NUMERICAL RELATIVITY SIMULATIONS OF BINARY BLACK HOLE MERGERS

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Korean Society of Computational Fluids Engineering Fall Meeting
KBNU, Daegu, Korea, Oct. 26, 2007

OUTLINE

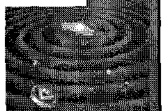
- Introduction
  - Black holes / Gravitational waves (GWs)
  - Binary black holes as a strong GW source
- Numerical relativity
  - Techniques
- Binary black hole merger simulations
  - Results (Gravitational waveforms/ Recoil kicks/ Final spins)
- Conclusion

INTRODUCTION

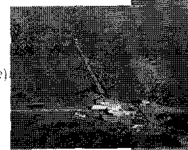
- Black holes are one of the most fascinating consequences of Einstein's theory of General Relativity.
  - Describes the final stages of unhalted gravitational collapse.
  - Physical singularity inevitably results (the singularity theorems of Penrose and Hawking).
  - Formed when enough mass-energy density is realized: $\frac{2MG}{Rc^2} > 1$
    - To Make BH out of the Sun, all the mass should be compressed within 3 km radius.
- There is mounting evidence for the existence of black holes.
  - Stellar mass black holes ----- X-ray binary observations; black hole can form as a result of collapse of massive stars in supernova explosion; can be detected thru influence on the companion star; "unseen" member is too massive to be explained by neutron star or any other known objects than black holes.
  - Supermassive black holes ----- Black holes in centers of galaxies. Detected thru the motions of near-by stars.
  - AGN (Active Galactic Nuclei) / Jets ----- Best explanation for such a small size engine powering AGN is a rotating supermassive black holes.

INTRODUCTION

- Fundamental features of general relativity are
  - (1) spacetime is *dynamic* and (2) gravitational effects are described by *curvature* of spacetime.



- Gravitational waves are the solutions that describe fluctuations in spacetime curvature which propagate as a wave.
  - Solutions to *linearized* Einstein eqn.
- Isolated black holes do not radiate.
  - Black holes have been observed from their *interactions* with surrounding matters thru the observations of E&M wave signals.



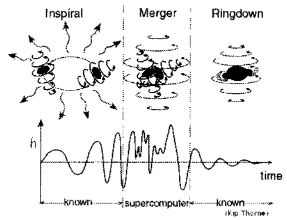
- Only *direct* way to see black holes is *thru gravitational waves* they emit when *interacting* with each other (i.e. binary black holes) or surrounding matter.
  - GW observation is the only way that we can study black hole systems even if there is no surrounding matters.
  - GWs seen indirectly (Hulse-Taylor binary pulsar, 1993 Nobel Prize)
  - Currently there are world-wide efforts to detect the GWs *directly* using laser interferometers (and bar detectors).

INTRODUCTION

- Scientific motivation for gravitational wave observation
  - Measure Gravitational Waves directly for the first time to confirm Einstein's prediction and study their properties.
  - Probe strong and dynamical region of spacetime that go beyond current observational limits.
  - Used as a tool for astronomy.
    - Certainly regions (e.g. deep inside the collapsing supernova or central regions of galaxies) are not accessible with E&M waves.
    - Characteristics of GWs is completely different from those of E&M waves. Astronomers are excited about this *new window* to the Universe that will open thru gravitational waves!
- What conditions give rise to strongest signals for GWs?
  - Quadrupole formula says for typical amplitude of the GWs $h \sim \frac{16\mu v^2}{r}$
  - This formula suggests the strongest source is simply the most massive (densest) objects moving the fastest (generating time dependent quadrupole moments).
  - Binary black hole systems offers one of the most promising candidate source for gravitational wave observation.

BINARY BLACK HOLE MERGER SIMULATIONS

- To facilitate the detection and interpretations of GW signals, we ultimately need *theoretical templates* based on source modeling.
  - Thus, the strongest motivating force behind binary black hole merger simulations is the gravitational wave observation.
- Binary black hole coalescence goes thru roughly three stages.
  - Gravitational waveforms are known in inspiral and ringdown stages via closed form approximations.
  - Merger stage (roughly lasting from the last few orbits thru the merger of two black holes.) requires full theory, and thus can only be studied with numerical relativity simulations.



