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# STABILITY REGION ESTIMATES FOR THE SDRE CONTROLLED ATTITUDE SYSTEMS IN SATELLITE FORMATION FLYING

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## ABSTRACT

The present work is to estimate the stability region of the State-Dependent Riccati Equation (SDRE) controlled system, which is used for a decentralized coordinated attitude control in satellite formation flying. In this research, currently emerging methods which estimate region of attraction for the SDRE controllers are introduced and the methods are applied to attitude control systems. The results guarantee the stability of the given decentralized coordinated attitude control system in satellite formation flying.

*Keywords*: stability region, SDRE, attitude control, decentralized control, formation flying

## 1. INTRODUCTION

The State-Dependent Riccati Equation (SDRE) control method is one of the nonlinear control methods which is introduced by James Cloutier (1997). Unlike other nonlinear control methods, it is easy to implement and systematic so that it has been applied to a large class of nonlinear systems. Despite these advantages of the SDRE controllers, global asymptotic stability is not guaranteed. Moreover, stability analysis is complicated because expression of the closed-loop form for the SDRE controlled system is typically not known. Researchers have tried to find the method to find the region of attraction for the SDRE controllers. In the current research, the methods of finding the stability region for the SDRE controllers are introduced, and the methods are applied to the decentralized coordinated attitude control system (Chang et al. 2007). An appropriate method is chosen to estimate the stability region of the control system.

## 2. COMPARISONS OF THE METHODS

Erdem & Alleyne (1999) found the analytic solution of the stability region estimation for the SDRE controllers. In order to use this method, however, the SDRE controlled system should have the two dimensional dynamic model matrix . If some system has higher order dimension, the method cannot be used.

Erdem & Alleyne (2002) suggested other method in order to satisfy the higher order dynamic system by using vector norms. By determining the overvaluing matrix for the dynamic system, the method shows the maximum boundary for the SDRE-controlled dynamic systems. However, this

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method is cumbersome from the computational standpoint for medium-high order systems (Bracci et al. 2006) such as the attitude control dynamic model (Chang et al. 2007).

Seiler (2003) found the method to estimate the stability region by turning the stability problem into a semidefinite programing problem (Parrilo 2002). By using the sum of square method, maximum values are found that satisfies the Lapunov stability condition. However, this method is optimized to the second order dynamic systems. If the dynamic model matrix is higher than three, it makes tremendous variables to find the maximum values for the Lyapunov function. The Windows based computer may not solve the problem even in third order dynamic system because of the memory problem.

Another method for estimating the region of attraction for the SDRE- controlled systems is suggested by Bracci et al. (2006). This procedure is an alternative to that proposed by Erdem & Alleyne (2002). In the next section, this method will be applied to estimate the stability region for the SDRE-controlled system whose objective is the decentralized coordinated attitude control in the satellite formation flying (Chang et al. 2007).

## 3. ESTIMATION OF THE STABILITY REGION

### 3.1 Bracci's Method

In the current paper, Bracci's method (2006) is applied to the estimation of the stability region for the decentralized attitude control problem (Chang et al. 2007). The procedure of the method is as follows:

Given the autonomous system $\dot{\mathbf{x}} = f(\mathbf{x})$ with $\mathbf{x} \in \mathbb{R}^n$. Suppose that the origin is an equilibrium point, then linearize the system in the neighborhood of the origin $(\mathbf{x}_0)$ obtaining

$$\mathbf{A} = \left.\frac{\partial f}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_0} \tag{1}$$

If $\mathbf{A}$ is Hurwitz, the Lyapunov function, $V(\mathbf{x})$, can be obtained by solving the Lyapunov equation which is defined as Eq. (2):

$$\mathbf{A}^{\mathbf{T}}\mathbf{M} + \mathbf{PA} = -\mathbf{Q} \tag{2}$$

$$V(\mathbf{x}) = \mathbf{x}^{\mathbf{T}}\mathbf{M}\mathbf{x} \tag{3}$$

where $\mathbf{M}$ is the solution of the Lyapunov equation Then the time derivative of the Lyapunov function can be expressed as follows:

$$\dot{V}(\mathbf{x}) = 2\mathbf{x}^{\mathbf{T}}\mathbf{M}f(\mathbf{x}). \tag{4}$$

The area satisfying the condition that (3) is positive and (4) is negative is belonging to the region of attraction accordingly to the Lyapunov theorem for local stability.

### 3.2 Application of Bracci's Method to the Attitude Control Problem

In this section, Bracci's method is applied to find the region of attraction for the SDRE controlled attitude system which was derived by Chang et al. (2007). The dynamic system has two different controllers; one is for absolute attitude motion and another is for relative attitude motion among other satellites.

