1차 지연시간 모델의 펄스응답기반 식별방식에 대한 강인성 해석

Robustness Analysis of Pulse Response based Identification Methods for First-Order Plus Time-Delay Model

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Abstract – A new approach on identifying a first-order plus time-delay (FOPTD) model using finite-duration pulse inputs has been presented recently [1]. The identification methods are very simple because it is enough to observe only two extremes and the time when they occur in the transient response to pulse input. However, when there is mismatch between actual system and FOPTD model, how sensitive the methods are has not been studied. In this paper, we investigate robustness issue of those identification algorithms in the presence of the model structure mismatch and uncertainties. Through an example we will demonstrate it.

Key Words : Robust analysis, Identification, FOPTD, Pulse Response, Model mismatch.

1. Introduction

There are many identification methods to obtain a FOPTD model which is very commonly used in chemical engineering and industrial applications [2-6]. Recently, new methods identifying parameters of a FOPTD using four different pulse inputs have been proposed [1]. The method provides exact analytical expressions for the steady-state gain, time constant and time delay of a FOPTD system. The approach uses only two relative extrema in transient response to finite-duration pulses of four different shapes so that can be simply implemented.

There are many cases that one cannot characterize real process by FOPTD. In this paper, our concern is to investigate how sensitive four methods developed in [1] are with respect to the order mismatch between actual process and FOPTD model. For the assessment of its accuracy, identification errors in both the time domain and the frequency domain are considered. Through an example, we will show the comparisons of four identification algorithms.

2. Preliminary Results of Pulse Response based Identification methods

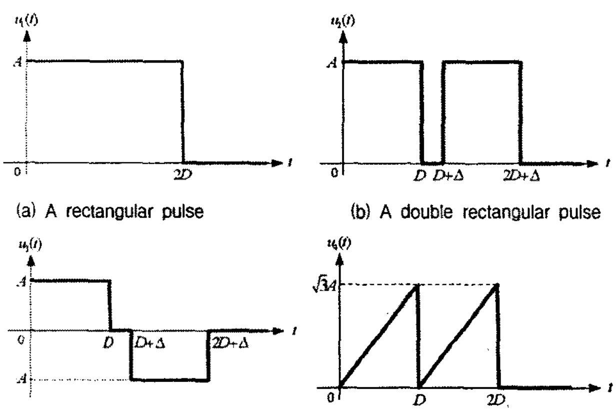
In this section, we give a brief summary of four identification algorithms of FOPTD based on pulse responses developed in [1].

Consider a FOPTD system

$$G(s)=\frac{Y(s)}{U(s)}=\frac{K}{1+Ts}e^{-Ls} \qquad (1)$$

where $K\neq 0$, $L\geq 0$, and $T>0$ respectively.

Most popular test input signals include pulse, pseudo random binary sequence, step, ramp and sinusoidal functions[6]. In [1], four different finite-duration pulse inputs shown in Fig. 1 are employed.



(a) A rectangular pulse (b) A double rectangular pulse

(c) A delayed rectangular doublet pulse (d) A double saw-tooth pulse

Fig. 1. Different finite-duration pulse inputs with the same energy.

Assumed that all pulses in Fig. 1 have the same total signal energy. Namely,

$$\int_0^{\infty} f_i^2(t)dt=2A^2D, \quad i=1,2,3,4. \qquad (2)$$

The inputs in Fig.1 satisfying (2) can be expressed as

$$u_1(t)=A[H(t)-H(t-2D)], \qquad (3)$$

$$u_2(t)=A[H(t)-H(t-D)] + A[H(t-D-\Delta)-H(t-2D-\Delta)], \qquad (4)$$

$$u_3(t)=A[H(t)-H(t-D)] - A[H(t-D-\Delta)-H(t-2D-\Delta)], \qquad (5)$$

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$$u_4(t) = \frac{\sqrt{3}A}{D}\left[tH(t) - (t-2D)H(t-2D)\right] - \sqrt{3}A\left[H(t-D) + H(t-2D)\right] \tag{6}$$

where $A \neq 0$, $D > 0$, and $\Delta > 0$ denote the pulse amplitude, pulse width and interval between two separate pulses respectively. And $H(t)$ is the unit step function.