NUMERICAL RELATIVITY

- Study of general relativity using numerical methods.
- Because of the complexities of Einstein field equations, solutions to only a very few situations (often highly symmetric) are known thru closed forms.
  - Physics in the most interesting situations can *only* be reliably studied via numerical methods.
  - Guidance given by approximations (e.g. perturbation, Post-Newtonian methods) has limitations.
- Scope of problems that numerical relativity is concerned with is larger than binary black hole spacetimes.
  - Critical phenomena
  - Properties of black hole singularities (inside the horizons)
- Here I go over some general issues in Numerical Relativity and then focus on binary black hole simulations.

NUMERICAL RELATIVITY

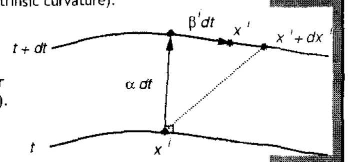
- *Form of the equations*
  - Constraint equations and evolution equations
- Coordinates
- Boundary conditions
- *Numerical techniques*
  - Finite differencing
  - AMR
  - (Iterative) Crank-Nicholson or Runge Kutta schemes
- Numerical Issues
  - Stability
  - Convergence
  - Consistency
  - Efficiency / Optimization

FORM OF THE EQUATIONS

- Unlike any other theories in physics, the form of the equations is *not* unique. Einstein's field equations are essentially a tensor equation and can be decomposed into many different ways.
  - Choice of dynamical variables are *not* unique either.
  - "Formalism" has been an issue in numerical relativity.
- Basic idea is to cast Einstein field equations (EFEs) as a Cauchy problem (i.e. IVP); use "3+1" formalism.
  - Foliate 4D spacetime as a stack of 3D-hypersurface labeled by time coordinate.
  - Solution to EFE is classical gravitational field that is the time history of geometry of a spacelike hypersurface.
  - Evolution variables (*spatial* metric and extrinsic curvature):

$$(\gamma_{ij}, K_{ij}) \qquad K_{ij} = -\frac{1}{2}\mathcal{L}_n \gamma_{ij}$$

  - By taking various projections of EFEs, we obtain a set of 10 coupled nonlinear PDEs (6 hyperbolic eqn./4 elliptic eqn.).



EQUATIONS

- Einstein field equations split into two kinds
  - Constraint equations

$${}^{(3)}R + K^2 - K_{ij}K^{ij} = 0$$
$$D_j(K^{ij} - \gamma^{ij}K) = 0$$

  - Evolution equations

$$(\partial_t - \mathcal{L}_\beta)\gamma_{ij} = -2\alpha K_{ij}$$
$$(\partial_t - \mathcal{L}_\beta)K_{ij} = -D_i D_j \alpha + \alpha(R_{ij} + K K_{ij} - 2K_{ik}K^k{}_j)$$

where ${}^{(3)}R$ is 3-dim Ricci scalar, D_i the covariant derivatives associated with the 3-dim metric, γ_{ij}, and K is trace of K_{ij}

- In our simulations, we use slightly modified set of equations called BSSN formulation.
- Basic strategy is we solve
  - At t=0, a set of constraint equations. In GR, we can not freely specify all the variables. Some of them are constrained via constraints.
  - At t>0, at set of evolution equations to evolve dynamical variables forward in time (and only monitor constraint violations).

EQUATIONS

- Ricci tensor: (note summation convention for repeated indices)

$$R_{ab} = R^c{}_{acb}$$
$$R_{abc}{}^d = \Gamma^d_{ac,b} - \Gamma^d_{bc,a} + \Gamma^e_{ac}\Gamma^d_{eb} + \Gamma^e_{bc}\Gamma^d_{ea}$$
$$\Gamma^a_{bc} = \frac{1}{2}\gamma^{ad}(\gamma_{db,c} + \gamma_{dc,b} - \gamma_{bc,d})$$

- Maple expressions
  - When expanded term-by-term, the full expressions for Ricci tensor components covers 100+ pages.

BSSN FORM OF EQUATIONS

- We typically use so-called BSSN formulation.

