The effect of 2D & 3D ionospheric model in interfrequency bias estimation

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Abstract

The radio signal in GNSS was intentionally designed with two frequencies in order to combat the dispersion error caused by trans-ionospheric propagation. By measuring the path delay independently at the two, widely spaced GPS frequencies, L1 & L2, the TEC along the path from satellite to receiver can be measured directly. The issue with dual frequency measurement of the ionosphere is the calibration of L1/L2 interfrequency biases. L1/L2 interfrequency biases are generated because physical electric signal paths of L1 and L2 circuits are different from each other for both satellites and receiver. Conventionally L1/L2 interfrequency bias is estimated and broadcasted by 2D ionospheric model.
In this paper, we estimated IFB (interfrequency bias) by 2D & 3D ionospheric models including real time filter methods and compared the result of those and concluded the merit of 3D tomography model to recover the problem of 2D thin shell model. We confirmed our conclusion by experimental data.

Keywords: Ionospheric delay, Tomography, IFB

1. Introduction

The Global Positioning System (GPS) is a relatively new tool for studying the ionosphere. This satellite system's original and continuing purpose is four dimensional radionavigation based on one way ranging. The radio signal was intentionally designed with two frequencies in order to combat the dispersion error caused by transionospheric propagation. For navigation the dispersion causes a nuisance delay in the range measurement. On the other hand the dual frequency ranging signal can probe the ionosphere if the transmitter and receiver are at known locations. Indeed GPS has become the most widely used sensor for ionospheric study.

While the L-band (-1575.42MHz and -1227.6MHz) frequency selection for the GPS signal was largely political it has a great deal of technical merit in balancing propagation loss against refraction. L-band signals allow reception by antennas commensurate with hand-held receivers and yet the phase path is well approximated as a straight line which keeps the effects linear.

One issue with dual frequency measurement of the ionosphere is the calibration of the dispersion within the antenna/cable/receiver. That is, the phase paths between the antenna and the correlation loops in the receiver are different. The difference corrupts the ionospheric measurements by introducing a bias. If this interfrequency bias (IFB) is not calibrated the receiver yields relative rather than absolute ionospheric measurements.

There are two primary difficulties in calibrating a receiver's IFB. First the antenna/cable/receiver installation cannot be altered after calibration since that will change the IFB. This precludes a laboratory calibration. Second, the IFB has a first order temperature dependence meaning that it is time varying when not under climate control. Previous work in the literature has neglected one or both of these complications. Here an adaptive filter is designed and implemented for estimating the IFB directly from the receiver's measurements of the GPS signals. The technique is general in that it will accept prior estimates if available and it produces estimates for any number of receivers inputting measurements. In fact the procedure improves with the number of receivers providing measurements.

Beginning with the index of refraction in a plasma the ionospheric measurements are derived in terms of the GPS observables available from the receiver. Given a measurement equation, the IFB calibration problem is cast in state space. Next section describes the real-time Kalman filter for implementation. The final section contains the output results and a comparison of the performance with 2D & 3D model. The limitation of 2D model and the merit of 3D model will be explained.

2. Paper Preparation

2.1 2D Modelling assumptions and inter-frequency bias

We have used Klobuchar's assumptions in our ionospheric time-delay model (Klobuchar, 1987). Figure 1 shows these assumptions;

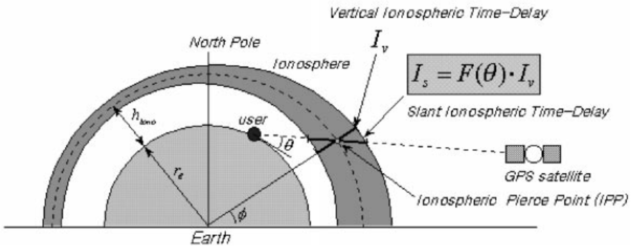

Figure 1. 2D Ionospheric time-delay modelling assumptions.

The ionosphere is assumed to be concentrated at the Ionospheric Pierce Point (IPP) and its average height (hiono) is 350 km~450km from the ground. This is the key concept of the 2D ionospheric model. The real delay of the GPS signal is a slant ionospheric time delay, but this is not appropriate for the 2D model because it varies according to satellite elevation angle; hence, the vertical ionospheric time delay should be used. Vertical and slant ionospheric time delays are related by an obliquity factor: Is =F x Iv , which is only a function of the satellite elevation angle: F = F(θ) (Qiu et al., 1994).

As ionosphere activity is dominated by local time and geomagnetic latitude, the ionospheric time-delay model should be expressed in the coordinate of local time (λ) and geomagnetic latitude (Φ) of the IPP. These can be calculated from GPS time, geographical latitude and longitude.