Fig. 2 illustrates four typical pulse responses with respect to the inputs (3)-(6) for $AK > 0$.

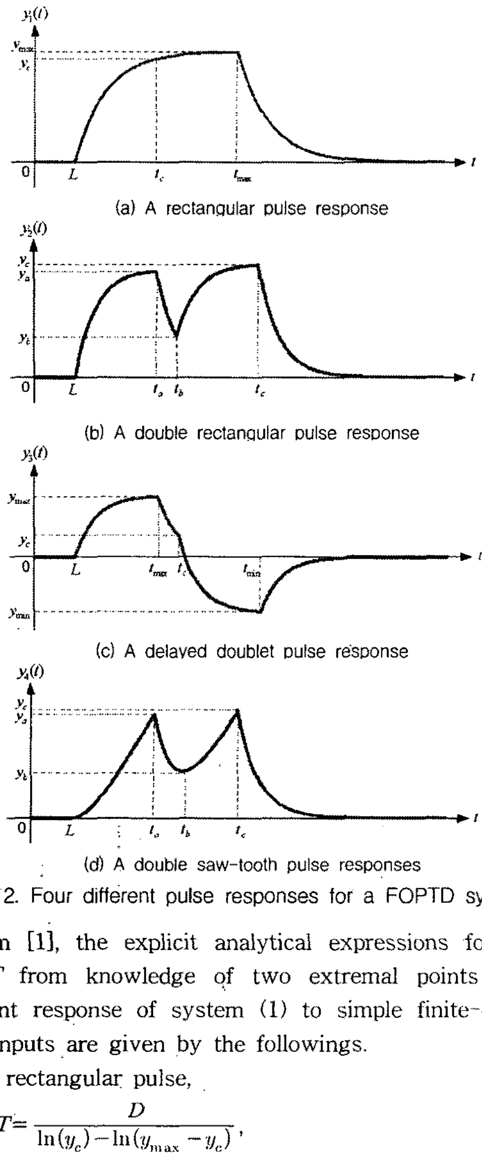

Fig. 2. Four different pulse responses for a FOPTD system.

From [1], the explicit analytical expressions for K, L and T from knowledge of two extremal points in the transient response of system (1) to simple finite-duration pulse inputs are given by the followings.

a) For rectangular pulse,

$$\text{(i)}\ T = \frac{D}{\ln(y_c) - \ln(y_{max} - y_c)}, \tag{7}$$

$$\text{(ii)}\ K = \frac{y_c^2}{A(2y_c - y_{max})}, \tag{8}$$

$$\text{(iii)}\ L = t_{max} - 2D. \tag{9}$$

b) For double rectangular pulse,

$$\text{(i)}\ T = \frac{\Delta}{\ln(y_a) - \ln(y_b)} = \frac{D+\Delta}{\ln(y_a) - \ln(y_c - y_a)}, \tag{10}$$

$$\text{(ii)}\ K = \frac{y_a}{A\left[1 - \left(\frac{y_b}{y_a}\right)^{\frac{D}{\Delta}}\right]} = \frac{y_a}{A\left[1 - \left(\frac{y_c - y_a}{y_a}\right)^{\frac{D}{D+\Delta}}\right]}, \tag{11}$$

$$\text{(iii)}\ L = t_a - D = t_b - D - \Delta = t_c - 2D - \Delta. \tag{12}$$

c) For delayed doublet pulse,

$$\text{(i)}\ T = \frac{D+\Delta}{\ln(y_{max}) - \ln(y_{min} + y_{max})} = \frac{\Delta}{\ln(y_{max}) - \ln(y_c)},\ if\ \Delta \neq 0 \tag{13}$$

