비계수 모델에 대한 PID 안정화기 전체 셋의 수치 해

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Numerical Solution of All Stabilizing PID Controllers for Non-Parametric Model

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Abstract - This paper presents a numerical solution of PID controller design. The complete set of PID controllers stabilizing an LTI plant that solely determined from the knowledge of the frequency response and the number of unstable poles of the plant. An illustrative example is given to describe implementation of the design algorithm and exhibit graphical displays of the feasible regions of the design parameters.

1. Introduction

It is always desirable to use a fixed-structure, low-order controller (e.g. PID or first-order controller) due to its simplicity and effectiveness in industrial applications. They can be designed by loop shaping, from the frequency response data, or a mathematical model (transfer function or steady-state model) of the plant to be controlled. Some results are developed to characterize the complete set of PID controllers for a given plant transfer function[1-4]. However, the analytical model is not easy to obtain in many practical situations without identification of the system.

Recently, a new design approach for PID controllers that stabilizes an LTI plant and achieves several performance specifications has been introduced[5]. In this method, knowledge of the frequency response and the number of RHP poles of the plant is solely available for design. However, it is hard to implement the design algorithm in software based on the numeric computation. This paper provides the numerical solution for such a control system design and show an effective visualization of the results, the feasible regions of the controller parameters, with 2-D and 3-D graphics.

2. PID Controller Design for Non-Parametric Model

Consider the feedback control system given by an LTI plant and a PID controller as shown in Fig. 1.

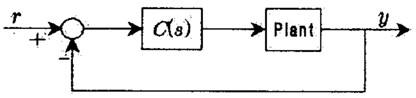

Fig. 1. A unity feedback system

Let the controller be

$$C(s)=\frac{K_d s^2+K_p s+K_i}{s}. \quad (1)$$

We first make some assumptions:

1. The plant has n poles and m zeros, but no poles or zeros in $j\omega$ axis.
2. The only information available for design is
 i) knowledge of the frequency response magnitude and phase, i.e.,
 $$G(j\omega)=|G(\omega)|e^{j\phi(\omega)}, \quad \text{for} \quad \omega\in[0,\infty). \quad (2)$$
 ii) knowledge of the number of the RHP poles p_r.

2.1 PID controller design algorithm

All stabilizing set of PID controllers can be computed by the following procedure.

Step 1: Determine the relative degree $n-m$ and the number of RHP zeros z_r from (a) and (b) respectively.

(a) The slope of the bode magnitude diagram in high frequency is $-(n-m)\times 20dB/decade$.

(b) The net phase change of $G(j\omega)$ is

$$\triangle_0^{\infty}(\phi)=-\frac{\pi}{2}\left[(n-m)-2(p_r-z_r)\right]. \quad (3)$$

Step 2: Pick up a fixed $K_p=K_p^*$ and solve

$$K_p^*=-\frac{\cos\phi(\omega)}{|G(\omega)|}. \quad (4)$$

Let the solution be $0<\omega_1<\omega_2<\cdots<\omega_{l-1}$ which denote the distinct finite frequencies. Also define $\omega_0=0$, $\omega_l=\infty$.

Step 3: Determine the admissible strings of integers $i_t\in(-1,0,1)$ for $t=0,1,\cdots,l$ and $j\in\{-1,1\}$ so that

$$\sigma=\begin{cases}\left[i_0+2\sum_{r=1}^{l-1}(-1)^r i_r+(-1)^l i_l\right]\cdot(-1)^{l-1}j, & n-m=even\\ \left[i_0+2\sum_{r=1}^{l-1}(-1)^r i_r\right]\cdot(-1)^{l-1}j, & n-m=\text{odd}\end{cases} \quad (5)$$

where $\sigma=n-m+2z_{r+1}$.

Step 4: Solve for the stabilizing (K_i, K_d) set,

$$\left[K_i-K_d\omega_t^2+\frac{\omega_t\sin\phi(\omega_t)}{|G(\omega)|}\right]\cdot i_t>0, \quad \text{for } t=0,1,\cdots,l. \quad (6)$$

Step 5: Repeat the step 2-4 by updating K_p over the pre-determined range.

The detailed derivation of the above result is referred to [5].

2.2 Description of the Matlab based implementation

Solving engineering and industrial problems by using computer algorithm is coming reality nowadays. Thus, we explain how we have practically implemented the PID controller design algorithm with Matlab software. The design frame is shown in Fig. 2.

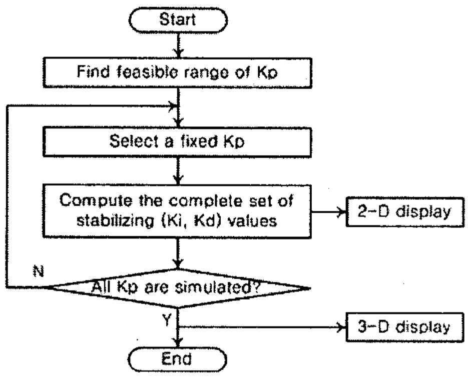

Fig. 2. A stabilizing PID controller design flow

Since the PID controller parameter K_p and (K_i, K_d) can be computed independently as in (4) and (6), it requires to sweep one of them K_p. And the feasible range of K_p is determined by the required signature for stability condition based on the generalization of the Hermite-Biehler Theorem[1]. However, the interval of K_p is theocratically not bounded and opens to infinity in some cases. Thus, it is necessary to give a limitation of the frequency for sweeping and provide the number of the K_ps to be simulated. To get the stabilizing (K_i, K_d) set, the numerical ranges of K_i, K_d gains for computing also need to be given previously.