In the implementation, the ionospheric vertical delay is modeled and expanded by k-th order spherical harmonics, i.e.

$$I_v = \sum_{n=0}^{k}\sum_{m=0}^{n}\{C_{nm}\cos(m\lambda) + S_{nm}\sin(m\lambda)\}P_{nm}(\sin(\phi)) \quad (1)$$

where P_{nm} is the legendre function.

For determining ionospheric model, we must solve coefficients (C_{nm} , S_{nm})

The measured slant ionospheric time delay (Is) contain L1/L2 inter-frequency biases (Wilson and Mannucci, 1994), which are generated because the electrical signal paths of the L1 and L2 circuits differ from each other. The biases exist both at the GPS satellite (Ib_TX) and the receiver (Ib_RX)(Wilson and Mannucci, 1993). Thus, Is can be formulated as Equation (2).

$$I_s = I_v \cdot F(\theta) + I_{b\_RX} - I_{b\_TX} + v \quad (2)$$

where v is the measurement noise.

If we declare that coefficients (C_{nm}, S_{nm}) are unknown vector $\underline{x}$ and Is is measurement z,

$$z = H\underline{x} + I_{b\_RX} - I_{b\_TX} + v \quad (3)$$

2.2 Ionospheric observation

The time delay of GPS radiowave propagating from transmitter to receiver through ionosphere is given by

$$\Delta t = \frac{1}{c}\int_{R(r)}^{SV(r)}(1-n)dl(r) \quad (4)$$

where c is the speed of light, SV is the transmitter location, R is the receiver location, n is the index of refraction, and r is a four dimensional position vector. The effect of the ionosphere is captured in the index of refraction, n, which is a function of both radiowave frequency and position along the phase path. The full expression for the complex index of refraction in a plasma such as the ionosphere is given by the Appleton-Hartree equation.

$$n^2 = \frac{X}{1 - iZ - \frac{Y_T^2}{2(1-X-iZ)} \pm \left[\frac{Y_T^4}{4(1-X-iZ)^2} + Y_L^2\right]^{\frac{1}{2}}} \quad (5)$$

where

$X = \frac{\mathrm{N(r)e^2}}{\varepsilon_0 m f^2} = \frac{f_N^2}{f^2}$: thermal motion of the electrons

$Y_L = \frac{eB_L}{mf} = \frac{f_H\cos\theta}{f}$: longitudinal component of the Lorentz force

$Y_T = \frac{eB_T}{mf} = \frac{f_H\sin\theta}{f}$: transverse component of the Lorentz force

$Z = \frac{v}{f}$: the ratio of collision frequency to radio frequency

N(r) is the local electric density of the plasma, e is the charge on an electron, ε_0 is the permittivity of free space, m is the mass of an electron, B_L and B_T terms are the longitudinal and transverse components of the geomagnetic field, f_H is the gyro (cyclotron) frequency and θ is the angle between the geomagnetic field vector and wave vector. Typically the local plasma frequency in the ionosphere is around 10 MHz, gyro frequency is around 1 MHz, and the collision frequency is around 10 kHZ. So, the L-band approximation to the Appleton-Hartree equation is

$$n \approx 1 - \frac{X}{2} \quad (6)$$

This is comparatively simple and yet good to better than 1% error. By substituting (6) into (4),

$$\Delta t = \frac{1}{c}\int_{R(r)}^{SV(r)}(1-n)dl(r) = \frac{1}{c}\int_{R(r)}^{SV(r)}\frac{X}{2}dl(r)$$
$$= \frac{a}{cf^2}\int_{R(r)}^{SV(r)}N(r)dl(r), \quad a = \frac{e^2}{8\pi^2\varepsilon_0 m} \quad (7)$$

If we apply equation (7) in L1,L2 and subtracting each other, we can get following equation.

$$\delta(\Delta t) = \Delta t_{L_2} - \Delta t_{L_1} = \frac{a}{c}\int_{R(r)}^{SV(r)}Ndl(r)\left(\frac{1}{f_{L_2}{}^2} - \frac{1}{f_{L_1}{}^2}\right) \quad (8)$$

Then. the total electron content (TEC) along the line of sight including IFB can be observed by dual frequency GPS receivers with the instantaneous code delay observation

$$TEC = \frac{f_{L_1}^2 \cdot f_{L_2}^2}{f_{L_1}^2 - f_{L_2}^2} \times \frac{c\cdot\delta(\Delta t)}{a} = \frac{f_{L_1}^2 \cdot f_{L_2}^2}{f_{L_1}^2 - f_{L_2}^2} \times \frac{\rho_2 - \rho_1}{a} - IFB \quad (9)$$

Typical IFBs can be as large as 15(m) which is unacceptable considering the ionospheric delay ranges from 2 to 30(m). The IFB depends on the antenna, pre-amp, cable, RF filters in the receiver and even the environment (temperature primarily), and the IFB is unique to every receiver installation.

