근접장에서 다각 평판에 대한 표적강도 이론식 개발 및 수중함의 근거리 표적강도 해석

Development of near field Acoustic Target Strength equations for polygonal plates and applications to underwater vehicles

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Key Words : Acoustic Target Strength(음향표적강도), Near Field(근접장), Sonar Cross Section, Theory of Boundary Diffraction Wave, Kirchhoff Approximation

ABSTRACT

Acoustic Target Strength (TS) is a major parameter of the active sonar equation, which indicates the ratio of the radiated intensity from the source to the re-radiated intensity by a target. In developing a TS equation, it is assumed that the radiated pressure is known and the re-radiated intensity is unknown. This research provides a TS equation for polygonal plates, which is applicable to near field acoustics. In this research, Helmholtz-Kirchhoff formula is used as the primary equation for solving the re-radiated pressure field; the primary equation contains a surface (double) integral representation. The double integral representation can be reduced to a closed form, which involves only a line (single) integral representation of the boundary of the surface area by applying Stoke's theorem. Use of such line integral representations can reduce the cost of numerical calculation. Also Kirchhoff approximation is used to solve the surface values such as pressure and particle velocity. Finally, a generalized definition of Sonar Cross Section (SCS) that is applicable to near field is suggested. The TS equation for polygonal plates in near field is developed using the three prescribed statements; the redection to line integral representation, Kirchhoff approximation and a generalized definition of SCS. The equation developed in this research is applicable to near field, and therefore, no approximations are allowed except the Kirchhoff approximation. However, examinations with various types of models for reliability show that the equation has good performance in its applications. To analyze a general shape of model, a submarine type model was selected and successfully analyzed.

1. Introduction

Acoustic Target Strength (TS) is a major parameter of the active sonar equation. Although it indicates the ratio of the intensities, developing a TS equation is a matter of solving the diffracted pressure field by a target. In solving the diffracted field, several numerical methods can be applied. The Boundary Element Method (BEM) and Finite Element Method (FEM) are the methods most widely used in solving the diffracted pressure field in low frequency.

As TS is a parameter that takes part in the active sonar equation, it deals with high frequency range. This limitation prevents a broader application of the FEM or BEM as a major method in the high frequency range. In high frequency range, these numerical methods require fine mesh sizes. In addition, the FEM involves volume integral and BEM involves surface integral. These properties increase the cost of numerical calculation.

The difficulties of the prescribed methods can be overcome applying the Theory of Boundary Diffraction Wave. The basic equations of the Theory of Boundary Diffraction Wave were developed by Maggi-Rubinowicz in the 19th century. However, their analyses were

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restricted to cases where the wave incident upon the aperture is plane or spherical. In the 1960's Miyamoto and Wolf (1962) developed an equation applicable to a general pressure distribution upon the aperture. The significance of the theory is in the idea of applying Stoke's theorem to Helmholtz-Kirchhoff formulae to reduce the order of integral. As a consequence, the Theory of Boundary Diffraction Wave requires only line integral.

In the 1970's, Gordon (1975) developed a far field approximation method of Theory of Boundary Diffraction Wave. This method involves no integration when the aperture is a polygon. As the primary focus of the instant research is in near field, the far field approximate method is not applicable here. Hence, the maximum reduction of the order of integration should be line integration.

In practice, the initial pressure distribution upon a surface plane (no more an aperture) is generated by the mechanism of reflection and transmission. The pressure field initially generated by a point source is reflected by the surface plane. To determine the initial pressure and particle velocity distribution, which are assumed as known terms of the surface plane, Kirchhoff approximation is applied. Kirchhoff approximation assumes that the factors $\Re$ and T, which are derived for reflection and transmission of an infinite plane wave at an infinite plane interface, can be used at every point of a rough surface interface (Medwin and Clay, 1998).

In far field, the distance from the source that generates the initial pressure field to the surface plane is so far that an approximation of constant distance is feasible. In contrast, when the source is positioned within a close distance from the surface plane, constant distance approximation is not practicable. This causes a critical problem in the process of developing near field TS equation. However, the problem is solved by an analogy of the logics between the far field and near field TS definitions. In the process, a generalized definition of SCS is suggested.

TS equation which is applicable to near field is developed by combining the three aspects previously stated which are the Theory of the Boundary Diffraction Wave, Kirchhoff approximation and generalized definition of SCS equation.

For validation of the equation, several models, such as those in plate and cylinder form, were examined in the condition of far field. Coinciding results with far field equations were observed for such models. Finally, as a practical case, a submarine model in near field was analyzed.

2. Development of near field Acoustic Target Strength equation for polygonal plates

2.1 Theory of Boundary Diffraction Wave

The generalized equation of Theory of Boundary Diffraction wave, which is applicable to general pressure field at the aperture, was developed by K. Miyomoto and E. Wolf. In practice, this equation is utilized for a polygonal surface plate. For simplicity, $\Re = 1$ is assumed throughout the entire polygonal surface plate. (Use of the reflection factor indicates that Kirchhoff approximation is applied to the entire surface plate.)

2.1.1 Line integral form of Helmholtz-Kirchhoff formulae

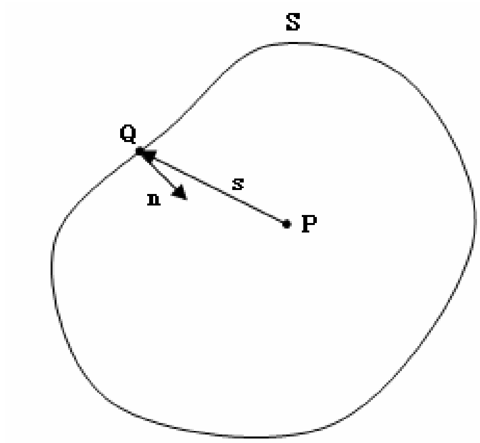

Fig. 1 Illustration of notations for the Helmholtz-Kirchhoff formulae

Assume U as the space-dependent term of the wave which satisfies the homogeneous Helmholtz equation as follows.

