Latin Hypercube Sampling Experiment와 Multiquadric Radial Basis Function을 이용한 최적화 알고리즘에 대한 연구

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Study on a Robust Optimization Algorithm Using Latin Hypercube Sampling Experiment and Multiquadric Radial Basis Function

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Abstract - This paper presents a "window-zoom-out" optimization strategy with relatively fewer sampling data. In this method, an optimal Latin hypercube sampling experiment based on multi-objective Pareto optimization is developed to obtain the sampling data. The response surface method with multiquadric radial basis function combined with $(1+\lambda)$ evolution strategy is used to find the global optimal point. The proposed method is verified with numerical experiments.

1. Introduction

For the optimization of electromagnetic devices, one of most popular techniques is to construct a surrogate model in design space [1], which is sufficiently accurate to replace the original code. In general, if the number of sampling data points is sufficient, the global optimal point can be obtained at one time. However, having sufficient sampling data requires huge computational efforts when the objective function value is involved with finite element analysis.

In this paper, a "window-zoom-out" optimization strategy with relatively fewer sampling data is presented. To insert the sampling data adaptively in successive "zoom-out" design space and improve the quality of distribution of sampling data, an optimal Latin Hypercube Sampling (LHS) strategy based on multi-objective Pareto optimization is developed. The response surface method (RSM) with multiquadric radial basis function combined with $(1+\lambda)$ evolution strategy is used to find the global optimal point. Numerical tests are applied to validate the performance and reliability of the proposed algorithm.

2. Optimization Strategy

The proposed optimization strategy described as "window-zoom-out" is summarized in Fig.1. At the initial iteration stage, the response surface is constructed at relatively small number of sampling points in the whole design space and $(1+\lambda)$ evolution strategy is started to find the optimal point, hereinafter referred to "pseudo-optimal point". Once the pseudo-optimal point is found, it is used as a central point for the definition of "zoom-out" design space, in which the additional sampling points are inserted by means of LHS experiment. Then, the response surface is constructed again and the procedure is iterated until the pseudo-optimal point found at two successive iterations does not change significantly. At that time, the pseudo-optimal point can be considered as the true global optimal point.

2.1 Initial Latin Hypercube Sampling Using Pareto Optimization

Latin hypercube sampling is a type of "space filling" experiment design strategies [2]. A Latin hypercube sampling is an n × k matrix where n is the number of sampling points data and k is the number of design variables. Each variable is divided into n intervals on the basis of equal probability and each of the k columns is a random permutation of {1, ⋯, n} which can be mapped onto the actual range of the variables. In practice, a LHS can be randomly generated, but a randomly selected LHS may have bad properties and act poorly in estimation and prediction, as shown in Fig. 2(b). An approach is to use optimal LHS designs according to some criteria to find especially good sampling data points.

A) Minimax Criterion

If design D_{MI} is a minimax criterion design, D_{MI} is defined as follows:

$$\min_{D}\max_{x} d(x, D) = \max_{x} d(x, D_{MI}) \quad (1)$$

$$d(x, D) = \min_{x_0 \in D} d(x, x_0) \quad (2)$$

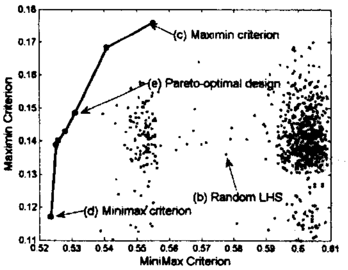
Where, $d(\cdot,\cdot)$ is a distance between any two points, D is a random LHS design, and x is any point in design space. Minimax criterion ensures that all sampling points are not too far from a design point.

B) Maximin Criterion

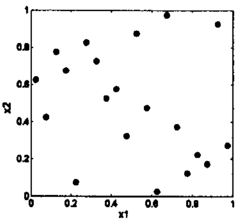
Design D_{MA} is a maximin criterion design if

Step 1. Generate initial sampling data using LHS in the whole design variable space.
Step 2. Construct the response surface with an optimal shape parameter and find a pseudo-optimal point using $(1+\lambda)$ evolution strategy.
Step 3. Check whether the pseudo-optimal point is convergent. If converged, stop and otherwise, go to next step.
Step 4. Generate additional sampling data within the reduced design space using the LHSmethod at the neighborhood of current pseudo-optimalpoint, and go to Step 2.

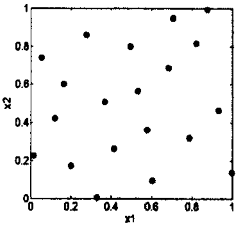
Fig. 1. Summary of the optimization strategy.