BSSN evolution equations for $\{\varphi, \tilde{\gamma}_{ij}, K, \tilde{A}_{ij}, \tilde{\Gamma}^i\}$:

$$\partial_t\varphi - \mathcal{L}_\beta\varphi = -\frac{1}{6}\alpha K$$
$$\partial_t K - \mathcal{L}_\beta K = -\gamma^{ij}D_iD_j\alpha + \alpha(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2) + \frac{1}{2}\alpha(\rho + S)$$
$$\partial_t\tilde{\gamma}_{ij} - \mathcal{L}_\beta\tilde{\gamma}_{ij} = -2\alpha\tilde{A}_{ij}$$
$$\partial_t\tilde{A}_{ij} - \mathcal{L}_\beta\tilde{A}_{ij} = e^{-4\varphi}[-D_iD_j\alpha + \alpha(R_{ij} - S_{ij})]^{TF} + \alpha(K\tilde{A}_{ij} - 2\tilde{A}_{il}\tilde{A}^l_j)$$
$$\partial_t\tilde{\Gamma}^i = \tilde{\gamma}^{jk}\partial_j\partial_k\beta^i + \frac{1}{3}\tilde{\gamma}^{ij}\partial_j\partial_k\beta^k + \beta^j\partial_j\tilde{\Gamma}^i - \tilde{\Gamma}^j\partial_j\beta^i + \frac{2}{3}\tilde{\Gamma}^i\partial_j\beta^j - 2\tilde{A}^{ij}\partial_j\alpha + 2\alpha(\tilde{\Gamma}^i_{jk}\tilde{A}^{jk} + 6\tilde{A}^{ij}\partial_j\varphi - \frac{2}{3}\tilde{\gamma}^{ij}\partial_j K)$$

$$\mathcal{L}_\beta\varphi = \beta^k\partial_k\varphi + \frac{1}{6}\partial_k\beta^k$$
$$\mathcal{L}_\beta K = \beta^k\partial_k K$$
$$\mathcal{L}_\beta\tilde{\gamma}_{ij} = \beta^k\partial_k\tilde{\gamma}_{ij} + \tilde{\gamma}_{ik}\partial_j\beta^k + \tilde{\gamma}_{kj}\partial_i\beta^k - \frac{2}{3}\tilde{\gamma}_{ij}\partial_k\beta^k$$
$$\mathcal{L}_\beta\tilde{A}_{ij} = \beta^k\partial_k\tilde{A}_{ij} + \tilde{A}_{ik}\partial_j\beta^k + \tilde{A}_{kj}\partial_i\beta^k - \frac{2}{3}\tilde{A}_{ij}\partial_k\beta^k$$

- Don't be fooled by compact form of expressions using tensor notation. Once we write out the equations term by term, the expressions are *unmanageable by hand*. *Need symbolic algebra* package such as Maple or Mathematica.

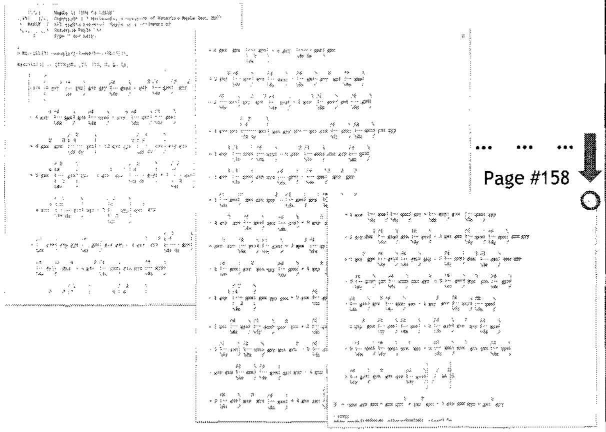
COORDINATES

- Again, unlike any other theory of physics, coordinates (on which we describe physics) themselves are *dynamic* in nature. There are *no unique or preferred choice of coordinates.*
- Of course, physics does not depend on coordinates systems as long as it covers the spacetime regions of interests.
  - But often "wrong" choice of coordinates lead to coordinate singularities or numerical instabilities.
- Choosing "right" coordinates has been a nontrivial task in numerical relativity.
  - There are no cure-all choices, i.e., ideal choice of coordinates is problem-dependent.
- Often coordinates themselves are *evolved* along with dynamical variables. For example, lapse (define time coord.) and shifts (define spatial coords.) are given by