$$\left(\nabla^2 - k^2\right)U = 0 \tag{2.1}$$

Here, $k = \omega / c$. Let S be any closed surface bounding volume V, throughout and on the boundary of which U has continuous first and second-order partial derivatives. According to Helmholtz-Kirchhoff integral the disturbance at any point P within V may then be expressed in the form of

$$U(P) = \iint_S V(Q,P) \cdot n dS \tag{2.2}$$

where,

$$V(Q,P)=\frac{1}{4\pi}\left\{\begin{array}{r}U(Q)\nabla_Q\dfrac{\exp(jks)}{s}\\ -\nabla_Q U(Q)\dfrac{\exp(jks)}{s}\end{array}\right\} \tag{2.3}$$

According to Helmholtz decomposition, a general vector field can be decomposed into a rotational term and an irrotational term. It will now be shown that the vector $V(Q,P)$ may always be expressed as the curl of a suitably chosen vector potential $W(Q,P)$. For this purpose, apply divergence to $V(Q,P)$ as follows..

$$\nabla_Q\cdot V(Q,P)=\frac{1}{4\pi}\left\{\begin{array}{r}U(Q)\nabla_Q{}^2\dfrac{\exp(jks)}{s}\\ -\nabla_Q{}^2U(Q)\dfrac{\exp(jks)}{s}\end{array}\right\} \tag{2.4}$$

Now both the functions $\dfrac{\exp(jks)}{s}$ and $U(Q)$ satisfy the homogeneous Helmholtz equation except the singular points of the domain. $\dfrac{\exp(jks)}{s}$ is singular at the field point P, and $U(Q)$ is singular at the position of self-generating sources. Unless the self-generating sources and the field point does not exist on the surface S, $\dfrac{\exp(jks)}{s}$ and $U(Q)$ always satisfy homogeneous Helmholtz equation. Even though the governing equation holds only in the domain but not on the boundaries, the property that $\dfrac{\exp(jks)}{s}$, $U(Q)$ and their second derivatives are continuous in the entire domain tells that the homogeneous Helmholtz equation is also satisfied on the boundary. Therefore on the boundary,

$$\nabla_Q{}^2\frac{\exp(jks)}{s}=-k^2\frac{\exp(jks)}{s} \tag{2.5}$$

$$\nabla_Q{}^2U(Q)=-k^2U(Q) \tag{2.6}$$

are satisfied. Substituting equation (2.5) and (2.6) for (2.4), equation (2.7) is obtained.

$$\nabla_Q\cdot V(Q,P)=0 \tag{2.7}$$

According to Helmholtz decomposition, it is possible to represent $V=\nabla\phi+\nabla\times W$. And as vector potential W identically satisfies $\nabla\cdot(\nabla\times W)=0$, V can always be expressed in terms of vector potential W in the form of,

$$V(Q,P)=\nabla_Q\times W(Q,P) \tag{2.8}$$

So the Helmholtz-Kirchhoff formula becomes

$$U(P)=\iint_S\nabla_Q\times W(Q,P)\cdot ndS \tag{2.9}$$

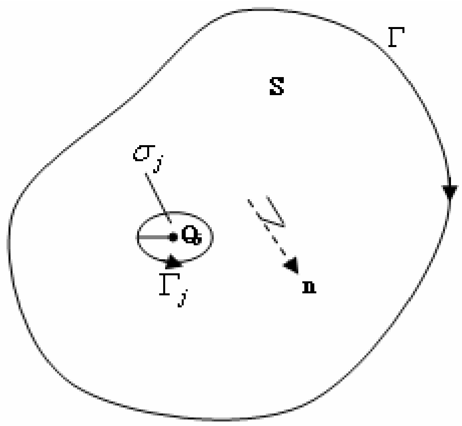

Fig. 2 Illustration of notations for the singular point of the vector potential

Now this form can be converted into a closed form of boundary line (single) integral representation by applying Stoke's theorem. For an opened surface S (Fig.2), equation (2.10) is obtained.

$$\begin{aligned}U(P)&=\iint_S\nabla_Q\times W(Q,P)\cdot ndS\\ &=\int_\Gamma W(Q,P)\cdot r'(\tau)d\tau\\ &\quad+\sum_j\lim_{\sigma_j}\int_{\Gamma_j}W(Q,P)\cdot r'(\tau)d\tau\end{aligned} \tag{2.10}$$

Here $r'(\tau)$ is the unit vector along the tangent to Γ and Γ_j. The second term of equation (2.10) is the line integration around the singular point, which arises when the vector potential $W(Q,P)$, is singular on the surface plane.

2.1.2 Singularity problem of the vector potential

In the previous section, derivation of a line (single) integral equation for the diffracted field, which is equation (2.10), was introduced. The first term of equation (2.10) does not have any problems in numerical integration. But the second term, which indicates the singularity of the vector potential $W(Q,P)$, causes a critical problem when integrating numerically. Even though the analytical solution of the singular point integration is known, it causes an enormous damage to the solution. To prevent this problem, line integral around the singular point should not be used. Instead, surface (double) integral should be applied around the singular point. Therefore, the integral equation, which is applied in practical cases, should be a hybrid form of line (single) integration and surface (double) integration as the following equation (2.11).

$$U(P)=\int_{\Gamma} W(Q,P)\cdot r'(\tau)d\tau + \sum_j \int_{\Gamma_j} W(Q,P)\cdot r'(\tau)d\tau + \iint_{S_j} V(Q,P)\cdot n dS \quad (2.11)$$

Here, $V(Q,P)$ is the vector, which is the integrand of Helmholtz-Kirchhoff formula. s_j is the surface around the singular point Q_j. Notice that the '$\lim_{\sigma_j}$' notation is not written in the second term of equation (2.11). This means that σ_j should be large enough to neglect the singular effect on the first term. Although equation (2.11) involves surface (double) integration, it provokes little increase in the numerical calculation cost. This is because only a small area of the surface around the singular point requires surface integral.

