

# Positioning using ZigBee and Ultrasound

\*Chansik Park<sup>1</sup>, Seung Beom Kim<sup>2</sup>, Dong-Youn Kang<sup>1</sup>,  
Hee Hak Yun<sup>1</sup>, En Jong Cha<sup>3</sup>, Sang Jeong Lee<sup>2</sup>

<sup>1</sup> School of Electrical & Computer Engineering, Chungbuk Nat'l Univ., CBITRC (E-mail: chansp@chungbuk.ac.kr)

<sup>2</sup> School of Electrical & Computer Engineering, Chungnam Nat'l Univ. (E-mail: eesjl@cslab.cnu.ac.kr)

<sup>3</sup> Department of the Biomedical Engineering, Chungbuk Nat'l Univ., CBITRC (E-mail: ejcha@chungbuk.ac.kr)

## Abstract

To find a location, GPS has been widely used. But, it is hard to use in indoor because of very weak signal level. To meet indoor requirements, there have been many studies applying wireless communication networks such as WLAN, UWB and ZigBee. Among these, ZigBee is widely adopted in many WSN applications because it has an advantage of low-power and low-cost. In ZigBee, the RSSI is used as range measurement for ad-hoc network. The RSSI are converted to ranges using the signal attenuation model and these ranges become inputs of positioning methods. The obtained position with RSSI has large error because of its poor accuracy. To overcome this problem, ultrasonic sensors are added in many researches. By measuring the arrival time difference of ZigBee and ultrasound as a range measurement, the precise position can be found. However, there are still many problems: scheduling of beacons to transmit signals in a correct order, addition and synchronization of beacons and low-rate positioning rate. At this paper, an efficient method to solve these problems is proposed. In the proposed method, a node transmits ZigBee and ultrasound signal simultaneously. And beacons find the range with the received signals and send it back to a node with ZigBee. The position is computed in a node with the received ranges. In addition, a new positioning algorithm to solve the risk of the divergence in the linearization method and the singularity problem in the Savarese method is presented. Both static and dynamic experimental results show 0.02m RMS errors with high output rate.

**Keywords:** Ultrasound, ZigBee, Positioning, WSN.

## 1. Introduction

WSN(Wireless Sensor Network) is the communication and network technology using ultra small sized and ultra low powered sensor devices, which can be applied to many fields like smart homes, health cares, military, delivery, ITS(Intelligent Transportation System), environment monitoring, robots and cars. ZigBee is used in many WSN applications because it meets the low cost and low-power requirements. The PHY and MAC layer of ZigBee is standardized in IEEE 802.15.4. In location awareness WSN applications, seamless positioning is required. GPS (Global Positioning System) is popular in an outdoor positioning, but it suffers from signal blockage by the obstacles like buildings, bridges and forest. Also high cost prevents a GPS receiver from wide spreading in WSN applications. On the other hand, RSSI (Received Signal Strength Indication) is inherently supplied for ad-hoc network in ZigBee, thus it can be used for positioning as in many studies like MoteTrack[2-5]. But the obtained position using RSSI has large errors because of its poor accuracy. RSSI is difficult to apply to many applications where precise position is required [6-7]. To improve the accuracy, other sensors are added to RSSI as: wireless LAN in RADAR [8]; infrared light in Active Badge [9]; UWB (Ultra Wide Band) in Ubisense [10]; ultrasound in Cricket [11] and U-SAT [12-14]. In ultrasound, accurate range can be found easily by measuring the arrival time difference between RF and ultrasound with simple hardware and software.

In Cricket and U-SAT, the beacon in the known coordinates transmits both RF and ultrasound signals at the same time. A node measures the arrival time difference and converts to the range measurements by multiplying the speed of ultrasound. The position can be found using these range measurements. Because

an effective multiple access method for ultrasound is not known yet, Cricket and U-SAT use TDMA (Time Division Multiple Access). Each node transmits signals during an allocated time slot  $T/m$  where  $T$  is the duration and  $m$  is the number of nodes. Time slot should guarantee the ultrasonic flying time from beacon to node. This structure has difficulties in adding, removing and synchronizing the beacons. For example, if there are 4 beacons and 1 node and maximum range is 30m, the minimum time slot is 90ms, and it takes 360ms to get the range measurements from 4 beacons, therefore the positioning rate is less than 3 Hz. Also, the faster user can not find correct position because the range measurements are sequentially collected with 90ms intervals not at the same time. Furthermore, the power consumption is high because beacons should transmit RF and ultrasound continuously, so it is not suitable for low-power applications like WSN [15]. Finally, the positioning algorithms have limitations: beacons should be installed on the different heights in U-SAT and only 2 dimensional positioning is possible in Cricket.

