Numerical Analysis for Fluid-Structure Interaction in Aircraft Structure Considering Uncertainty

불확정성을 고려한 항공기 구조물의 유체-구조간 상호 간섭 현상의 수치 해석

ChanHoon Chung† and SangJoon Shin* (Seoul National University)

정 찬훈†, 신 상준* (서울대학교)

Key Words: Aeroelasticity, Uncertainty, Unsteady aerodynamics, Cantilevered wing.

ABSTRACT

For the modern aircraft, uncertainty has been an important issue to its aeroelastic stability. Therefore, many researches have been conducted regarding this topic. The uncertainties in the aeroelastic system may consist of the structural and aerodynamic uncertainty. In this paper, we suggest a parametric uncertainty modeling and conduct the aeroelastic stability analysis of a typical wing including the uncertainty.

1. Introduction

The modern aircrafts, especially the military aircrafts, are generally required to have higher performance and maneuverability while they perform the missions. However, there are still some limitations in those aircrafts. The aeroelastic phenomenon, flutter, is one of the important situations which limit the aircraft flight speed. Such aeroelastic issue was early founded by Wright Brothers and many researchers have studied it to improve the performance of the aircrafts.

On the other hand, when the aircraft operates in a high speed flight, there may exist many uncertainties in its structural and aerodynamic characteristics. For example, a slight change of the wing structural mode may induce a variation of its aerodynamic forces. These uncertainties will ultimately influence the flutter speed characteristics of the aircraft. Thus, an accurate prediction of the flutter speed including those uncertainties will be quite important to the aircraft safety.

Lind at the NASA Dryden Flight Research Center suggested a match point solution method about a robust flutter prediction [1].

He examined the variation of the aircraft flutter speed in terms of the variation of the altitude. Borglund suggested a μ-k method for a robust aeroelastic stability analysis [2]. However, those examinations have not considered an uncertainty in the generalized forces and the structural mode. When an aeroelastic interaction occurs on the aircraft structure, its deflection modes may have a significant change. Furthermore, if the structural mode changes, the aerodynamic force acting on it should be different as well.

Danowsky suggested a flutter prediction method including these uncertainties [3]. He used a NASTRAN structural model and a doublet lattice aerodynamics. However, this method required a large number of discretized panels for an accurate analysis.

In the present paper, we suggest an analysis of aeroelastic stability boundary in frequency domain when varying the natural frequencies. We apply the developed analysis to a three-dimensional wing which was analyzed by Goland [4]. For aerodynamics, the analysis is based on the lifting line theory. For uncertainty, we use parametric structural and aerodynamic uncertainties.

† 서울대학교 공과대학 기계항공공학부
E-mail : chan3241@snu.ac.kr
Tel : (02) 880-1901.

* 서울대학교 공과대학 기계항공공학부

2. Aeroelastic Model

2.1 Three-dimensional Wing

For a preliminary study, we conducted an analysis about a two−dimensional airfoil section. However, in such an airfoil analysis, generalized mass, stiffness and aerodynamic effects were not included. The airfoil analysis also needed more assumptions. Thus, for more accurate prediction, we need to consider a three−dimensional wing model. Figure 2.1 represents a three−dimensional wing. This is the model examined by Goland [4]. We assume that the present wing is a uniform and cantilevered wing

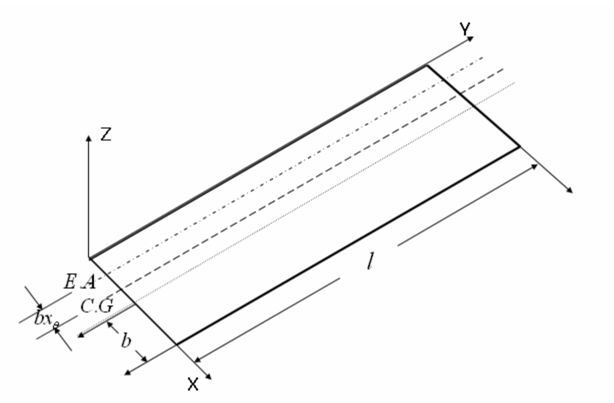

Figure 1. Three−dimensional wing model

2.2 Governing Equations

Aeroelastic equations can be represented as a typical mechanical vibration governing equation as follows.