2.1.3 Development of vector potential $W(Q,P)$ for polygonal plates associated with a spherical wave

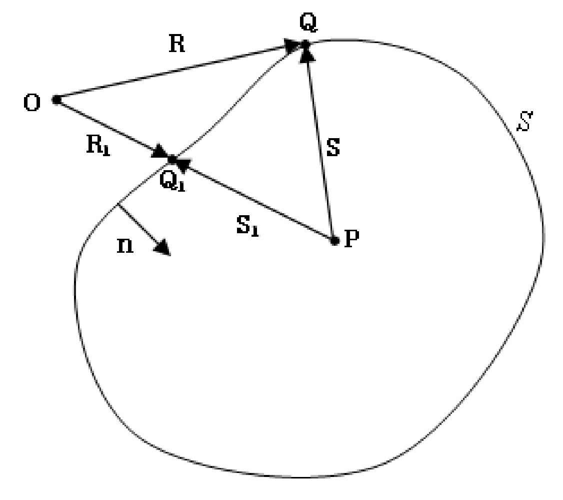

Fig. 3 Illustration for the vector potential where the initial source is a spherical wave

According to K. Miyamoto and E. Wolf, the vector potential $W(Q,P)$ of a spherical wave is

$$W(Q,P)=\frac{\exp(jk|S|)}{4\pi|S|}\hat{S} \times \left[\frac{R}{|R|+R\cdot\hat{S}}\frac{\exp(jk|R|)}{|R|}\right]+W_\infty \quad (2.11)$$

Here, $\hat{S}$ is the unit vector in the direction of S. Geometrical meanings of other variables are illustrated in Fig.3. As a divergent spherical wave follows the sommerfeld radiation condition, W_∞ is zero. Therefore, vector potential $W(Q,P)$ of a spherical wave is,

$$W(Q,P)=\frac{1}{4\pi}\frac{\exp(jk|R|)}{|R|}\frac{\exp(jk|S|)}{|S|}\frac{S\times R}{|S||R|+S\cdot R} \quad (2.13)$$

Apply equation (2.13) to a monostatic & polygonal plate case.

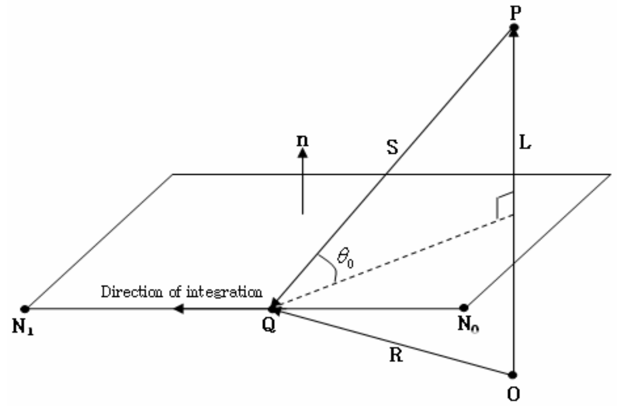

Fig. 4 Illustration of monostatic & polygonal plate case

P : Position of real source & receiver (monostatic)

O : Position of mirror image source. The Origin of calculation

Q : Point position of the boundary of polygonal plate

N_0 : Position of starting vertex of an edge of polygonal plate

N_1 : Position of starting vertex of an edge of polygonal plate

R : Vector $\overrightarrow{OQ}$

S : Vector $\overrightarrow{PQ}$

L : Vector $\overrightarrow{OP}$

θ_0 : Angle between $\overline{PQ}$ and the polygonal plate

It is also assumed that $\Re = 1$ for simplicity.

As the case is monostatic, equation (2.13) becomes

$$W(Q,P)=\frac{1}{4\pi}\frac{\exp(jk2|S|)}{|S|^2}\frac{S\times R}{|S|^2+S\cdot R} \quad (2.14)$$

The angle between S and R equals two times θ_0. Therefore equation (2.14) becomes,

$$W(Q,P)=\frac{1}{4\pi}\frac{\exp(jk2|S|)}{|S|^2}\frac{|S|^2\sin\theta}{|S|^2(1+\cos\theta)}\hat{W} \quad (2.15)$$

Where, $\theta = 2\theta_0$ and $\hat{W}=\dfrac{S\times R}{|S\times R|}$.

Substituting $\theta = 2\theta_0$ to equation (2.15),

$$W(Q,P) = \frac{1}{4\pi}\frac{\exp(jk2|S|)}{|S|^2}\tan\theta_0 \hat{W} \tag{2.16}$$

Where,

$$\tan\theta_0 = \frac{|L|}{|2R-L|}$$

$$R = \frac{1}{2}\tau(N_1 - N_0) + \frac{1}{2}(N_1 + N_0) \quad , -1 \le \tau \le 1$$

$$|S| = |R| = \left|\frac{1}{2}\tau(N_1 - N_0) + \frac{1}{2}(N_1 + N_0)\right| \quad , -1 \le \tau \le 1$$

The front three terms of equation (2.16) can be represented as a function of τ by substituting three equations above to equation (2.16).

$$\begin{aligned} &\frac{1}{4\pi}\frac{\exp(jk2|S|)}{|S|^2}\tan\theta_0 \\ &= \frac{1}{4\pi}\frac{\exp(jk|(N_0 + N_1) + (N_1 - N0)\tau|)}{|1/2(N_0 + N_1) + 1/2(N_1 - N0)\tau|^2} \\ &* \frac{|L|}{|(N_0 + N_1) + (N_1 - N0)\tau - L|} \end{aligned} \tag{2.17}$$

Also for line integration, $\hat{W} \cdot r'(\tau)$ should be evaluated where, $r'(\tau) = \frac{1}{2}(N_1 - N_0)$. Therefore,

$$\hat{W} \bullet r'(\tau) = \frac{1}{2}\frac{\left\{\frac{1}{2}(N_o + N_1) + \frac{1}{2}(N_1 - N_o)\tau - L\right\} \times \left\{\frac{1}{2}(N_o + N_1) + \frac{1}{2}(N_1 - N_o)\tau\right\}}{\left|\left\{\frac{1}{2}(N_o + N_1) + \frac{1}{2}(N_1 - N_o)\tau - L\right\} \times \left\{\frac{1}{2}(N_o + N_1) + \frac{1}{2}(N_1 - N_o)\tau\right\}\right|} \bullet (N_1 - N_o) \tag{2.18}$$