$$\text{(ii)}\ K = \frac{y_{max}}{A\left[1 - \left(\frac{y_{min} + y_{max}}{y_{max}}\right)^{\frac{D}{D+\Delta}}\right]} = \frac{y_{max}}{A\left[1 - \left(\frac{y_c}{y_{max}}\right)^{\frac{D}{\Delta}}\right]}\ if\ \Delta \neq 0 \tag{14}$$

$$\text{(iii)}\ L = t_{max} - D = t_{min} - 2D - \Delta = t_c - D - \Delta. \tag{15}$$

d) For double saw-tooth pulse,

$$\text{(i)}\ T = \frac{D}{\ln(y_a) - \ln(y_c - y_a)}, \tag{16}$$

$$\text{(ii)}\ K = \frac{y_a^2\left[\ln(y_a) - \ln(y_c - y_a)\right]}{\sqrt{3}A\{(y_c - 2y_a) + y_a\left[\ln(y_a) - \ln(y_c - y_a)\right]\}}, \tag{17}$$

$$\text{(iii)}\ L = t_a - D = t_c - 2D. \tag{18}$$

3. Robustness Analysis

From the viewpoint of practical implementation, the simple identification methods in [1] may have significant estimation errors when the response data is measured with noise and the actual process is of higher order. In this section, the accuracy of estimated model against mismatched model order will be investigated. The assessment of accuracy is performed in both the time domain and the frequency domain. For the time domain accuracy, the integrated squared error (ISE) and integrated absolute error (IAE) to the step responses of the estimated model and real process are employed.

$$ISE := \int_0^{t_s} e(t)^2 dt, \tag{19}$$

$$IAE := \int_0^{t_s} |e(t)| dt. \tag{20}$$

It is well known that the good fitness of identified model in time domain does not always guarantee the good matching in frequency response. Let the estimated models of (1) be $\hat{G}(s)$. The frequency domain estimation error can be measured by the following worst case error [6].

$$E := \max_{\omega \in [0, \omega_c]}\left\{\left|\frac{\hat{G}(j\omega) - G(j\omega)}{G(j\omega)}\right| \times 100\%\right\} \tag{21}$$

where ω_c is the frequence such that $\angle G(j\omega_c) = -\pi$.

Now let us examine the robustness of four identification methods by using a numerical example.

Example: Consider a real process of third-order,

$$G(s) = \frac{50}{(s+1)(s+5)(s+10)}e^{-s}. \tag{22}$$

Then we have $K = 1$ and $L = 1$.

The parameters defining the different pulse inputs are assumed to be $A = 1$, $D = 2$ and $\Delta = 0.1$ respectively.

To show the robustness of four methods to uncertainty, the zero mean white noise with variance $\sigma_n = 0.01K$ is introduced into the process output.

Therefore, the measured output of the system be

$$y(t) = y_r(t) + y_n(t). \tag{23}$$

where $y_r(t)$ and $y_n(t)$ represent the pulse responses of the real process and noise.

Using the identification algorithms in section 2, we have obtained four methods shown in Table 1.

Table 1. The identified model parameters, *ISE*, *IAE* and *E* of four identification algorithms.

	$\hat{K}$	$\hat{T}$	$\hat{L}$	*ISE*	*IAE*	*E*
rectangular pulse	1.025	1.230	1.036	0.129	2.271	2.46%
double rectangular pulse	1.016	1.173	1.137	0.034	1.265	1.57%
delayed doublet pulse	1.017	1.169	1.123	0.053	1.469	1.74%
double saw-tooth pulse	0.966	1.212	1.166	0.165	3.678	3.41%

Fig. 3 shows the step responses of real process with noise and identified model to four different pulse inputs. Due to the difference of the order between the actual system and estimated models, the estimation errors cannot be avoided.

The step responses of the actual process and four estimated models are compared in Fig. 4.