$$M\ddot{q} + Kq = F_G \tag{1}$$

Hodges suggested to use Eq. (1) based on the generalized modes [5]. In Eq. (1), q denotes generalized bending and torsion modes.

$$q = \begin{bmatrix} \eta \\ \theta \end{bmatrix} \tag{2}$$

Three bending and three torsional mode shapes are used in the present analysis as follows.

$$\Psi_i = \cosh(\alpha_i y) - \cos(\alpha_i y) - \beta_i[\sinh(\alpha_i y) - \sin(\alpha_i y)] \tag{3}$$

$$\alpha_i = \frac{i\pi}{l} \tag{4}$$

$$\Theta_i = \sqrt{2}(\gamma_i y) \tag{5}$$

$$\gamma_i = \frac{\pi(i - \frac{1}{2})}{l} \tag{6}$$

In Eq. (1), the mass and stiffness matrix are the generalized mass and stiffness matrix, which can be described as follows.

$$M = ml\begin{bmatrix} [\Delta] & -bx_\theta[A]^T \\ -bx_\theta[A] & b^2r^2[\Delta] \end{bmatrix} \tag{7}$$

$$K = \begin{bmatrix} \frac{EI}{l^3}[B] & [0] \\ [0] & \frac{GJ}{l}[T] \end{bmatrix} \tag{8}$$

In above equation, Δ matrix means an identity matrix, and elements of matrix A represent coupling between the modes and can be expressed as follows.

$$A_{ij} = \frac{1}{l}\int_0^l \Theta_i \Psi_i dy \tag{9}$$

Matrices B and T are diagonal matrices given as follows.

$$B_{ii} = (\alpha_i l)^4 \tag{10}$$

$$T_{ii} = (\gamma_i l)^4 \tag{11}$$

In Eq. (1), F_G denotes the generalized aerodynamic forces. They consist of the generalized lift and pitching moment. Hodges also suggested the generalized aerodynamic forces as follows.

$$F_G = \begin{bmatrix} \Xi_\omega \\ \Xi_\theta \end{bmatrix} \quad (12)$$

The present three-dimensional aerodynamic forces are developed based on the two-dimensional lift and moment. In Eq. (12), Ξ_w and Ξ_θ denotes the generalized lift and moment, respectively, and which are formulated as follows.

$$\Xi_{w_i} = \int_0^l \Psi_i L' dy \quad (13)$$

$$\Xi_{\theta_i} = \int_0^l \Theta_i [M'_{1/4} + (1/2 + a) bL'] dy \quad (14)$$

where the two-dimensional lift and moment are obtained from the previous study as follows.

$$L' = 2\pi\rho UbC(k)[U\theta - \frac{\partial w}{\partial t} + b(\frac{1}{2} - a)\frac{\partial \theta}{\partial t}] + \pi\rho b^2 (U\frac{\partial \theta}{\partial t} - \frac{\partial^2 w}{\partial t^2} - ba\frac{\partial^2 \theta}{\partial t^2}) \quad (15)$$

$$M'_{1/4} = -\pi\rho b^3 [U\frac{\partial \theta}{\partial t} - \frac{1}{2}\frac{\partial^2 w}{\partial t^2} + b(\frac{1}{8} - \frac{a}{2})\frac{\partial^2 \theta}{\partial t^2}] \quad (16)$$

The aerodynamic force is now constructed as a matrix equation as follows.