Consequently, using equation (2.17) and (2.18),

$$W(Q,P) \cdot r'(\tau) = \frac{1}{4\pi}\frac{\exp(jk2|S|)}{|S|^2}\tan\theta_0 * \hat{W} \bullet r'(\tau) \tag{2.19}$$

$*$: multiplication
$\bullet$: dot operator
$\times$: cross operator

2.2 Generalized definition of SCS

Definitions of SCS and TS have identical physical meanings. The definition of SCS is the ratio of the squared value of amplitude of radiated acoustic pressure from the source to the squared value of amplitude of re-radiated acoustic pressure by the target. Similarly, the definition of TS indicates the ratio of the radiated intensity from the source to the re-radiated intensity by the target. It is obvious that intensity is proportional to squared value of the amplitude of acoustic pressure, unless the source or receiver is close to the target compared to the wavelength.

Near field, which is the focus of the instant research, refers to the distance between the source or receiver and the target. And the phrase “near field” used in the instant research, is much larger than the wavelength and has subequal order with the characteristic length of the target.

Thus, the development of a generalized definition of SCS is, in other words, the development of a generalized definition of TS. The established definition of SCS is applicable only when the source and receiver are both far from the target. As the focus of the instant research is in the near field range, it is necessary to define a generalized representation of SCS.

2.2.1 Far field definition of SCS

Physically, SCS is the cross sectional area of the target which intercepts the initially generated pressure field and its’ notation is σ in general. To consider the phase information of the re-radiated wave, $\sqrt{\sigma}$, which is the square root value of σ will be used instead of σ .

$$\sqrt{\sigma} \equiv 2\sqrt{\pi}R\frac{U_{ref}}{|U_{inc}|} \tag{2.20}$$

Here, R is the distance from the target to the receiver. U_{ref} is the acoustic pressure at the receiver position which is re-radiated by the target. $|U_{inc}|$ is the incident acoustic pressure generated from the source. Note that $|U_{inc}|$ is the acoustic pressure at the target position.

In far field ranges, the distance between the source (receiver) and the target is much larger than the characteristic length of the target. Therefore, R and $|U_{inc}|$ can be approximated as constant values through out the entire target. Consequently, solving U_{ref} is the sole problem in evaluating SCS.

However, the approximations which were prescribed above are not valid in near field ranges. As the order of distance from the source (receiver) to the target is similar to that of the characteristic length of the target, R and $|U_{inc}|$ cannot be approximated as constant values.

2.2.2 Generalized definition of SCS applicable to near field

In near field ranges, R and $|U_{inc}|$ varies depending on the position of the selected point of the target. Consequently, $\sqrt{\sigma}$ varies even when the source position, the receiver position and the target position are fixed.

One of the important characteristics of SCS is that SCS is independent to distance. SCS only depends on the relative position of the source, receiver and target. Relative position indicates the (θ, ϕ) coordinate out of (r, θ, ϕ) spherical coordinate. Therefore, it is necessary to define a compatible definition of SCS that can be applied to near field ranges.

In the range of high frequency, it is obvious that the creeping wave (surface wave) effect is negligible. Hence, in far field,

$$\sqrt{\sigma}_{total} = \sqrt{\sigma}_1 + \sqrt{\sigma}_2 + \cdots \tag{2.21}$$

Where, $\sqrt{\sigma}_{total}$ is the total SCS of a model and $\sqrt{\sigma}_j$ is the SCS of portions of the model. This property of SCS provides a clue of generalized definition of SCS.

The generalized definition of SCS should satisfy the following conditions.

(1) The generalized SCS definition should be independent of distance from the target to receiver.
(2) The generalized SCS definition should be identical to the definition of SCS in far field ranges.
(3) The generalized SCS definition should be dependent only to the relative position of the source, receiver and the target.

Condition (1) and (3) have synonymous meanings.

Satisfying these conditions, the generalized definition of SCS is,

$$\sqrt{\sigma} \equiv \int d\sigma = \iint_S 2\sqrt{\pi} R(Q) \frac{V(Q,P)\cdot n}{|U_{inc}(Q)|} dS \tag{2.22}$$

Where, $R(Q)$ is the distance between the point of the target surface and the receiver. $V(Q,P)$ is the integrand of the Helmholtz-Kirchhoff formula. $|U_{inc}(Q)|$ is the incident acoustic pressure amplitude on the surface of the target which is generated from the source. And n is the normal vector of the surface S.

2.3 Kirchhoff approximation

As Kirchhoff approximation is a well known theory in the art, and is applied to a wide range of fields in engineering, it will be introduced briefly in this section. Kirchhoff approximation assumes that the factors $\Re$ and T, which are derived for reflection and transmission of an infinite plane wave at an infinite plane interface, can be used at every point of a rough surface interface. In other words, Kirchhoff approximation assumes wave as a ray in representing the reflected and transmitted waves at the point where the ray strikes the plane surface. As most of the underwater vehicles' surfaces are not rough, this approximation is valid. In this research, $\Re = 1$ is assumed for simplicity.

2.4 Development of near field TS equation for polygonal plates

A vector potential $W(Q,P)$ for monostatic & polygonal plate case was developed in section 2.1. And in section 2.2, a generalized definition of SCS that is applicable to near field has been suggested. According to these sections, it is possible to develop a generalized representation of TS that is applicable to near field.

2.4.1 Modification of the generalized definition of SCS

According to equation (2.22), the evaluation of SCS in near field involves a surface (double) integral. As in section (2.1), it may be possible to reduce the surface integral to line integral form if and only if the divergence of the integrand of equation (2.22) identically equals zero. Unfortunately, this is not true. Hence, it is impossible to reduce the order of integration in a mathematical procedure.

