# 영역별 배전계통 운용을 위한 Non-Interior Point OPF 알고리즘

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# Non-Interior Point Optimal Power Flow Algorithm for Sectional Distribution System Operation

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**Abstract** - 본 논문에서는 구역별 배전계통운영을 위하여 Non-interior point 배전용 최적조류계산(Distribution non-interior point optimal power flow: NIPDOPF) 알고리즘을 소개한다. NIPDOPF 알고리즘은 향후 지역이나 구역으로 분산전원이 도입될 경우 이를 대비한 영역별 최적조류계산 알고리즘으로 이용할 수 있다.

## 1. 서 론

조류계산 및 최적조류계산은 계통의 재배치와 복구, 전압조정, Var/전압 협조, 고장분석을 위하여 필요하다. 특히 개인발전사업자나 민간발전사업자가 배전계통에 지역 및 구역 발전기를 도입할 경우 이를 중심으로 구역별 최적조류계산 (optimal power flow: OPF)이 필요하게 된다. 본 논문에서는 smoothing non-interior point method (NIPM)를 이용한 최적조류계산 알고리즘을 기술한다. Primal dual IPM과 같이, 제안된 기법은 부등식제약조건을 다루기 위하여 slack 변수, Lagrangian 함수를 형성하고 KKT 최적 조건을 적용하기 위하여 dual 변수, 합성 수식을 풀기 위한 Newton 기법을 사용한다. 제안된 기법은 complementary 조건을 다루기 위하여 logarithm barrier대신에 smoothing 함수를 사용한다. 이는 변수들의 제약조건들을 극복하여 모든 반복시에 feasible 공간의 interior에 있도록 할 수 있다.

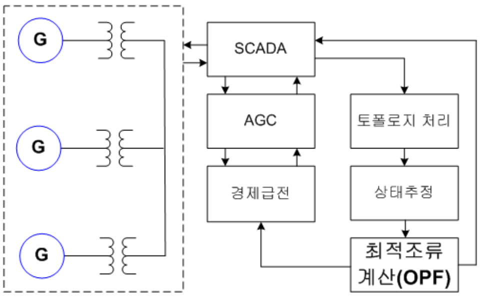


그림 1. 최적조류계산 (Optimal Power Flow) 블록선도

## 2. Non-Interior Point OPF 문제 [1]

비선형 최적화 목적함수 $\min f(x)$ (1)

등식제약 $h(x)=0$ (2)

부등식제약 $g_{\min} \le g(x) \le g_{\max}$ (3)

$$g(x)-z_l-g_{\min}=0 \quad (4)$$

$$g(x)+z_u-g_{\max}=0 \quad (5)$$

$$z_l, z_u \ge 0 \quad (6)$$

Lagrangian 함수는

$$L=f(x)-\lambda^T h(x)-\pi_l^T(g(x)-z_l-g_{\min})-\pi_u^T(g(x)+z_u-g_{\max}) \quad (7)$$

KKT 최적조건은

$$\triangle_x L=\nabla f(x)-\nabla h(x)^T\lambda+\nabla g(x)^T\pi_l-\nabla g(x)^T\pi_u=0 \quad (8)$$

$$\triangle_\lambda L=-h(x)=0 \quad (9)$$

$$\triangle\pi_l L=-(-g(x)+z_l+g_{\min})=0 \quad (10)$$

$$\triangle\pi_u L=-(-g(x)+z_u-g_{\max})=0 \quad (11)$$

$$\triangle_{zl}L=Z_l\pi_l=0 \quad \pi_l, Z_l>0 \quad (12)$$

$$\triangle_{zu}L=Z_u\pi_u=0 \quad \pi_u, Z_u>0 \quad (13)$$

Perturbation factor

$$\triangle Z_l\pi_l+Z_l\triangle\pi_l=-Z_l\pi_l \quad (14)$$

$$\pi_{li}\triangle z_{li}+z_{li}\triangle\pi_{li}=-\pi_{li}z_{li} \quad (15)$$

$$L_\mu=f(x)-\mu\sum(\ln z_l+\ln z_u)-\lambda^T h(x) -\pi_l^T(g(x)-z_l-g_{\min})-\pi_u^T(g(x)+z_u-g_{\max}) \quad (16)$$