$$\begin{bmatrix} \Xi_\omega \\ \Xi_\theta \end{bmatrix} = -\pi\rho b^2 l \begin{bmatrix} [\Delta] & ba[A]^T \\ ba[A] & b^2(a^2 + 1/8)[\Delta] \end{bmatrix} \ddot{q}$$

$$-\pi\rho bUl \begin{bmatrix} 2C(k)[\Delta] & -b[1+2(1/2-a)C(k)][A]^T \\ 2b(1/2+a)C(k)[A] & b^2(1/2-a)[1-2(1/2+a)C(k)][\Delta] \end{bmatrix} \dot{q}$$

$$-\pi\rho bU^2 l \begin{bmatrix} [0] & -2C(k)[A]^T \\ [0] & -b(1+2a)C(k)[\Delta] \end{bmatrix} q \quad (17)$$

We assume a simple harmonic motion.

$$\eta = \overline{\eta} \exp^{iwt} \quad (18)$$

$$\theta = \overline{\theta} \exp^{iwt} \quad (19)$$

$$\mu = \frac{m}{\pi\rho b^2} \quad (20)$$

Then, the aeroelastic governing equation is finalized as follows.

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \overline{\eta} \\ \overline{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (21)$$

3. Uncertainty Modeling

3.1 Uncertainty in the Aeroelastic System

Livne reported that the uncertainty problem would be one of the important future topics in aeroelasticity [6]. In the present aeroelastic stability analysis, we add an uncertainty which consists of the structural and aerodynamic uncertainty. Structural uncertainty is induced by change of the stiffness and mass.

Such variation of stiffness and mass influences the natural frequency of the wing. And of the varied natural frequencies induce a variation in the generalized mass and stiffness. Finally, these changes influence aerodynamic forces acting on the wing. Therefore, in this paper, we consider these uncertainties and suggest aeroelastic stability analysis including them.

3.2 Structured Parametric Uncertainty

In order to include uncertainty, the previous aeroelastic equations need to be converted into an uncoupled form. For that, we use the modal mass and stiffness matrices obtained as follows.

$$M' = E^T M E \quad (22)$$

$$K' = E^T K E \quad (23)$$

Then, the modal mass and stiffness matrices become diagonalized. Now, we assume that the modal stiffness has some uncertainty weight values defined as follows.

$$K_w = W \times K' \quad (24)$$

where the uncertainty weight matrix W can be represented as follows.

$$W = diag(w_1, w_2, diag(0)) \qquad (25)$$

3.3 Aerodynamic Uncertainty

When the structural mode shape of the wing has an unsteady variation, then the resulting aerodynamic force exhibits a difference in its magnitude and phase. Furthermore, if the aircraft is operated in either a compressible or incompressible flight regime, its unsteady aerodynamic forces will also have a different value in its magnitude or phase. Such variation in the aerodynamic forces may be represented by that in the Theodorsen lift deficiency function. So, we consider the aerodynamic uncertainty based on Theodorsen's function. The Theodorsen's function is generally defined as

$$C(k) = F(k) + G(k)i \qquad (26)$$

Then we assume its real and imaginary part has its respective uncertainty weight value. Now the Theodorsen's function can be represented as follows.

$$C_w(k) = w_1 F(k) + w_2 G(k)i \qquad (27)$$

4. Numerical Results.

4.1 Goland's Wing

We use the Goland's three-dimensional wing, and Table 1 shows the characteristic values of it.

Table 1. Characteristic values of the wing

Wing Span	$20\,ft$	Static Imbalance	$0.447 slug/ft$
Chord	$6\,ft$	EI	$m \times 31.7 \times 10^6$ $lb \cdot ft^3/slug$
Radius of gyration	25% chord	GJ	$I \times 1.23 \times 10^6$ $lb \cdot ft/slug$
Spanwise elastic axis	33% chord	Mass moment of inertia	1.943 $slug \cdot ft^2/ft$
Center of gravity	43% Chord	Mass of unit length	$0.743 slug/ft$
Semi chord	$3\,ft$		

4.2 Numerical Results

Table 2 shows the results of the structural natural frequencies.