However, it is possible to reduce the cost of numerical calculation by modifying the generalized definition of SCS. As the three conditions prescribed in section 2.2.2 are the only requirements of the generalized definition of SCS, SCS may be defined differently as the followings.

$$\sqrt{\sigma} \equiv 2\sqrt{\pi} \frac{U_{ref}}{\left(\frac{|U_{inc}|}{R}\right)_{rep}} \tag{2.23-1}$$

$$\left(\frac{|U_{inc}|}{R}\right)_{rep} = \frac{\iint_S \left.\frac{|U_{inc}|}{R}\right|_Q dS}{S} \quad (2.23\text{-}2)$$

$$\sqrt{\sigma} \equiv 2\sqrt{\pi}\left(\frac{R}{|U_{inc}|}\right)_{rep} U_{ref} \quad (2.24\text{-}1)$$

$$\left(\frac{R}{|U_{inc}|}\right)_{rep} = \frac{\iint_S \left.\frac{R}{|U_{inc}|}\right|_Q dS}{S} \quad (2.24\text{-}2)$$

Where, U_{ref} is the re-radiated acoustic pressure which can be evaluated by equation (2.11) and (2.19). R is the distance between the receiver and the target.

These two types of definitions (2.23-1,2) and (2.24-1,2) satisfy the three conditions stated in section 2.2.2. $\left(\frac{|U_{inc}|}{R}\right)_{rep}$ and $\left(\frac{R}{|U_{inc}|}\right)_{rep}$ which are defined as above, will be called "the representative reference". $\left(\frac{|U_{inc}|}{R}\right)_{rep}$ and $\left(\frac{R}{|U_{inc}|}\right)_{rep}$ are representative values through the entire target surface that cancel out the dependency of distance of target-receiver.

Even though the generalized definitions of SCS (2.23) and (2.24) also involve surface integral similar to that of definition (2.22), the cost of numerical integration is not high. This is because the integrands of the surface integral in definitions (2.23) and (2.24) vary monotonically. Therefore, fine mesh of the target is not necessary, and this leads to little effect to numerical cost.

2.4.2 Development of near field TS equation for polygonal plates

Near field TS equation for polygonal plates can be developed by a combination of the previously developed equations and definitions. The TS equation is developed using equations (2.11), (2.19), definitions (2.22), (2.23) and the simple relation between TS and SCS.

$$TS = 10\log\left(\frac{\sigma}{4\pi}\right)$$

$$\sqrt{\sigma} \equiv 2\sqrt{\pi}\,\frac{U_{ref}}{\left(\frac{|U_{inc}|}{R}\right)_{rep}}$$

$$\left(\frac{|U_{inc}|}{R}\right)_{rep} = \frac{\iint_S \left.\frac{|U_{inc}|}{R}\right|_Q dS}{S}$$

$$U(P) = \int_\Gamma W(Q,P)\cdot r'(\tau)d\tau + \sum_j \int_{\Gamma_j} W(Q,P)\cdot r'(\tau)d\tau + \iint_{S_j} V(Q,P)\cdot n dS$$

$$\begin{aligned}
&W(Q,P)\cdot r'(\tau)\\
&= \frac{1}{8\pi}\frac{\exp\left(jk\left|(N_0+N_1)+(N_1-N0)\tau\right|\right)}{\left|1/2(N_0+N_1)+1/2(N_1-N0)\tau\right|^2}\\
&* \frac{|L|(N_1-N_o)}{\left|(N_0+N_1)+(N_1-N0)\tau-L\right|}\\
&\bullet \frac{\left\{\frac{1}{2}(N_o+N_1)+\frac{1}{2}(N_1-N_o)\tau-L\right\} \times \left\{\frac{1}{2}(N_o+N_1)+\frac{1}{2}(N_1-N_o)\tau\right\}}{\left|\left\{\frac{1}{2}(N_o+N_1)+\frac{1}{2}(N_1-N_o)\tau-L\right\} \times \left\{\frac{1}{2}(N_o+N_1)+\frac{1}{2}(N_1-N_o)\tau\right\}\right|}
\end{aligned} \quad (2.25)$$

$*$: multiplication
$\bullet$: dot operator
$\times$: cross operator

3. Validation of the near field TS equation for polygonal plates

3.1 Rectangular plate

As general models are composed of polygonal plates, the inconsistency of far field equation and near field equation should be examined. As shown below, the order of characteristic length compared to the order of distance affects the inconsistency.

3.1.1 x = 1m, y = 1m, Frequency = 1000Hz

For a $1m \times 1m$ rectangular plate, the difference between the far field and near field TS equations is negligible in the entire range. This is because the characteristic length of the plate is much shorter than the distance.

Table 1 Comparison of maximum TS value for a x=1m, y=1m rectangular plate

Distance	20m	100m	10000m
FF eq.	-3.5 dB	-3.5 dB	-3.5 dB
NF eq.	-3.5 dB	-3.5 dB	-3.5 dB

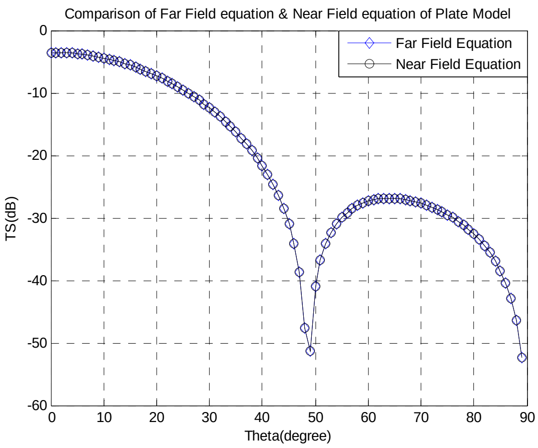

Fig. 5 Comparison of the far field equation and near field equation for a x=1m, y=1m rectangular plate - Distance = 10000m