$$\triangle_{zl}L_\mu=Z_l\pi_l-\mu e=0 \quad (17)$$

$$\triangle_{zu}L_\mu=Z_u\pi_u-\mu e=0 \quad (18)$$

Newton's direction

Perturbed KKT 식은

$$\begin{bmatrix} \pi_l & 0 & Z_l & 0 & 0 & 0 \\ 0 & \pi_u & 0 & Z_u & 0 & 0 \\ -Z_l & 0 & 0 & 0 & \nabla g(x)^T & 0 \\ 0 & -Z_u & 0 & 0 & -\nabla g(x)^T & 0 \\ 0 & 0 & \nabla g(x) & -\nabla g(x) & \nabla_x^2 L_\mu & -\nabla h(x)^T \\ 0 & 0 & 0 & 0 & -\nabla h(x) & 0 \end{bmatrix} \times \begin{bmatrix} \triangle Z_l \\ \triangle Z_u \\ \triangle\pi_l \\ \triangle\pi_u \\ \triangle x \\ \triangle\lambda \end{bmatrix} = - \begin{bmatrix} \triangle_{zl}L_\mu \\ \triangle_{zu}L_\mu \\ \triangle_{\pi l}L_\mu \\ \triangle_{\pi u}L_\mu \\ \triangle_x L_\mu \\ \triangle_\lambda L_\mu \end{bmatrix} \quad (19)$$

여기서 $\nabla_x^2 L_\mu=\nabla_x^2 f(x)-\nabla_x^2 h(x)^T\lambda+\nabla_x^2 g(x)^T\pi_l-\nabla_x^2 g(x)^T\pi_u$ (20)

$$\begin{bmatrix} H & -J_h^T \\ -J_h & 0 \end{bmatrix} * \begin{bmatrix} \triangle x \\ \triangle\lambda \end{bmatrix} = - \begin{bmatrix} \psi \\ h(x) \end{bmatrix} \quad (21)$$

$$\triangle Z_l=\nabla g(x)^T\triangle x-\nabla_{\pi l}L_\mu \quad (22)$$

$$\triangle Z_u=-\nabla g(x)^T\triangle x+\nabla_{\pi u}L_\mu \quad (23)$$

$$\triangle\pi_l=Z_l^{-1}(-\pi_l\triangle Z_l-\nabla_{zl}L_\mu) \quad (24)$$

$$\triangle\pi_u=Z_u^{-1}(-\pi_u\triangle Z_u+\nabla_{zu}L_\mu) \quad (25)$$

$$H=\nabla_x^2 L_\mu+\nabla g(x)(Z_l^{-1}\pi_l-Z_u^{-1}\pi_u)\nabla g(x)^T \quad (26)$$

$$J_h=\nabla h(x) \quad (27)$$

$$\psi=-\nabla_x L_\mu-\nabla g(x)(Z_l^{-1}\pi_l-Z_u^{-1}\pi_u)\nabla g(x)^T +Z_l^{-1}\nabla_{zl}L_\mu-Z_u^{-1}\nabla_{zu}L_\mu \quad (28)$$

$$x^{k+1}=x^k+\alpha_p^k\triangle x \quad (29)$$

$$\lambda^{k+1}=\lambda^k+\alpha_d^k\triangle\lambda \quad (30)$$

$$z_l^{k+1}=z_l^k+\alpha_p^k\triangle z_l \quad (31)$$

$$\pi_l^{k+1}=\pi_l^k+\alpha_d^k\triangle\pi_l \quad (32)$$

$$z_u^{k+1}=z_u^k+\alpha_p^k\triangle z_u \quad (33)$$

$$\pi_u^{k+1}=\pi_u^k+\alpha_d^k\triangle\pi_u \quad (34)$$

$$\alpha_p^k=\min\left\{1, \gamma\min\left\{-\frac{z_l^k}{\triangle z_l}/\triangle z_l<0, -\frac{z_u^k}{\triangle z_u}/\triangle z_u<0\right\}\right\} \quad (35)$$

$$\alpha_d^k=\min\left\{1, \gamma\min\left\{-\frac{\pi_l^k}{\triangle\pi_l}/\triangle\pi_l<0, -\frac{\pi_u^k}{\triangle\pi_u}/\triangle\pi_u<0\right\}\right\} \quad (36)$$

$$\mu^{k+1}=\sigma^k\frac{\rho^k}{2p} \quad (37)$$

$$\rho^k=(Z_l^k)^T\pi_l^k+(Z_u^k)^T\pi_u^k \quad (38)$$

부록에서 표시한 NCP에다 Smoothing 함수를 결합한 KKT 조건은 다음과 같다.