Table 2. Natural frequency results (rad/s)

	1st B	1st T	2nd B	2nd T	3rd B	3rd T
Goland [4]	50.0	87.0	–	–	–	–
Present method (with S)	47.8	91.6	333.9	249.0	1235.9	429.2
Present method (without S)	48.5	87.1	310.1	261.3	868.4	435.5
Uncoupled beam analysis	49.5	87.1	308.4	261.3	868.2	435.5

In Table 2, S denotes the static imbalance, and we compute the difference between two cases, in which one includes the static imbalance and the other without it. By Eq. (7), S can be expressed as follows.

$$S = mx_\theta \qquad (28)$$

Table 3 shows the results about the flutter speeds both at 20,000 ft above the sea level and at the sea level.

Table 3. Flutter speed and frequency results

	Sea level		20,000 ft above the sea level	
	Quasi Steady	Unsteady	Quasi steady	Unsteady
Flutter speed	476 ft/sec	465 ft/sec	579 ft/sec	576 ft/sec
Flutter frequency	87 rad/s	85 rad/s	86.5 rad/s	88 rad/s

In Table 3, we use the atmospheric density equation as follows.

$$\rho = \rho_0 (1 - 0.000006875h)^{4.2561} \qquad (29)$$

To analyze the influence on the flutter characteristics due to the uncertainty, we use a structured parametric uncertainty weight value conducted as follows. Figures 2 and 3 show the influence on the first bending and torsion natural frequencies by the weight values of the coupled modes.

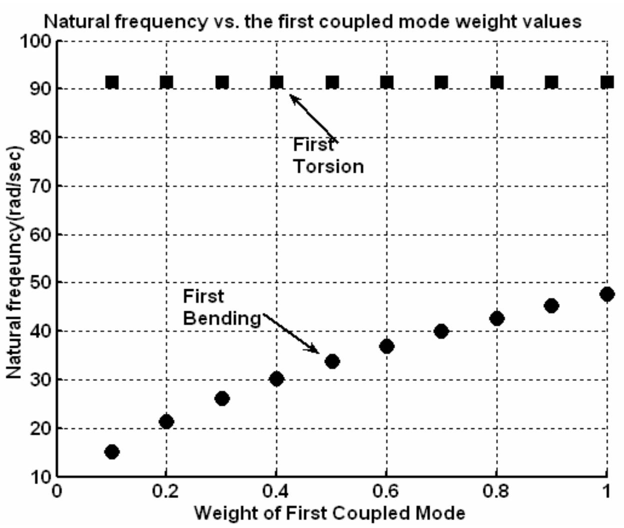

Figure 2. Natural frequency vs. the first coupled mode weight values

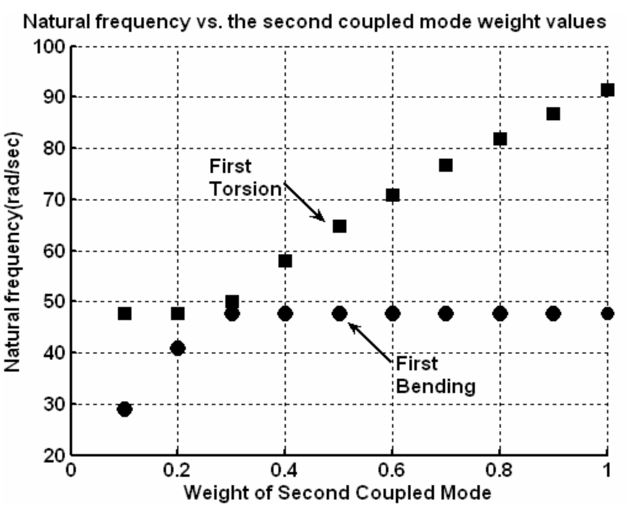

Figure 3. Natural frequency vs. the second coupled mode weight values

From Figures 2 and 3, an uncertainty of the first coupled mode influences the first bending modes and that of the second coupled mode influences the first torsion mode. However, in the previous result, it is found that such a high coupling does not influence much upon the natural frequency. The first coupled mode is a combination of the first pure bending and the first pure torsion mode, but dominated by the bending mode. Thus, it influences a lot on the bending frequency. The second coupled mode is also a mixture of the same modes, but dominated by the torsion mode. Then, it influences much on the torsional frequency. Figures 4 and 5 represent the influence of the weight values at each coupled mode upon the flutter speed and frequency.