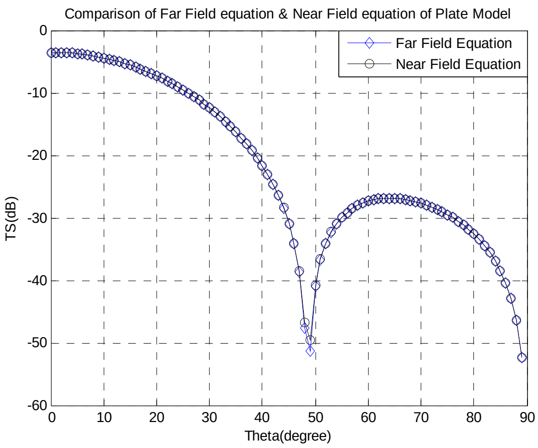

Fig. 6 Comparison of the far field equation and near field equation for a x=1m, y=1m rectangular plate - Distance = 10000m

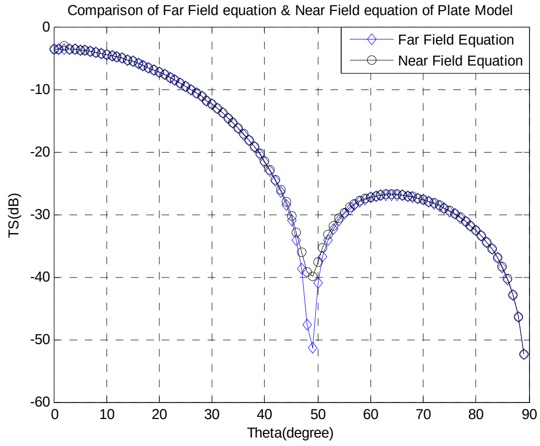

Fig. 7 Comparison of the far field equation and near field equation for a x=1m, y=1m rectangular plate - Distance = 20m

3.1.2 x = 10m, y = 0.5m, Frequency = 1000Hz

For a $10m \times 0.5m$ plate, the inconsistency of the near field equation and the far field equation is more obvious than that of a $1m \times 1m$ plate.

Specifically, in Fig.10, the effect of near field becomes larger as the orders of distance and the characteristic length become subequal.

Table 2 Comparison of maximum TS value for a x=10m, y=0.5m rectangular plate

Distance	20m	100m	10000m
FF eq.	10.5 dB	10.5 dB	10.5 dB
NF eq.	5.6 dB	10.0 dB	10.5 dB

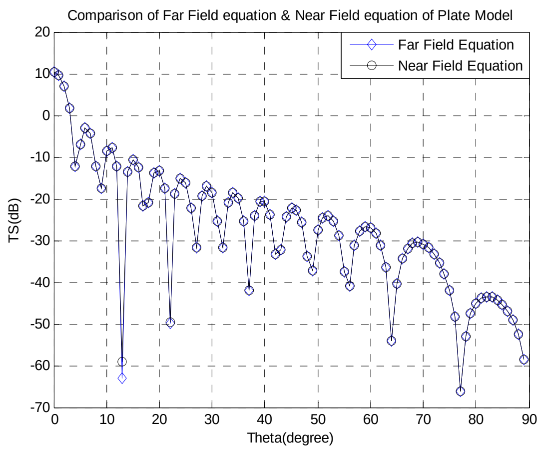

Fig. 8 Comparison of the far field equation and near field equation for a x=10m, y=0.5m rectangular plate - Distance = 10000m

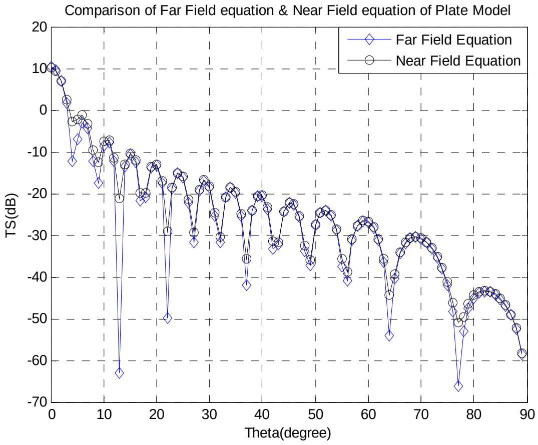

Fig. 9 Comparison of the far field equation and near field equation for a x=10m, y=0.5m rectangular plate – Distance = 100m

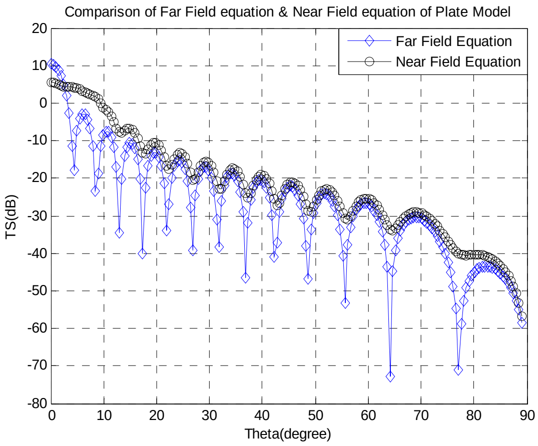

Fig. 10 Comparison of the far field equation and near field equation for a x=10m, y=0.5m rectangular plate – Distance = 20m

3.2 Cylinder

The purpose of examining a cylinder is to determine as to whether the curvature affects the inconsistency between the near field equation and the far field equation. The examination to cylinder shows that the curvature affects the inconsistency around the normal direction even in the far field.

3.2.1 R=5m, H=10m, Frequency=1000Hz

As shown in the previous examination of the plates, the relative characteristic length of the cylinder to the distance affects the inconsistency. Moreover, as shown in Fig.12, the curvature of the cylinder affects the inconsistency around the normal direction even in the far field ranges.