$$\theta(y)=\begin{bmatrix} \varphi_\mu(\pi_l, Z_l) \\ \varphi_\mu(\pi_u, Z_u) \\ -(-g(x)+z_l+g_{\min}) \\ -(g(x)+z_u-g_{\max}) \\ \nabla f(x)-\nabla h(x)^T\lambda+\nabla g(x)^T\pi_l-\nabla g(x)^T\pi_u \\ -h(x) \end{bmatrix}=0 \quad (39)$$

smoothing variable $\mu$($\mu=0$최적상태)를 도입하면,

$$\theta'(y)\triangle y=-\theta(y) \quad (40)$$

$$\theta_\mu(y,\mu)=\begin{bmatrix}\varphi_\mu(\pi_l,Z_l)\\ \varphi_\mu(\pi_u,Z_u)\\ -(-g(x)+z_l+g_{\min})\\ -(g(x)+z_u-g_{\max})\\ \nabla f(x)-\nabla h(x)^T\lambda+\nabla g(x)^T\pi_l-\nabla g(x)^T\pi_u\\ -h(x)\\ \mu\end{bmatrix}=0 \quad (41)$$

뉴톤기법을 위식에 대입하면,

$$\begin{bmatrix}D_{zl} & 0 & D_{\pi l} & 0 & 0 & 0 & D_{\mu\varphi(Z_l,\pi_l)}\\ 0 & D_{zu} & 0 & D_{\pi u} & 0 & 0 & D_{\mu\varphi(Z_u,\pi_u)}\\ -Z_1 & 0 & 0 & 0 & \nabla g(x)^T & 0 & 0\\ 0 & -Z_u & 0 & 0 & -\nabla g(x)^T & 0 & 0\\ 0 & 0 & \nabla g(x) & -\nabla g(x) & H & -\nabla h(x)^T & 0\\ 0 & 0 & 0 & 0 & -\nabla h(x) & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}\times\begin{bmatrix}\triangle Z_l\\ \triangle Z_u\\ \triangle\pi_l\\ \triangle\pi_u\\ \triangle x\\ \triangle\lambda\\ \triangle\mu\end{bmatrix}=-\begin{bmatrix}\varphi_\mu(\pi_l,Z_l)\\ \varphi_\mu(\pi_w,Z_u)\\ -(-g(x)+z_l+g_{\min})\\ -(g(x)+z_u-g_{\max})\\ \nabla f(x)-\nabla h(x)^T\lambda+\nabla g(x)^T\pi_l-\nabla g(x)^T\pi_u\\ -h(x)\\ \mu\end{bmatrix} \quad (42)$$

여기서, $D_{zl}=\dfrac{\partial\varphi_\mu(\pi_l,z_l)}{\partial z_l}$, $D_{\pi l}=\dfrac{\partial\varphi_\mu(\pi_l,z_l)}{\partial\pi_l}$, $D_{\mu\varphi(z_l,\pi_l)}=\dfrac{\partial\varphi_\mu(\pi_l,z_l)}{\partial\mu}$

$D_{zu}=\dfrac{\partial\varphi_\mu(\pi_w,z_u)}{\partial z_u}$, $D_{\pi u}=\dfrac{\partial\varphi_\mu(\pi_w,z_u)}{\partial\pi_u}$, $D_{\mu\varphi(z_w,\pi_u)}=\dfrac{\partial\varphi_\mu(\pi_w,z_u)}{\partial\mu}$

임의의 $\sigma\in(0,1)$에 대하여 $\mu\to\sigma\mu$로 대치하면

$$\begin{bmatrix}D_{zl} & 0 & D_{\pi l} & 0 & 0 & 0 & D_{\mu\varphi(Z_l,\pi_l)}\\ 0 & D_{zu} & 0 & D_{\pi u} & 0 & 0 & D_{\mu\varphi(Z_u,\pi_u)}\\ -Z_1 & 0 & 0 & 0 & \nabla g(x)^T & 0 & 0\\ 0 & -Z_u & 0 & 0 & -\nabla g(x)^T & 0 & 0\\ 0 & 0 & \nabla g(x) & -\nabla g(x) & H & -\nabla h(x)^T & 0\\ 0 & 0 & 0 & 0 & -\nabla h(x) & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}\times\begin{bmatrix}\triangle Z_l\\ \triangle Z_u\\ \triangle\pi_l\\ \triangle\pi_u\\ \triangle x\\ \triangle\lambda\\ \triangle\mu\end{bmatrix}=-\begin{bmatrix}\varphi_\mu(\pi_l,Z_l)\\ \varphi_\mu(\pi_w,Z_u)\\ -(-g(x)+z_l+g_{\min})\\ -(g(x)+z_u-g_{\max})\\ \nabla f(x)-\nabla h(x)^T\lambda+\nabla g(x)^T\pi_l-\nabla g(x)^T\pi_u\\ -h(x)\\ \sigma\mu\end{bmatrix} \quad (43)$$

