

The optimal guarantee value of the cost functional is

$$J^0=\min_{\pi}\max_{w(\cdot)}\left(\int_0^{t_1}u_1^2(t|z_0,u_0(\cdot))dt+\int_{t_1}^{\overline{t_2}}u_2^2(t|z_1,u_1(\cdot))dt\right) \quad (8)$$

Then, let us define the admissible disturbance

$$\Omega=\left\{w(\cdot):\int_0^{t_1}w^2(t)dt\le v_1;\int_{t_1}^{t_*}w^2(t)dt\le v_*\right\} \quad (9)$$

Considering the (7), for the close-loop worst-case optimal-feedback control policy, we can see that the relationship will became

$$\delta_*^2\ge v_*\lambda_{\max}(Q_*) \quad (10)$$

which is necessary for (6) satisfying (7).

$$Q_*=\int_{t_1}^{t_*}F(t_*,t)g(F(t_*,t)g)^T dt, \quad (11)$$

$F(t,\tau)$ is the fundamental solution matrix of the system $\dot{x}(t)=Ax(t)$

To simplify the notation, we assume that the equality always holds in (10)

$$\delta_*^2=v_*\lambda_{\max}(Q_*) \quad (12)$$

To derive the worst-case optimal policy, we distinguish two systems, in first interval $t\in[t_0,t_1]$

The real system subject to a disturbance

$$\dot{z}(t)=Az(t)+bu(t-h)+gw(t), z(t_0)=z_0; t\in[t_0;t_1] \quad (13)$$

The other is nominal system without disturbance

$$\dot{x}(t)=Ax(t)+bu(t-h), x(t_0)=z_0, t\in[t_0;t_1] \quad (14)$$

Lemma 1 The follow relationship happens(which is proved in [4])

$$\| z_1-x_1\|_{Q^{-1}}^2\le v_1 \quad (15)$$

Then, apply the two systems in the second interval $t\in[t_1,t_*]$. Based on the information, we may determine the state $x_2=x(t_2)$ of the nominal system (14) at the moment t_2 and $x_3=x(t_3)$ at final time:

$$x_2=F(t_2,t_1)z_1+\int_{t_1}^{t_2}F(t_2,t)bu_1(t-h)dt, \quad (16)$$

$$x_3=F(t_*,t_2)x_2+\int_{t_2}^{t_*}F(t_*,t)bu(t-h)dt \quad (17)$$

According to Cauchy's formula, terminal state real state $z_3=z(t_*)$ of real system (5) can be expressed:

$$\begin{aligned}z_3&=F(t_*,t_2)z_2+\int_{t_2}^{t_*}F(t_*,t)bu_1(t-h)dt+\int_{t_2}^{t_*}F(t_*,t)gw(t)dt\\ &=x_3+\int_{t_1}^{t_*}F(t_*,t)gw(t)dt\end{aligned} \quad (18)$$

If the assumption that equation (12) holds true, the terminal real system state $z(t_*)$ will satisfy the condition (7) only that follow equation takes place

$$x_3=x_* \quad (19)$$

From [5], the cost functional (8) may be rewritten in the form

$$J_1(x_1)=\min_{\phi_1}\max_{z_1}(\phi_1^T G\phi_1+\| x_*-F_*(Fz_1+G\phi_1)\|_{G_*^{-1}}^2), \quad (20)$$

$$G=\int_{t_0}^{t_1}F(t_1,t)b(F(t_1,t)b)^T dt,\ G_*=\int_{t_1}^{\overline{t_2}}F(\overline{t_2},t)b(F(\overline{t_2},t)b)^T dt$$

$$\phi_1=\phi_1(x_1)=G^{-1}(x_2-Fx_1).$$

$$\text{s.t.}\ \| z_1-x_1\|_{Q^{-1}}^2\le v_1$$

Let $\phi_1^0=\phi_1^0(x_1)$ be a solution of the problem (20). Then, the optimal control law

$$\begin{aligned}&u_1^0(t|x_1)=\phi_1^{0T}(x_1)F(t_1,t)b, t\in[t_0,t_1],\\ &u_2^0(t|x_2)=(x_*-F_*x_2)^T G_*^{-1}F(\overline{t_2},t)b, t\in[t_1,\overline{t_2}],\end{aligned} \quad (21)$$

The problem (20) is a 1-level min-max problem for the decision variables $\phi_1\in R^n, z_1\in R^n$. In the simulation section, we consider an approximation of the problem (21), which is equivalent and easily computed.

4. Simulation and experiment result

We consider the DC motor with the following nominal parameters:

$U_{initial}$=110 V, L_a=0.16H, C_r=0.58 Vs/rad, C_m=0.58Nm/s, J_r=0.028Nms^2/rad, ρ=0.01Nms/rad. The desired angle speed is ω_d=50rad/s and desired current is i_d=5A, which means that the given final state is: $x_*=\begin{bmatrix}50\\ 5\end{bmatrix}$.