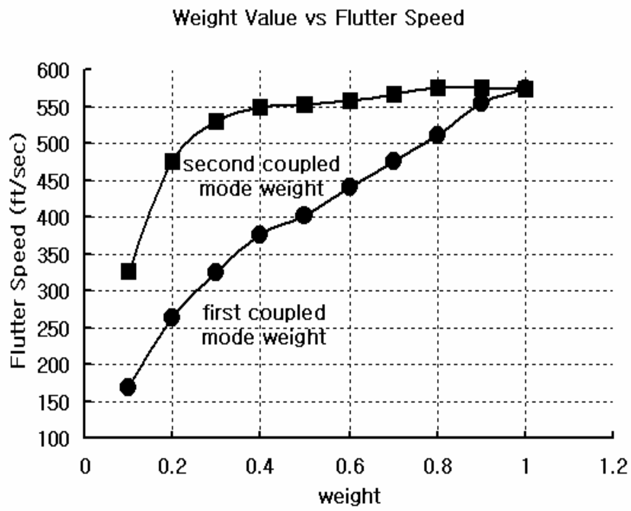

Figure 4. Flutter speed in terms of the weight on each coupled mode

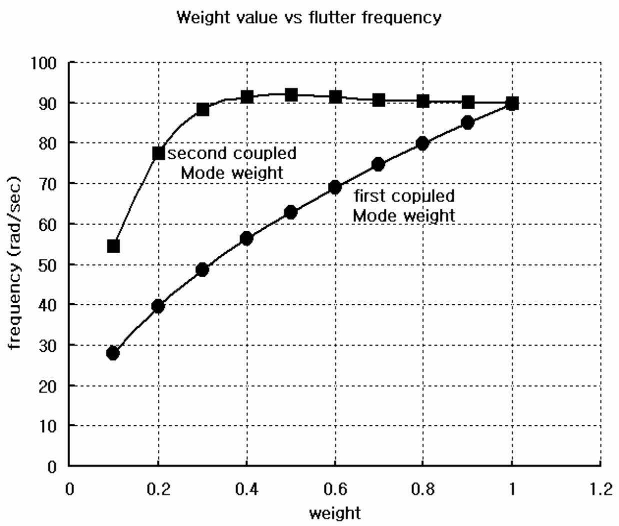

Figure 5. Flutter frequency in terms of the weight values on each coupled mode

As shown in Figures 4 and 5, an uncertainty on the first coupled mode influences much more significantly than that on the second coupled mode does upon the flutter speed of the aircraft. In the wing flutter analysis, an influence by the first mode pure bending motion is usually most significant. In the present analysis, the weight on the first coupled mode signifies a variation of the first mode pure bending motion. Thus, its influence is most significant on the flutter results.

For an uncertainty in unsteady aerodynamic forces, we apply a weight value upon the real and imaginary parts in the Theodorsen's function. Figures 6 and 7 show the flutter speed and frequency results in terms of the weight values included in the real and imaginary parts.