This paper proposes a new positioning method using ZigBee and ultrasound. A node transmits ZigBee and ultrasound simultaneously, and beacons measure a range using the arrival time difference in the proposed method. A beacon sends the range measurements back to the node by ZigBee communication channels. Finally, node determines the position using returned range measurements. The proposed method can solve many problems such as beacon installations, position rates and power consumption. Also, the positioning algorithm which computes 3 dimensional positions without installation constraints is proposed. The evaluations of the proposed structure and algorithm are performed by static and dynamic experiments using a real system. The proposed method can be easily applied to all applications where both indoor and outdoor precise locations are required.

## 2. Positioning using ZigBee and Ultrasound

### 2.1 System Architect

The process of ranging using the arrival time difference of ZigBee and ultrasound is shown in Figure 1. In the figure, node is a device which computes a 3 dimensional position. And beacon is a device installed on the known position ( $B^i = [X^i, Y^i, Z^i]$ ). If a node transmits ZigBee and ultrasound simultaneously, the ZigBee signal at the light speed arrives very faster than the ultrasound at the speed  $v_{US} = 331.5 + 0.60714T_C$  [m/s] where  $T_C$  represent temperature in Celsius. Thus beacon measures the interval ( $t_{US} - t_{RF}$ ) and converts to the range by multiplying the speed of ultrasound as shown in eq. (1) where  $u_A = [x_A, y_A, z_A]$  is a unknown node position and  $w_A^i$  is AWGN (Additive White Gaussian Noise) measurement noise with mean 0 and a variation  $\sigma^2$ .

$$\rho_A^i = \frac{(t_{US} - t_{RF})v_{US}}{\sqrt{(X^i - x_A)^2 + (Y^i - y_A)^2 + (Z^i - z_A)^2 + w_A^i}} \quad (1)$$

The measurement noise is affected by many factors: ultrasound speed error after temperature compensation, the noise of ultrasound detection circuits and so on. The temperature influence is not negligible because its error increases as the range is long. Fortunately, most nodes in WSN have temperature sensors and can use them to calibrate the ultrasound speed. If there is no temperature sensor in the WSN, it is desirable that some reference beacons have temperature sensors and transmit measured temperature to other nodes using network.

To determine node's 3 dimensional positions, the range measurements from more than 3 beacons are needed. If node moves on the plane, 2 dimensional positions can be found with 2 range measurements. Generally, more than 3 beacons are used to determine uninterrupted position even if some beacons are blocked by obstacles. Figure 2 shows the example of a beacon constellation. The corner of the ceiling might be a good alternate when beacons are installed in a room.

There are two methods for measuring the range between node and beacon in the Figure 2: a beacon transmits RF and ultrasound at the same time and node measures the ranges to each beacon (method 1); and a node transmits RF and ultrasound, beacons measure the ranges, and beacons send the measured ranges back to the node (method 2). Method 1 is similar to GPS; it has advantage of measuring range at node; and it is applied to Cricket and U-SAT. However, in method 1, only 1 beacon can transmit signals to avoid collision. To the best of authors knowledge, there is no efficient multiple access method except TDMA in ultrasound. Figure 3 illustrates a possible timing diagram of method 1 using TDMA. 4 beacons divide a cycle with an equal time slot and each beacon transmits RF and ultrasound to node at an allocated time slot. A node measures the range by measuring the time difference of RF and ultrasound. The size of time slot depends on the speed of ultrasound, the number of beacons and the size of an application area. If the maximum range in an application area is  $R_{max}$ , the minimum time slot  $R_{max}/v_{US}$  should be guaranteed in order to avoid a collision. For example, maximum range is about 28 m and minimum time slot is 83 ms in U-SAT. Therefore, using method 1, 3 Hz positioning results can be obtained in this environment [12-14]. Also, method 1 has problems such as adding / removing / synchronizing beacons, power consumption, erroneous position for fast moving node, and low output rate.

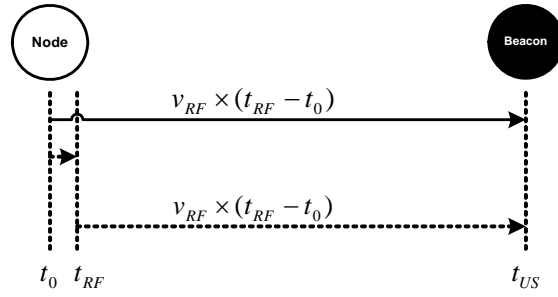


Figure 1. Process of ranging using RF and Ultrasound.