$$(\partial_t - \beta^i \partial_i)\alpha = -2\alpha K$$

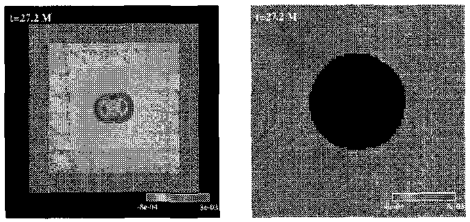
$$\partial_t \beta^i = \frac{3}{4} B^i, \quad \partial_t B^i = \partial_t \tilde{\Gamma}^i - \eta B^i$$

MAPLE OUTPUT



NUMERICAL TECHNIQUES

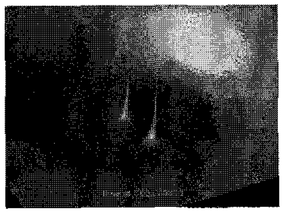
- At least in vacuum, we expect that the solutions (gravitational fields) are *smooth* functions (no shocks or sharp discontinuities) *except* near *black hole singularities.*
  - One way of dealing with these infinities (either physical or coordinate) is to *excise* regions that that include physical singularities from the computational domain.
  - OK because of event horizons.
  - Nontrivial to implement. Treatment of numerical instability around excision boundaries.



(Pretorius)

NUMERICAL TECHNIQUES

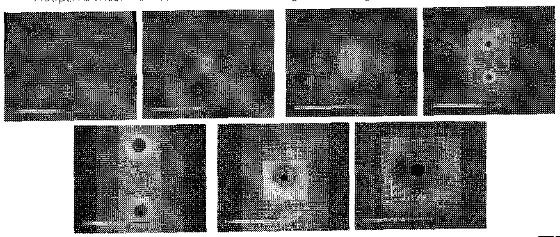
- Moving puncture technique
  - One simplest prescription avoids physical singularities with clever choice of slicing but there are still some sharp features (i.e. large gradients) in the solution due to *coordinate* singularities.
  - Yet, with proper choices of coordinates, remarkably, sharp figures (and also non-convergent region of space) do remain within the event horizon. Note that BH horizon is causal boundary between interior and exterior.



(UTB/RIT group)

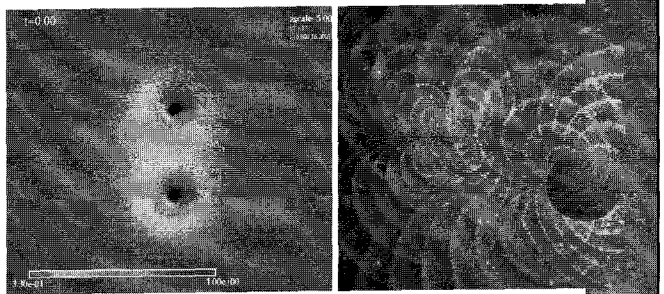
NUMERICAL TECHNIQUES

- Finite differencing method on a structured grid
  - Spatial derivative terms are replaced by finite difference operators. Centered differencing is used except for advection terms where upwind differencing is used.
- Initial Data
  - Use Multi-Grid method to solve a set of (up to) 4 elliptic equations.
- Evolution
  - Iterative Crank-Nicholson or Runge-Kutta Scheme.
  - Adaptive mesh refinement based on Berger-and-Oliger algorithm is used.



SIMULATION RESULTS

- Movies of merger of two equal-mass non-spinning black holes. (used moving puncture technique)
  - LEFT: Lapse function, α, on the Z=0 plane.
  - RIGHT: Movie of gravitational waves (isosurface of the imaginary part of ψ_4) generated from another simulation of equal mass and non-spinning binary.