Table 3 Comparison of maximum TS value for a R=5m, H=10m cylinder

Distance	20m	100m	10000m
FF eq.	13.5 dB	21.4 dB	22 dB
NF eq.	22.2 dB	24.2 dB	24 dB
Analytical	22.2 dB	22.2 dB	22 dB

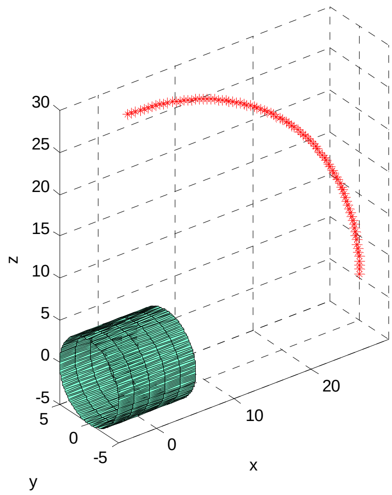

Fig. 11 R=5m, H=10m Cylinder model

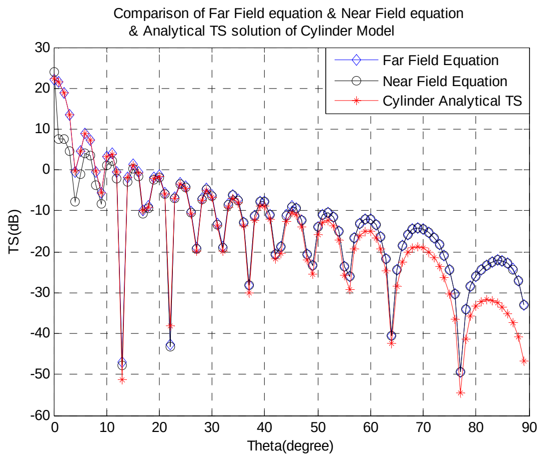

Fig. 12 Comparison of the far field equation, near field equation and analytical TS solution for a R=5m, H=10m cylinder model – Distance=10000m

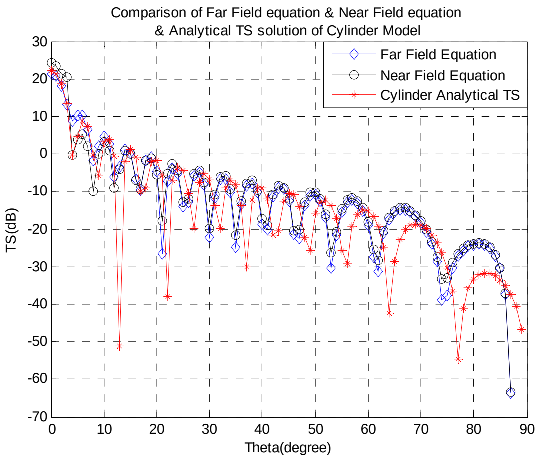

Fig. 13 Comparison of the far field equation, near field equation and analytical TS solution for a R=5m, H=10m cylinder model – Distance=100m

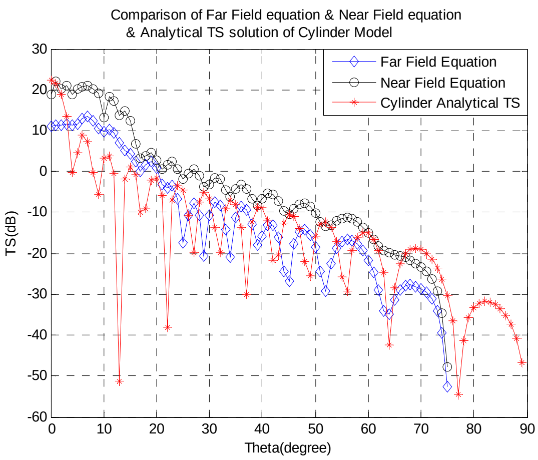

Fig. 14 Comparison of the far field equation, near field equation and analytical TS solution for a R=5m, H=10m cylinder model – Distance=20m

3.2.2 R=5m, H=40m, Frequency=1000Hz

As the characteristic length of the cylinder increases, the inconsistency increases accordingly. It becomes more obvious that the curvature of the cylinder affects the inconsistency near the normal direction even in the far field range.

Table 4 Comparison of maximum TS value for a R=5m, H=40m cylinder

Distance	50m	100m	10000m
FF eq.	18.3 dB	21.8 dB	34.2 dB
NF eq.	25.4 dB	24.6 dB	35.5 dB
Analytical	34.3 dB	34.3 dB	34.3 dB

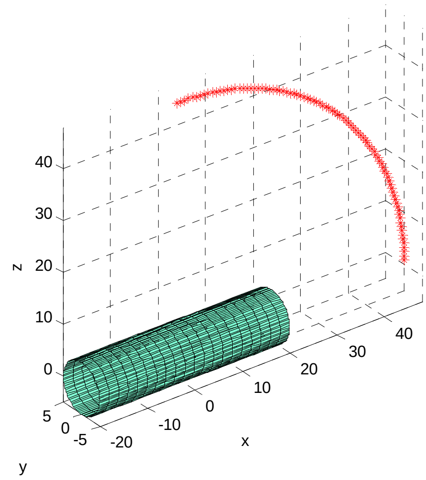

Fig. 15 R=5m, H=40m Cylinder model

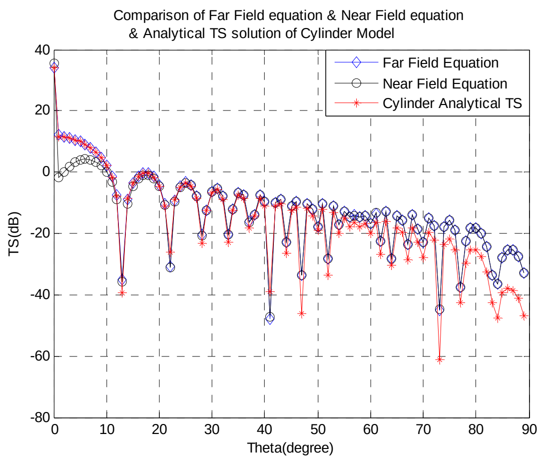

Fig. 16 Comparison of the far field equation, near field equation and analytical TS solution for a R=5m, H=40m cylinder model – Distance=10000m