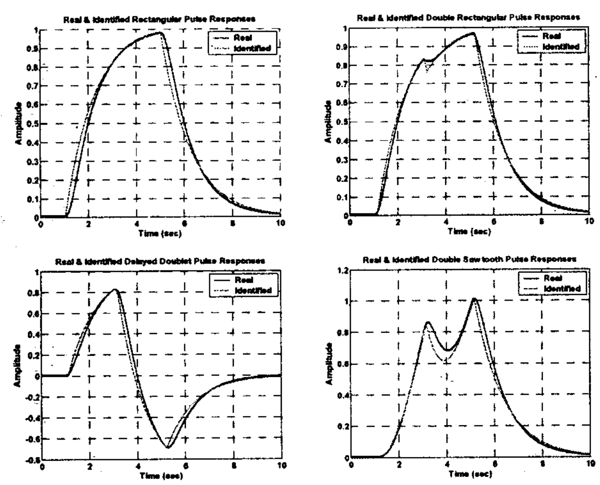

Fig. 3. Pulse responses for real process and identified model.

In Table 1, we also present the corresponding *ISE*, *IAE* and *E* by computing (19)-(21).

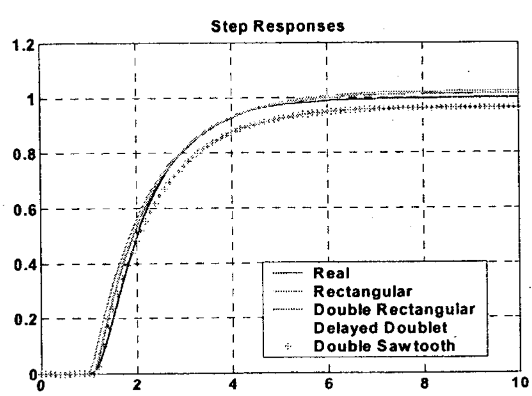

Fig. 4. Step responses for real process and identified models.

As a result, we conclude that the double rectangular pulse response based identification gives rise to the best accuracy and followed by the delayed doublet pulse and rectangular pulse methods. The double saw-tooth pulse results in the worst accuracy relatively.

4. Conclusion

Recently, a new identification approach of FOPTD using finite-duration pulse responses has been proposed. Since the method requires only two extremal values and their occurrence time in transient pulse responses, it is so simple to estimate such a FOPTD. In this paper, we have investigated the robustness of the methods against model mismatch. For time domain accuracy, the *ISE* and *IAE* have been used, while for frequency domain accuracy, the worst case error defined by the maximum value of frequency response error between actual and estimated models over a specific frequency range was applied. As a result of numerical analysis, we conclude that the double rectangular pulse method results in the best accuracy and followed by the delayed doublet pulse, the rectangular pulse, and the double saw-tooth pulse respectively.

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References

[1] B.A. León de la Barra, L.H. Jin, Y.C. Kim, and M. Mossberg, "Identification of First-Order Time-Delay Systems using Four Different Finite-Duration Pulse Inputs," *Submitted for the 46th IEEE Conference on Decision and Control*, 2007.

[2] K.J. Åström and T. Hägglund, Advanced PID Control, *ISA*, pp.49-51, 2006.

[3] L. Ljung, "Identification for control: simple process models," *Proceedings of the 41st IEEE Conference on Decision and Control*, vol.4, pp.4652-4657, 2002

[4] S.H. Hwang and S.T. Lai, "Use of two-stage least-squares algorithms for identification of continuous systems with time delay based on pulse responses," *Automatica*, vol.40, no.9, pp.1561-1568, 2004.

[5] P. Banerjee and S.L. Shah, "Robust Stability of GPC as Applied to a First order Model with Delay," *Proceedings of the 1995 American Control Conference*, vol.4, pp.2884-2889, 1995.

[6] Q.G. Wang and Y. Zhang, "Robust identification of continuous systems with dead-time from step responses," *Automatica*, vol.37, pp.377-390, 2001.