2.3 3D ionospheric tomography and IFB

The goal of ionospheric tomography is to find 3D function N(r). N(r) is the electron distribution function as latitude, longitude and height in ionosphere. N(r) consists of the tensor product of horizontal function (Spherical Harmonics Function) and radial function (Empirical Orthogonal Function) like equation (10).

$$N(r) = \Gamma(h) \otimes Y(\theta,\phi)$$
$$\text{where, } \Gamma(h) = \sum a_k\Gamma_k(h), \quad Y(\theta,\phi) = \sum b_l Y_l(\theta,\phi) \quad (10)$$

Consequently, we must estimate coefficients (a, b) of those functions to solve function N(r). k is the number of EOF and l is the number of SHF terms. For example, if we use 3 EOF and 2nd order SHF, k=1~3 and l=1~9.

Putting equation (10) to (7), equation can be calculated like equation (11). (k=1~n, l=1~m)

$$\int_{R(r)}^{SV_i(r)}N(r)dl(r) = \int_{R(r)}^{SV_i(r)}\Gamma(h)\otimes Y(\theta,\phi)dl(r)$$
$$= \int_{R(r)}^{SV_i(r)}\sum a_k\Gamma_k(h)\otimes\sum b_l Y_l(\theta,\phi)dl(r)$$
$$= H_{i11}a_1b_1 + H_{i12}a_1b_2 + \cdots + H_{inm}a_nb_m$$
$$= [H_{i11}\ H_{i12}\cdots H_{i1m}\ H_{i21}\cdots\cdots H_{inm}]\underline{x} = z_i \quad (11)$$

where,

$$H_{kl} = \int_{R(r)}^{SV(r)}\Gamma_k(h)\cdot Y_l(\theta,\phi)dl(r)$$
$$\underline{x} = [a_1b_1\ a_1b_2 \ldots a_1b_m\ a_2b_1 \ldots a_nb_m]^T \quad \underline{x} \in R^{n\times m}$$

The subscript i denotes each measurement. If we consider IFB and measurement noise,

$$z_i = [H_{i11} \quad H_{i12} \quad \cdots \quad H_{inm}]\underline{x} + I_{b\_RX} - I_{b\_TX} + v_i \quad (12)$$

Accumulating p measurements, we cam make matrix equation (13).

$$\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_p \end{bmatrix} = \begin{bmatrix} H_{111} & H_{112} & \cdots & H_{1nm} \\ H_{211} & H_{212} & \cdots & H_{2nm} \\ \vdots & & & \vdots \\ H_{p11} & H_{p12} & \cdots & H_{pnm} \end{bmatrix} \underline{x} + \underline{I_{b\_RX}} - \underline{I_{b\_TX}} + \underline{v_i}$$

$$\underline{Z} = H\underline{x} + \underline{I_{b\_RX}} - \underline{I_{b\_TX}} + \underline{v_i} \tag{13}$$

where $\underline{Z}$ is the stacked vector of measurements TEC + IFB, $H\underline{x}$ is the ionospheric model. Then, we can estimate x, IFBs and reconstruct ionosphere from basis functions with coefficients x.

If the true electron density distribution, Ne(r), were known then the calibration would be easy. Of course there would then be no need to make ionospheric measurements. Although the ionosphere is a distributed medium it is a physical process with a great deal of spatial and temporal correlation which we can leverage against our bias calibration task. Assuming statistical independence of the IFBs on different reference stations, an adaptive noise cancellation scheme can be used to remove the ionosphere (i.e noise with respect to the calibration) and leave the IFB (i.e. signal we seek).

In place of the true Ne(r) we posit a model of the ionosphere to capture its spatial and temporal correlation. The model is then filtered and subtracted from the TEC measurements to leave an estimate of the IFBs. Because all of the GPS signals received at the antenna from each satellite pass through the same signal path to the correlation loops, the IFB is constant across all measurements made at the same instant. This means that for M satellites in view of the receiver we have M noisy measurements of the same IFB, where the ionosphere is a correlated noise process.

2.4 IFB estimation using KALMAN Filter

This section provides an introduction to the Kalman filter method. The problem is to optimally update the solution to a linear least squares problem given time dependent observations and a prior model estimate of the solution. The unknowns, which in this case, represent the ionospheric electron density field and IFB, are stored in a state vector, x. Associated with the state is a covariance matrix, P, which is updated by the filter each iteration.

We use two-state kinematic filter [Loomis,et al]. The first state is the ionospheric model coefficients x and the second state is the rate of x. We suppose that IFB states are constants in filter dynamics.