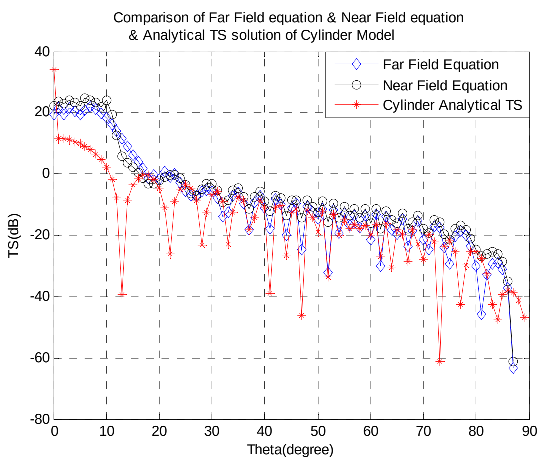

Fig. 17 Comparison of the far field equation, near field equation and analytical TS solution for a R=5m, H=40m cylinder model – Distance=100m

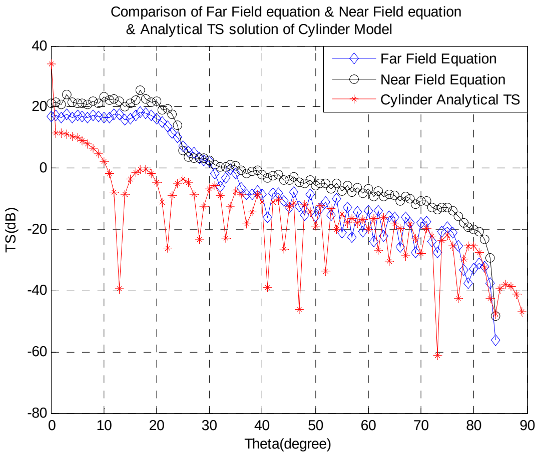

Fig. 18 Comparison of the far field equation, near field equation and analytical TS solution for a R=5m, H=40m cylinder model – Distance=50m

4. Application to an underwater vehicle (Submarine)

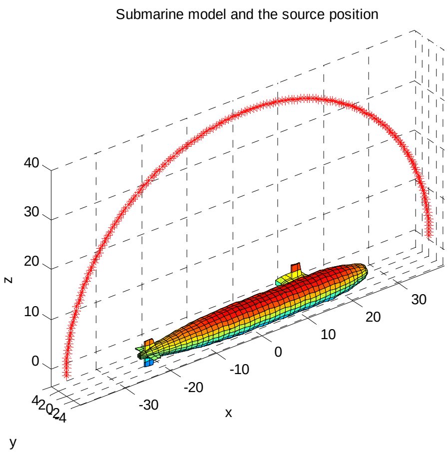

Fig. 19 Submarine Model & Source position

4.1 Far field analysis

In the previous section (2.2.2), the generalized definition of SCS satisfies the three conditions. Specifically, as condition (2) is satisfied, TS evaluated using either the far field equation or the near field equation would have subequal values. However, as shown in the previous section (3.2), curvature affects inconsistency between the far field equation and the near field equation. Therefore, similar tendency of the curve, but not identical values are observed in the results.

Table 5 Comparison of maximum TS values of the far field equation and near field equation Distance = 10000m, Frequency = 5000Hz

Maximum TS value	
FF eq.	31.8 dB
NF eq.	34.2 dB

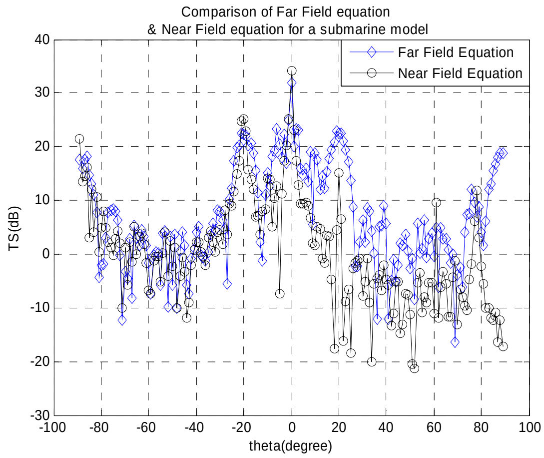

Fig. 20 Comparison of the far field equation and near field equation and analytical TS solution for a submarine model Distance = 10000m, Frequency = 5000Hz

4.2 Near field analysis

As shown in the examinations for plates and cylinders, TS values evaluated by the near field equation and far field equation are far inconsistent. Hence, it is obvious that inconsistent results will be shown in the near field analysis.

Table 6 Comparison of maximum TS values of the far field equation and near field equation Distance = 10000m, Frequency = 5000Hz

Maximum TS value	
FF eq.	25.0 dB
NF eq.	27.2 dB

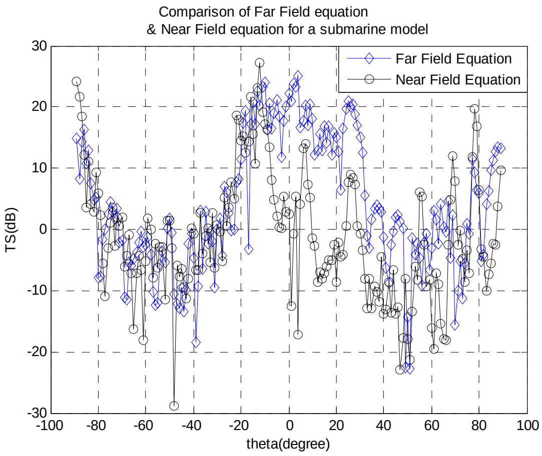

Fig. 21 Comparison of the far field equation and near field equation and analytical TS solution for a submarine model

$Distance = 120m \quad Frequency = 5000Hz$

5. Conclusions

By introducing a generalized definition of SCS and a vector potential for polygonal plates in monostatic conditions, a new TS equation applicable to near field has been developed. Because this equation assumes a spherical source, it solves the problems of previously developed methods. Such methods assume far field, so that spherical wave could be assumed as a plane wave in the aspect of phase. In contrast, the method developed in the instant research does not assume any approximations. Therefore, the equation developed in the instant research is applicable to near field as well as far field; the instant method produces better analysis results with a relatively higher precision. Also, as the equation's major component is line integration, it is possible to reduce the cost of numerical calculation.

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