The policies were tested on the set of correction points: $t_1=h=0.07s, t_2=0.14s$, $t_*=0.2s, \overline{t_2}=t_*-h=0.13s$.

The corresponding set of admissible disturbance Ω is defined through (9) where

$$|w(t)|\le\alpha, t\in[0,t_*],$$

$$v_1 = \alpha^2 h, v_* = \alpha^2 (t_* - h), \alpha = 3/4$$

We get the guarantee value of the cost functional from (20), which is a 1-level min-max problem. Now, we use an approximation method to (20), which is a convex mathematical programming problem with only one decision variable. From [5], the approximate value $I_1^0 = J_1^0$.

Disturbances $w(\cdot)$	$J(\widetilde{\pi^0}, w(\cdot))$
α	153.5349
$\alpha/2$	149.4615
$\alpha\lvert\cos(\pi \cdot t/h)\rvert$	150.3159
$\alpha\lvert\sin(\pi \cdot t/h)\rvert$	150.2916
$\alpha\lvert\cos[(\pi \cdot t/h)^2]\rvert$	150.6801
$\alpha\lvert\sin[(\pi \cdot t/h)^2]\rvert$	149.8328
$\alpha\lvert\cos(\pi \cdot \sqrt{t}/h)\rvert$	150.0314
$\alpha\lvert\sin(\pi \cdot \sqrt{t}/h)\rvert$	150.6168

Table.1

Table.1 shows the value of cost functional for approximate optimal policies applied to the dynamic system (5) on admissible disturbance. We get the guarantee value of the cost functional $I^0 = 153.5997$. Note that there is no admissible disturbance with $J(\widetilde{\pi^0}, w(\cdot)) > I^0$.

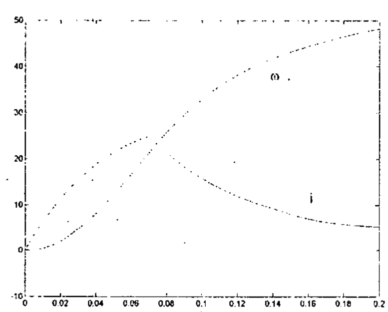


Figure.1

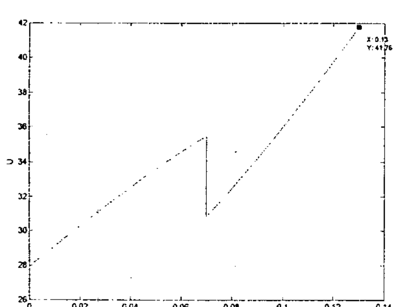


Figure.2

Figure.1 show the motor speed ω and current i on admissible disturbance $w(\cdot) = \alpha\lvert\cos(\pi \cdot t/h)\rvert, t \in [0, t_*]$. As the result, the final nominal state $x(t_*) = [50\ \ 5]^T = [\omega_d; i_d]$ (19), and the final real state $z(t_*) = [48.1834\ \ 5.2330]^T$. From rule (12), $\delta_*^2 = v_* \lambda_{max}(Q_*) = 9.2498$. Note that at the final time $\| z(t_* | \pi, w(\cdot)) - x_* \|_2^2 = 3.3544 < \delta_*^2$. The corresponding optimal control variable of the input voltage in the interval $[0, \overline{t_2}]$ is indicated in Figure.2.

5. Conclusion

This paper indicates a possibility to obtain optimal control policy of an electrical drive system, which has a constant input time-delay and subject to unknown but bounded disturbance. The result obtained by numerical simulation show that the value of cost functional on any admissible disturbance were guaranteed by the approximate value. And the trajectory of the motor's angle velocity ω and current i, at the final real time, was also in the guarantee neighborhood. The worst-case optimal policy is proved to be a adapt control method for terminal linear dynamic system with delay subject to unknown, but bounded disturbance.

참 고 문 헌

[1] Michael Basin Jesus Rodriguez-Gonzalez. "A closed-form optimal control for linear system with equal state and input delays" Automatica,Volume 41, Issue 5,May 2005,Pages 915-920

[2] Guo-Ping Cai Jin-Zhi Huang Simon X. Yang. "Anoptimal control method for linear systemswith time delay" Computers & Structures,Volume 81, Issue 15,July 2003,Pages 1539-1546

[3] Bahrin, V.; Haba, C.G.; Cojocaru-Filipiuc, C, " Power Electronics and Drive Systems, 2003. PEDS 2003. The Fifth International Conference on Volume 2,17-20 Nov. 2003 Page(s):1619 - 1623 Vol.2"

[4] Kostina EA, Kostyukova OI. "Robust optimal feedback for terminal linear-quadratic control problems under disturbances." Mathematical Programming 2006.

[5] Kil To Chong, Olga Kostyukova, Mariya Kurdina. "Worst-case control policy for linear-quadratic control problems with delay"

[6] S. H. Yang, X. Chen and J. L. Alty. "Design issues and implementation of internet-based process control systems" Control Engineering Practice,11, Issue 6,e 2003,709-720