$$\theta'_\mu(y,\mu)\begin{bmatrix}\triangle y\\ \triangle\mu\end{bmatrix}=-\theta_\sigma(y,\mu) \quad (44)$$

Step length

$$t_k=\max\{\zeta^l | l=0,1,2,\cdots\} \quad (45)$$

$$\|\Phi(z_k+t_k\triangle z)\|\le\beta(1-\sigma_k t_k)\mu_k \quad (46)$$

여기서,

$$z=(Z_l,\pi_l,Z_u,\pi_u) \quad (47)$$

$$\Phi(z)=\begin{bmatrix}\varphi(Z_l,\pi_l)\\ \varphi(Z_u,\pi_u)\end{bmatrix} \quad (48)$$

$$\beta\ge 2\psi(y_0)/\mu_0 \quad (49)$$

발전기의 연료비용을 최소화한 목적함수는

$$\text{minimize } f=\sum_{i=1}^{g}\left(\frac{1}{2}a_iP_{gi}+b_iP_{gi}+c_i\right) \quad (50)$$

등식제약조건은

$$(P_{gi}-P_{li}-P_i)=0 \quad (51)$$

$$(Q_{gi}-Q_{li}-Q_i)=0 \quad (52)$$

부등식제약조건은

$$v_{\min}\le v_i\le v_{\max} \quad (53)$$

$$P_{gmin}\le P_g\le P_{gmax} \quad (54)$$

$$Q_{gmin}\le Q_g\le Q_{gmax} \quad (55)$$

$$t_{\min}\le t_p\le t_{\max} \quad (56)$$

$$(S_{km})_j^2\le S_{\max}^2 \quad (57)$$

slack 변수를 추가한 후 부등식 제약조건은

$$-P_{gi}+z_{pl}+P_{gmin}=0,\ z_{pl}>0 \quad (58)$$

$$P_{gi}+z_{ph}-P_{gmax}=0,\ z_{ph}>0 \quad (59)$$

$$-(Q_i+Q_l)+z_{ql}+Q_{gmin}=0,\ z_{ql}>0 \quad (60)$$

$$Q_i+Q_l+z_{qh}-Q_{gmax}=0,\ z_{qh}>0 \quad (61)$$

$$-V_i+z_{vl}+V_{\min}=0,\ z_{vl}>0 \quad (62)$$

$$V_i+z_{vh}-V_{\max}=0,\ z_{vh}>0 \quad (63)$$

$$S_{km}^2+z_s-S_{\max}^2=0,\ z_s>0 \quad (64)$$

$$-t_p+z_{tl}+t_{\min}=0,\ z_{tl}>0 \quad (65)$$

$$t_p+z_{th}-t_{\max}=0,\ z_{th}>0 \quad (66)$$

여기서 $z$는 slack variables이다.

Lagrangian 함수는

$$\begin{aligned}L=&f-\sum_{i=1}^{n}\lambda_{pi}(P_{gi}-P_{li}-P_i)-\sum_{i=1}^{n}\lambda_{qi}(Q_{gi}-Q_{li}-Q_i)\\ &-\sum_{l=1}^{g}\pi_{ph}(P_{gi}+z_{ph}-P_{gmax})-\sum_{i=1}^{g}\pi_{pl}(-P_{gi}+z_{pl}-P_{gmin})\\ &-\sum_{l=1}^{g}\pi_{qh}(Q_i+Q_l+z_{qh}-Q_{gmax})\\ &-\sum_{l=1}^{g}\pi_{ql}(-Q_i-Q_l+z_{ql}+Q_{gmin})\\ &-\sum_{i=1}^{n}\pi_{vh}(V_i+z_{vh}-V_{\max})-\sum_{i=1}^{n}\pi_{vl}(-V_i+z_{vl}-V_{\min})\\ &-\sum_{i=1}^{l}\pi_s(S_{km}^2+z_s-S_{\max}^2)-\sum_{i=1}^{t}\pi_{th}(t_{pi}+z_{th}-t_{\max})\\ &-\sum_{i=1}^{l}\pi_{tl}(-t_{pi}+z_{tl}-t_{\min})\end{aligned} \quad (67)$$