State X = [coefficients x | the rate of x | receiver IFB;i=1~TRS number | satellite IFB;j=1~SV number]

The sequence of steps in updating the filter may be defined as follows: First the state vector, x, is projected into the future (the minus superscript implies prior estimates)

$$x = Ax^{-} + B \tag{14}$$

For the method presented in this paper the matrices A and B are generated from prior model estimates of the electron density field

The next stage in the update of the filter involves projecting the error covariance matrix

$$P = AP^{-}A^{T} + Q \tag{15}$$

The Q matrix defines the variance as being a constant fraction of an average of the background model and projected state estimates.

Given a set of line integral observations, z, with covariance R, and path integrals defined by, H, the Kalman Gain is given by

$$K = PH^{T}\left(HPH^{T} + R\right)^{-1} \tag{16}$$

Finally, using the Kalman gain, the state vector and its covariance are updated

$$x^{+} = x + K(z - Hx)$$

$$P^{+} = (I - KH)P \tag{17}$$

3. Experiment

3.1 Data

The data used in this paper were obtained from Korean DGPS stations. For experiment, we selected 6 stations (Socheng-do, Eocheng-do, Mara-do, Seoimal, Ulleng-do, Jeojin) for reference station and 1 station(Youngju) for user at July 16th, 2005 (24 hours) and used IONEX value for satellite inter-frequency bias. We compare WADGPS errors in case of each algorithm (No IFB, 2D IFB, 3D IFB).

3.2 Result & analysis

Figure 2 show WADGPS position error (2drms:2.8752) by no IFB correction and Figure 3 show WADGPS position error (2drms:0.9311) by 2D model IFB correction value and Figure 4 show WADGPS position error (2drms:0.6451) by 3D model IFB correction value. We can confirm that the estimated IFB value by 3D model reduce WADGPS error from results.

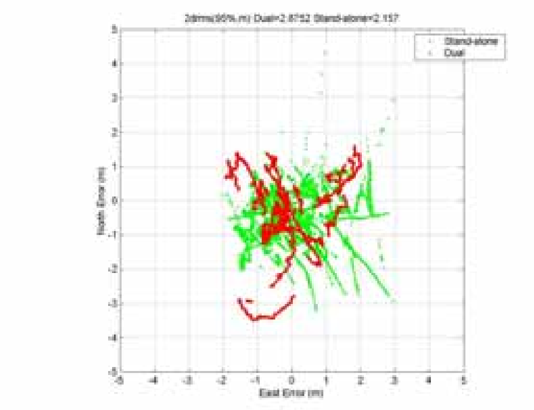

Figure 2. WADGPS horizontal position error with No IFB correction

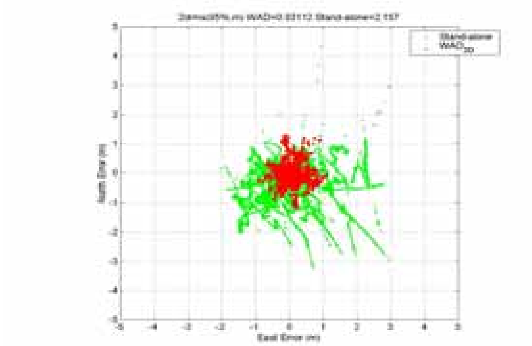

Figure 3. WADGPS horizontal position error with 2D IFB

correction

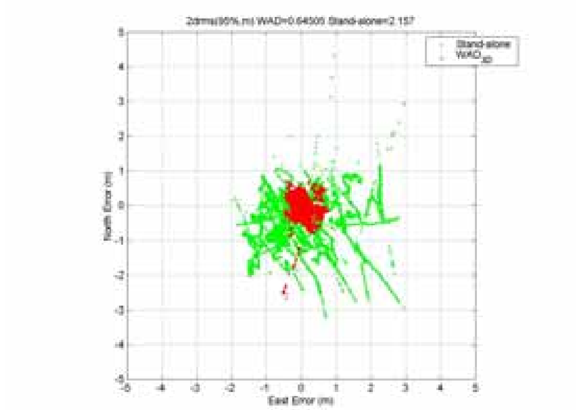

Figure 4. WADGPS horizontal position error with 3D IFB correction

4. Conclusion

In this paper, we estimated IFB (interfrequency bias) by 2D & 3D ionospheric models including real time filter methods. Because 2D shell model would squash vertical variation into TEC values at a pre-determined shell height, it has some modelling error reducing accuracy. The IFB estimation of 3D tomography model can recover the problem of 2D thin shell model. We confirmed our conclusion by simulation and experimental data.

Acknowledgement

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