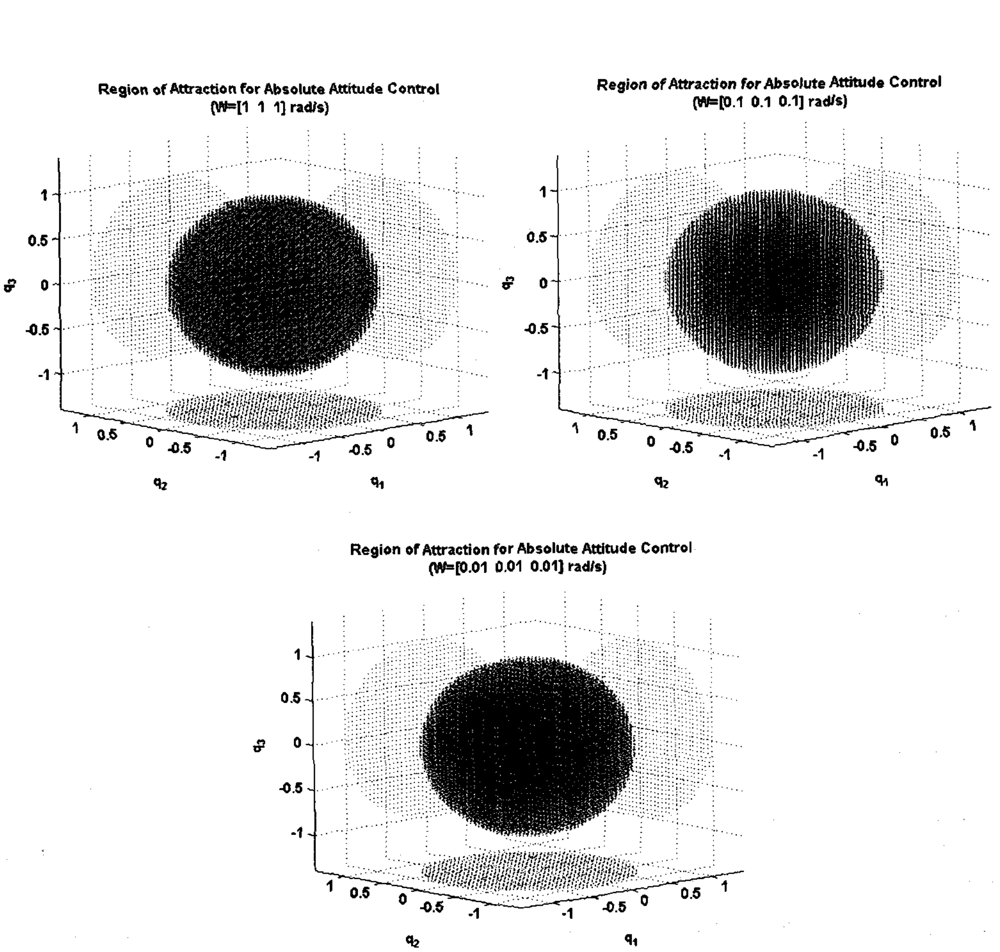


Figure 1. Region of attraction for absolute attitude control system under different angular velocities.

### *3.2.1 Absolute Attitude Control*

Figure 1 shows the results for the estimation of the region of attraction for the SDRE-controlled absolute attitude control system under different angular velocities ($\omega^{bo} = [1 \quad 1 \quad 1]^{\mathrm{T}}$, $[0.1 \quad 0.1 \quad 0.1]^{\mathrm{T}}$, $[0.01 \quad 0.01 \quad 0.01]^{\mathrm{T}}$ rad/s). Because the system has six variables, the results cannot express in three dimensional space. So under assumption that all angular velocities are constant, the quaternions are depicted. The figures in Figure 1 consist of sphere and circles which denote region of attraction of the system and projected areas of the region of attraction, respectively. As can be seen, the quaternions have almost all values ($-0.95 \sim +0.95$) in all cases. Hence, the SDRE controller for the absolute attitude motion is guaranteed the local asymptotic stability.

### *3.2.2 Relative Attitude Control*

Because of the selective control strategy (Chang et al. 2007) for the relative attitude control, this controllers are peformed when relative Euler angles are less than 0.01 deg (about 0.001 in quaternion). Figure 2 shows the stability region estimate for the relative SDRE-controlled attitude system. In the figure, the sphere and circles denote the permissible boundary for the relative attitude controller and the projection of the boundary, respectively as mentioned in the absolute attitude controller. In Figure 2, the quaternions have almost all values ($-0.95 \sim +0.95$). Hence, the SDRE

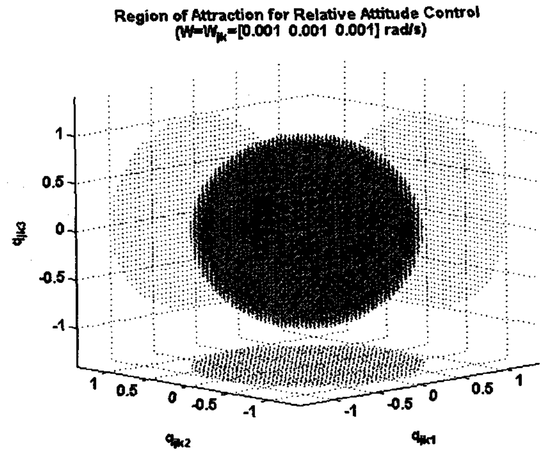


Figure 2. Region of attraction for relative attitude control system.

controller for the relative attitude motion is also guaranteed the local asymptotic stability.

## 4. CONCLUSIONS

The stability region for the SDRE-controlled attitude systems are estimated in the current paper. Bracci's method is less computational load than other methods to find the region of attraction for the SDRE controllers. By using the results in the current paper, the stability regions are assured in local areas of the absolute as well as relative quaternions ($-0.95 \sim +0.95$) for the decentralized coordinated attitude control in satellite formation flying suggested by Chang et al. (2007)

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