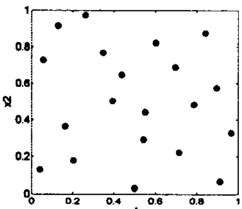
(a) Pareto front in the fitness space



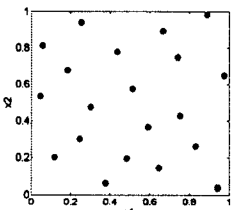
(b) Random LHS.



(c) Maximin criterion



(d) Minimax criterion



(e) Pareto-optimization

Fig. 2. Latin hypercube sampling design of size n=20 with two de sign variables.

$$\max_{D} \min_{x_1, x_2 \in D} d(x_1, x_2) = \min_{x_1, x_2 \in D_{MM}} d(x_1, x_2) \tag{3}$$

Maximin criterion designs ensure that no two sampling points are too close to each other.

C) Pareto-optimization

In the minimax criterion designs, it often happens that the distance between two sampling points is obviously closer than that between others, as shown in Fig. 2(d). On the contrary, the maximin criterion designs always try to increase the distance between two sampling points so that some sampling points locate unexpectedly on the boundary of the design space, as shown in Fig. 2(c). In order to consider the two design criterions simultaneously, a multi-objective Pareto optimization is presented to select good LHS design towards the Pareto front rather than towards an absolute minimum or maximum. In our case, the final Pareto curve represents the solutions from considering maximin criterion to considering minimax criterion, as shown in Fig. 2(a). This allows us to choose an acceptable tradeoff between the two goals by picking a point somewhere along the Pareto front. Fig. 2(e) is selected as a good LHS design with the property of more uniform distribution.

2.2 Adaptively Inserted LHS Data Based on Gaussian distribution

Based on Gaussian distribution function, by making the mean value μ correspond to the current pseudo-optimal point, we can make the inserted

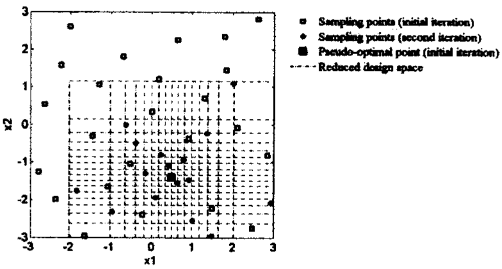


Fig. 3. The insertion of the sampling points using LHS design.

sampling data distributed around the pseudo-optimal point, and by modifying the standard deviation σ, we can control the degree of closeness of the new sampling data to pseudo-optimal point. A user selectable flag can be used to control the size of the "window-zoom-out" design space. The use of the selectable flag and σ increases the robustness of the method, because, especially during the first iterations, the procedure is able to scan the design space broadly, without remaining trapped in narrow regions. At this stage, the Pareto optimization is also applied to generate additional sampling data. Fig. 3 shows the distribution of sampling points both the initial stage and the second iteration.

2.3 RSM with Multiquadric Radial Basis Function

In the global interpolation strategy of the objective function, the RSM using multiquadric radial basis functions is one of the most impressive from the viewpoint of its smoothness and fitting ability with a limited number of sampling points in the design space. With given sampling data, the response surface is constructed as follows [3]:

$$S(\mathbf{x}) = \sum_{i=1}^{N} \beta_i \left(\| \mathbf{x} - \mathbf{x}_i \|^2 + h^2 \right)^{1/2} \tag{4}$$

$$X = \{ (\mathbf{x}_i, f(\mathbf{x}_i)), i = 1, 2, \cdots, N \} \tag{5}$$

where $\| \cdot \|$ is Euclidean distance, $\mathbf{x}$ is the design parameter vector, β_i is the coefficient corresponding to the i-th sampling point $\mathbf{x}_i$, X is the set of sampling data of size N, and h is the shape parameter.

The shape parameter, h, has an effect on the smoothness and accuracy of the interpolation. In the present work, we employ (1+λ) evolution strategy to find the optimal h that minimizes the interpolation error of response surface, described as the "leave-one-out" method [3]

3. Numerical Experiment

In order to illustrate graphically the proposed procedure, a test case with an objective function depending on two design variables is presented. The objective function and constraints are given as:

$$\text{Minimize } F(\mathbf{x}) = 3(1-x_1)^2 e^{[-x_1^2-(x_2+1)^2]} \tag{6}$$

$$-10\left(\frac{x_1}{5} - x_1^3 - x_2^5\right) e^{-x_1^2-x_2^2} - \frac{1}{3} e^{-(x_1+1)^2-x_2^2}$$