complementary 조건

$$\pi_{vl}z_{vl}=0 \quad \pi_{vh}z_{vh}=0 \quad (68)$$

$$\Phi(\pi_{vl},z_{vl})=0 \quad \Phi(\pi_{vh},z_{vh})=0 \ \cdots \ \Phi(\pi_{th},z_{th})=0 \quad (69)$$

smoothing 기법에 의한 complementary 조건

$$\Phi_\mu(\pi_{vl},z_{vl},\mu)=0 \ \Phi_\mu(\pi_{vh},z_{vh},\mu)=0 \ \cdots \ \Phi_\mu(\pi_{th},z_{th},\mu)=0 \quad (70)$$

$\pi_{vh},z_{vh}$를 고려하면, smoothing 함수는

$$\varphi_\mu(\pi_{vh},z_{vh})=\pi_{vh}+z_{vh}-\sqrt{\pi_{vh}^2+z_{vh}^2+2\mu^2} \quad (71)$$

**<알고리즘>**

Step 1 : (초기화) $y_0$ 및 $\mu_0(>0)$, $\sigma_0\in(0,1)$, $\zeta\in(0,1)$, $\epsilon>0$,
$\beta\ge 2\psi(y_0)/\mu_0$, 반복수 $k=0$

Step 2 : (완료 조건) $\psi(y_k)>\epsilon$이면, 완료

Step 3 : 해 $(\triangle y_k,\triangle\mu_k)$

Step 4 : (선로 탐색 및 갱신) $t_k=\max\{\zeta^l | l=0,1,2,\cdots\}$
$\|\Phi(z_k+t_k\triangle z)\|\le\beta(1-\sigma_k t_k)\mu_k$
$y_{k+1}=y_k+t_k\triangle y_k$
$\mu_{k+1}=\mu_k+t_k\triangle\mu_k=\mu_k+t_k\sigma_k\mu_k=(1-\sigma_k t_k)\mu_k$

Step 5 : (갱신) $\sigma_{k+1}=\delta\sigma_k$
만일 $\sigma_{k+1}\le\rho_{\min}$, $\sigma_{k+1}=\rho_{\min}$
만일 $\sigma_{k+1}\ge\rho_{\max}$, $\sigma_{k+1}=\rho_{\max}$
$k=k+1$, step 2로 간다.

<부록>

NCP : Nonlinear Complementary Problem

(1) Fischer-Burnmeister 함수

$$\varphi(a,b)=a+b-\sqrt{a^2+b^2} \quad (A1)$$

(2) Minimum 함수

$$\varphi(a,b)=a+b-\sqrt{(a-b)^2} \quad \theta_0(y)=0 \quad (A2)$$

여기서

$$\theta_0(y)=\begin{bmatrix}\varphi(\pi_l,Z_l)\\ \varphi(\pi_u,Z_u)\\ -(-g(x)+z_l+g_{\min})\\ -(g(x)+z_u+g_{\max})\\ \nabla f(x)-\lambda\nabla h(x)-\pi_l\nabla g(x)-\pi_u\nabla g(x)\\ -h(x)\end{bmatrix} \quad (A3)$$

$$\varphi_\mu(a,b)=a+b-\sqrt{a^2+b^2+2\mu^2} \quad (A4)$$

(4) Chen-Harker-Kanzow-Smale smoothing 함수

$$\varphi_\mu(a,b)=a+b-\sqrt{(a-b)^2+4\mu^2} \quad (A5)$$

## 3. 결 론

본 논문에서는 구역별 배전계통 운영을 고려한 Non-interior point 배전용 최적조류계산(Distribution non-interior point optimal power flow: NIPOPF) 알고리즘을 소개하였다. NIPOPF 알고리즘은 향후 지역이나 구역으로 분산전원이 도입될 경우를 대비하여 이를 이용할 수 있을 것이다.

## [참 고 문 헌]


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[2] IEEE Tutorial Course, "Optimal Power Flow: Solution Techniques, Requirements, and Challenges", IEEE Power Engineering Society, 1996.



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