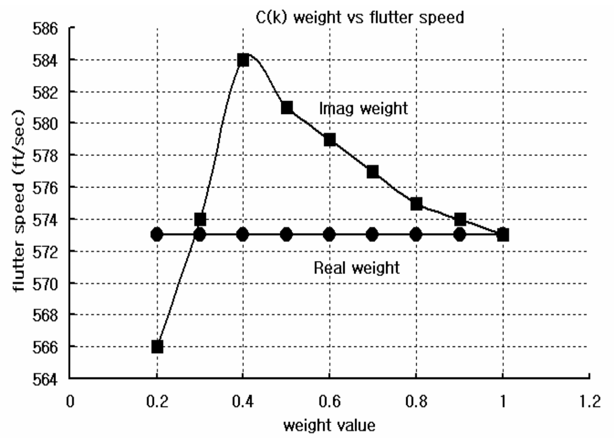

Figure 6. Flutter speed in terms of the weight values in the Theodorsen's function

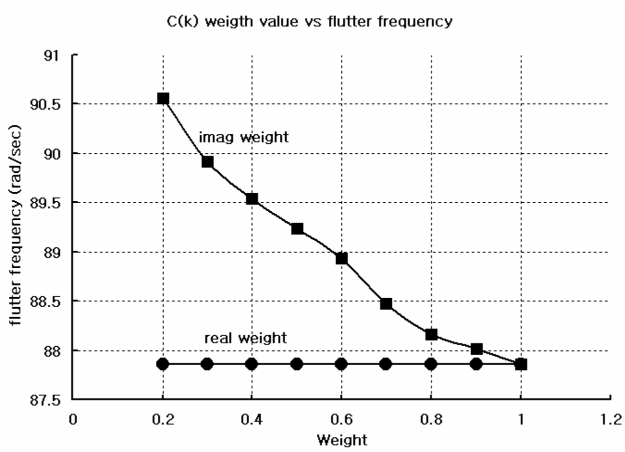

Figure 7. Flutter frequency in terms of weight values in the Theodorsen's function

As shown in Figures 7 and 8, an uncertainty in the imaginary part influences much upon the flutter characteristics of the aircraft, On the contrary, an effect of uncertainty in the real part almost does not influence any on them. In the Theodorsen's function, the imaginary part is a phase lag of the unsteady aerodynamic forces with respect to the wing motion. Therefore, it is concluded that a phase variation on the unsteady aerodynamic forces induces a much more significant change in the flutter characteristics of the aircraft.

5. Conclusions

In this paper, we study the uncertainty effects which exist in the structural and aerodynamic aspects. We predict the flutter boundary and frequency of the aircraft including the uncertainty. We suggest a quite simple model to predict its stability which contains the parametric structural and aerodynamic uncertainties. From the numerical results obtained, it is found that the first coupled mode uncertainties are more influential than the second coupled mode uncertainty is on the flutter characteristics. For an aerodynamic force uncertainty, we assume that the uncertainty exists in the Theodorsen' s function. The uncertainty in the imaginary part is much more influential than the real part uncertainty is. In the future, we will verify the present stability boundary results with the doublet lattice method. And, we will examine an active aeroelastic control device on the aircraft and design its control laws to augment the stability including the uncertainty.

Acknowledgment

This work was supported by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD, Basic Research Promotion Fund) (KRF−2006−331−D00086).

References

[1] Lind, R., "Match−Point Solutions for Robust Flutter Analysis," *Journal of Aircraft*, Vol. 39,No. 1, 2002, pp.91−99.

[2] Borglund, D., "The μ−k Method for Robust Flutter Solutions," *Journal of Aircraft*, Vol. 41,No. 5, 2004, pp. 1209−1216.

[3] Danowsky, B, P., and Chavez, F, K., "Formulation of an Aircraft Structural

Uncertainty Model for Robust Predictions," Proceedings of the 45th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Materials Conference, 2004, pp.1−15.

[4] Goland, M., "The Flutter of a Uniform Cantilever Wing," *Journal of Applied Mechanics*, Vol. 12,No. 4, 1945, pp.A197−A208.

[5] Hodges, D. H., *Introduction to Structural Dynamics and Aeroelasticity*, Cambridge University Press, 2002.

[6] Livne, E., "Future of Airplane Aeroelasti−city," *Journal of Aircraft*, Vol. 40,No. 6, 2003, pp.1066−1092.