$$\text{Subject to } -3.0 \le x_i \le 3.0 \tag{7}$$

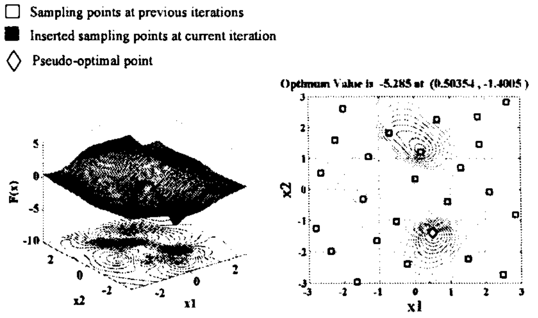
The global optimal point is located at (0.2282, -1.6256) with the corresponding function value -6.5511[4]. Fig. 4 shows a comparison of the response surfaces with the pseudo-optimal points during the iterations for optimization. At the initial iteration, a response surface is constructed by using 25 sampling points generated in the whole design space using the LHS design. By applying the suggested optimization algorithm, a pseudo-optimal point $(x_1, x_2) = (0.5035, -1.400)$ is obtained, as shown in Fig. 4(a).

At the second iteration, 16 additional sampling points are inserted at the neighborhood of the previously obtained pseudo-optimal point in the reduced design space by a factor of 0.85. By repeating the iterations for optimization, the pseudo-optimal point moves to $(x_1, x_2) = (0.2432, -1.6299)$ at second iteration, $(x_1, x_2) = (0.2292, -1.6278)$ at the third iteration with 11 additional sampling points, and finally $(x_1, x_2) = (0.2278, -1.6254)$ at fourth iteration with 6 additional sampling points, as shown in Fig. 4. Fig. 5 shows the convergency of the objective function value during the iterations for optimization. It can be seen that the pseudo-optimal point can be converged to the true global optimal point as the number of inserted sampling points and the size of "window-zoom-out" design space reduce gradually.

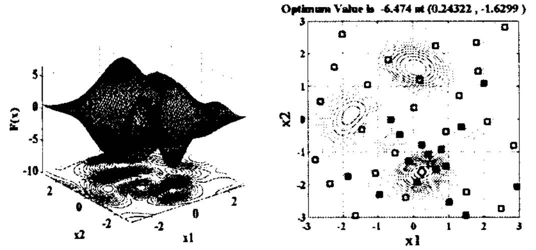
4. Conclusion

A global optimization algorithm is suggested by combining a type of "space filling" experiment designs and response surface function approximation. An optimal LHS design based on multi-objective Pareto optimization is developed to sample the design space uniformly. A "window-zoom-out" optimization strategy with relatively fewer sampling data is presented. The RSM with multiquadric radial basis function combined with $(1+\lambda)$ evolution strategy is used to find the global optimal point. The numerical results from the test example reveal that the global optimal point can be found accurately.

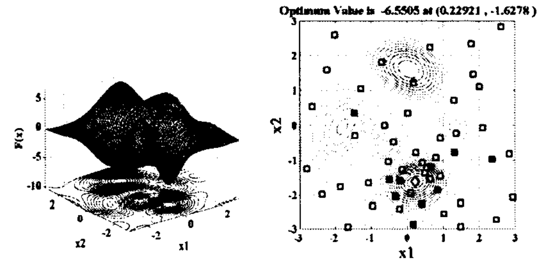
Further activity is under the way to show the ability of significantly reducing the computation cost of finite element analysis for the optimization design of the electromagnetic devices in comparison with other optimization strategies.



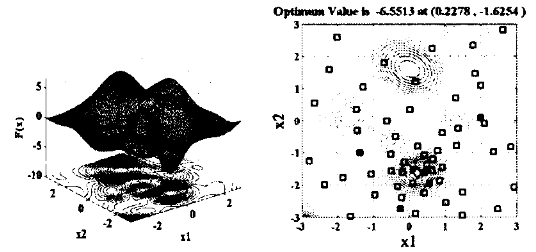
(a) initial response surface with 25 sampling data



(b) response surface with 41 sampling data

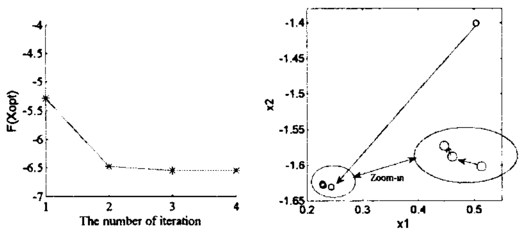


(c) response surface with 52 sampling data



(d) response surface with 58 sampling data

Fig. 4. Response surfaces and movement of the pseudo-optimal point.



(a) (b)

Fig. 5. (a) The convergency of the objective function value during the iterations for optimization. Xopt means pseudo-optimal point. (b) Trajectory of Xopt.

[Reference]

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