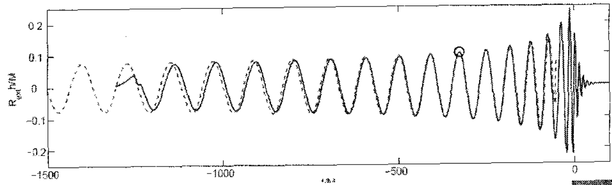


SIMULATION RESULTS

- Binary black hole merger simulations started in mid-70s and after long struggling years, the breakthrough finally came in 2005.
  - Since then, results are coming out at a rate that is unseen before.
  - NR results are stimulating interactions with Post-Newtonian people, as well as astronomers.
- Main contributions so far:
  - Gravitational waveforms.
  - GW-induced recoil kicks.
  - Final spins of the merged black hole.
- For binary black holes, parameter space to explore is 7-dimensional.
  - Mass ratio.
  - Spin vector of BH 1.
  - Spin vector of BH 2.
  - Difficult to cover this huge parameter space only by direct simulations (although it IS necessary to a certain extent).
    - Efforts are underway to develop BBH merger models with a closed form formula capturing the basic physics, by combining NR simulation results and closed-form considerations.

GRAVITATIONAL WAVEFORMS

- Gravitational waveforms from equal-mass and non-spinning binary merger.
  - Blue: NR waveform from a 7-orbit merger.
  - Red: PN waveform with 2.5PN amplitude and 3.5PN phasing.



GRAVITATIONAL WAVE INDUCED RECOIL KICKS

- (Seen from the center of mass frame) generic binary BH merger results in loss of linear momentum in addition to orbital energy and angular momentum except in situations with very special symmetry in the configuration such as equal mass non-spinning black holes.
- Integrated over time, there is net overall linear momentum loss due to GW. Momentum conservation implies the merged (single) black hole receives the linear momentum of negative sign ➔ recoil velocity.
- A simple configuration such as equal mass binaries with initially anti-aligned spins in the orbital plane generates kicks in the 1000s of km/s. (e.g. Gonzalez, *et al*, PRL 98, 231101 (2007); Campanelli, *et al*, gr-qc/0702133)

TABLE I. Initial puncture parameters and final kick velocity.

Run	X	P_y	m_i	S_x	v_{kick}(km/s)
MI	±3.257	±0.133	0.363	±0.2	2450 ± 250
MII	±4.0	±0.1125	0.287	±0.2	2650

(Gonzalez, et al PRL 7)

FINAL SPIN OF THE MERGED BLACK HOLE

- Knowledge about the final spin of black hole produced by the merger may have a direct impact on studies of SMBH evolutions and dynamics of compact objects in dense stellar systems (e.g. globular clusters).
- Active study on this issue most recently by combining NR simulations and closed-form considerations.
  - E.g. Initially non-spinning BBH merger: ➔

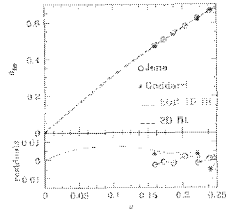


FIG. 3.— *Upper panel:* Comparison of the numerical-relativity data with the 2D fit through (6) in the case of nonspinning binaries, i.e., a = 0. Empty circles indicate the Jena data (Berti et al. 2007), stars the Goddard data (Buonanno et al. 2007a)], a long-dashed line the quadratic EOB fit (Damour & Nagar 2007) and a short-dashed line our 2D fit. *Lower panel:* residuals between the different estimates and the 2D fit.

(L. Rezzolla, et al 2007)

CONCLUDING REMARKS

- With upcoming gravitational wave observations for the next decade, understanding of GW signals (e.g. generated from binary black hole mergers) will provide
  - a completely new view on the universe.
  - new insights to the most elusive force in the universe, gravity.
- Numerical relativity simulations of binary black hole merger play a crucial role in the emerging field of gravitational wave astrophysics.
  - Only way to investigate reliably strong *and* dynamical gravitational fields.
  - Simulations already started to produce interesting results.
    - Merger waveforms / Recoil kicks / Final spins.
    - Still big portion of parameter space left being unexplored.
- NR is still in its early stage in terms of its full potential for discovering new physics.
  - Algorithms and tools are not sophisticated yet; potentially lots of room for improvement.
  - Can benefit from interactions with experts in the areas of numerical analysis, applied math, comp. sci., etc. as well as theoretical physicists.