The Matlab scripts for finding the complete set of stabilizing PID gains are given as follows.

```
>>[sigma, nm, zr]=GetSigma(Mag, Phase, W, pr);
>>[KpRange, mag, phase, w]=GetFeasibleKp(Mag, Phase, W, MaxOmega, sigma, nm);
>>Kpr=User2KpRange(KpRange);
>>KpMin=Kpr(1); KpMax=Kpr(2); KpNo=Kpr(3); Kp=linspace(KpMin, KpMax, KpNo);
```

```
>>for Index=1:KpNo
      kp=Kp(Index);
      [pmag, theta, wi]=GetWi(mag, phase, w, sigma, nm, kp, MaxOmega);
      A=GetStringSet(sigma, nm, phase, wi);
      [ki, kd]=Kp2KiKd(pmag, theta, wi, nm, kp, MaxKi, MaxKd, A);
  end
```

Herein, the input parameters are the frequency(W) and its response magnitude(Mag), phase(Phase), and the number of the RHP poles(pr) of the plant. And the maximum value(MaxOmega) of the frequency for sweeping and the limitation of the K_i, K_d values(MaxKi, MaxKd) are also given for simulation. The output is the design PID gain(kp, ki, kd).

Finally, the result of the stabilizing region in (K_i, K_d) plane for a fixed value K_p is depicted as 2-D graphics. And the entire stabilizing PID gains for the selected number of K_ps in the feasible range are described to be a 3-D visualization. It is important to note that the 3-D graphic is composed by a set of convex polygonal slices.

3. Numerical Example

In this section, we give a numerical example to show the PID controller design procedure.

Example 1: Consider a stable plant whose bode diagram is given in Fig. 2.

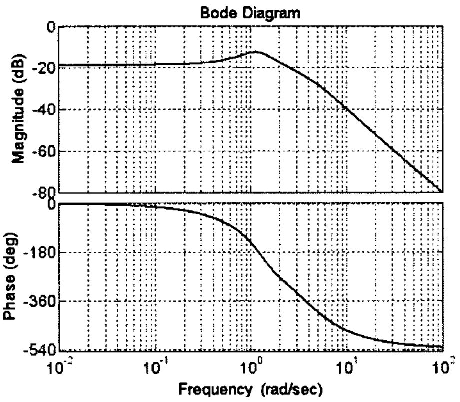

Fig. 2. Bode diagram of the plant

Since the plant is stable, the number of RHP poles is $p_r = 0$. Step 1 provides the knowledge of the plant and thus the feasible range of the sweeping parameter is given in Fig. 3. In this example, the feasible range is found to be $K_p \in (-8.497, 4.233)$.

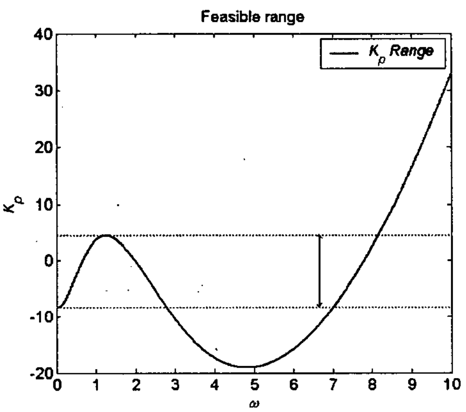

Fig. 3. Feasible range of K_p

When a specific value of $K_p = 1$ is fixed, we obtain the admissible region of (K_i, K_d) values by performing step 2-4. The 2-D region is shown in Fig. 4. And step 5 provides the complete set of stabilizing PID controllers. The controller gains are depicted as a set of convex polygon slices in Fig. 5.

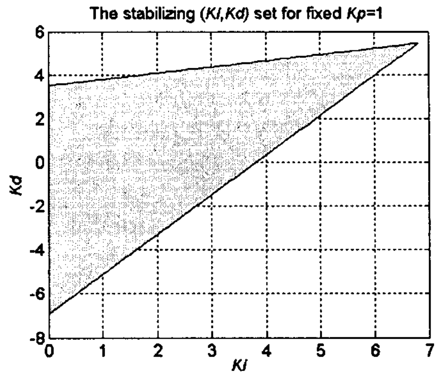

Fig. 4. The stabilizing (K_i, K_d) set for fixed $K_p = 1$

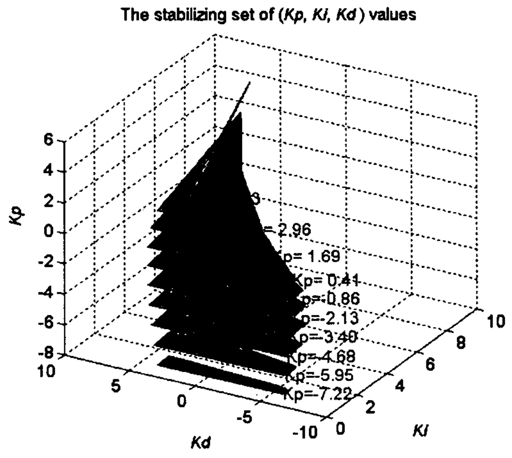

Fig. 5. All stabilizing set of (K_p, K_i, K_d) values

4. Concluding Remarks

In this paper, a new design methodology of computing the complete set of stabilizing PID controllers for non-parametric model has been implemented. The proposed design procedure dose not need any mathematical plant model, but solely require the knowledge of the frequency response and non-minimum phase poles of the plant. The obtained all stabilizing set of PID controllers are illustrated by the 2-D and 3-D graphics so as to be useful for computer-aided design. The extension to the implementation of multi-objective design for several performance requirements, such as gain and phase margin, and H_∞ margin, is now underway.

Acknowledgement

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