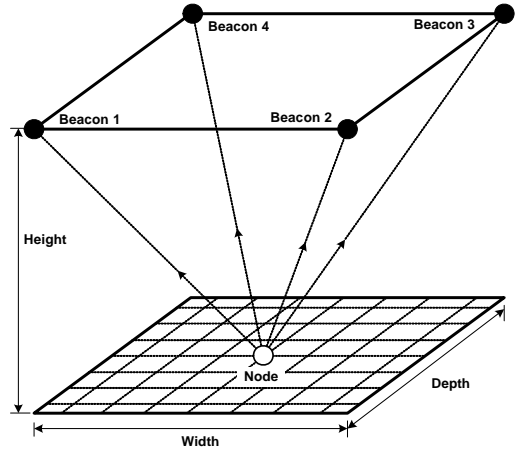


Figure 2. Deployment of Beacons and a node

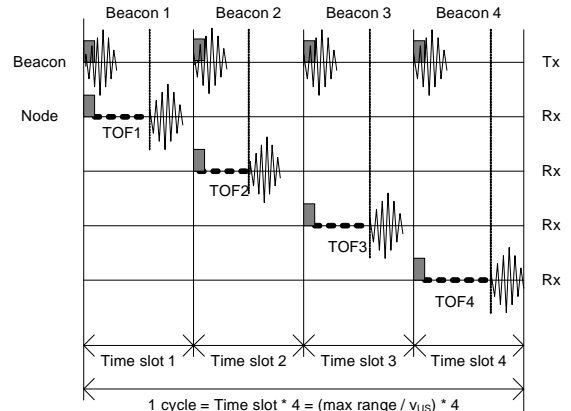


Figure 3. The timing diagram of method 1.

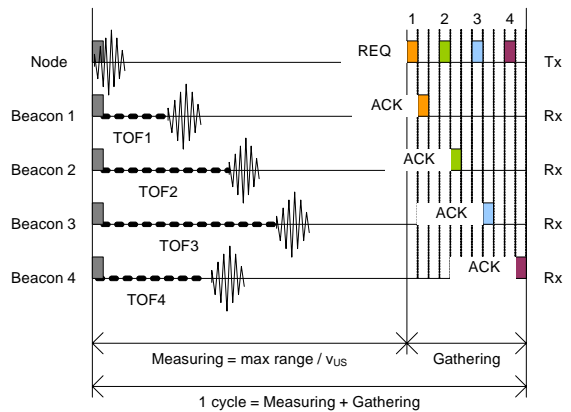


Figure 4. The timing diagram of method 2.

The time diagram of method 2 is given in Figure 4. A node transmits RF and ultrasound without considering the time slot and all beacons receive them. And then node requests to each beacons to return the measured ranges. A simple implementation where the node position is found at a beacon is also possible. It is applied to a patient monitoring at a hospital and in this case, the measured data gathering process can be omitted. However, in order to keep privacy, in this paper, the measured range is returned to a node thru the ZigBee network to compute a position. There are two methods to gather the range measurements: The first, beacons transmit the range measurements as soon as they measure them. The second, beacons keep the range until node requests it. In the first method, it is difficult to avoid the packet loss from collision. To prevent packet loss, the second method is chosen in this paper. The gathering process starts after guaranteed travel time of ultrasound, and a retry mechanism in application layer is added to prevent packet loss.

## 2.2 Positioning Algorithm

In order to determine node position in three dimensions, the nonlinear equation (1) is solved. A linearization method using Taylor series is reprehensive [16] but it has the significant problem that a solution may not converge to the correct value when the line-of-sight is rapidly changing. The other method named Savarese [17, 18] does not require an initial guess, it calculates a position by subtraction after squaring the measurements. But it has the singularity problem when the heights of all beacons are the same. In this paper the linearization method for 2 dimensions or 3 dimensions position is given. Also the modified Savarese method is given, where the weights from the covariance of the range measurements noise are included and the limit of same beacon's height is removed.

### 2.2.1 Linearization Method

If a node has the range measurements from  $m(\geq 3)$  beacons, a position can be found as following. Linearization of eq. (1) with respect to the nominal position  $u_0 = (x_0 \ y_0 \ z_0)^T$  gives eq. (2), or in a simple form of eq. (3).

$$\begin{bmatrix} \rho_A^1 - \rho_0^1 \\ \vdots \\ \rho_A^m - \rho_0^m \end{bmatrix} = \begin{bmatrix} h_x^1 & h_y^1 & h_z^1 \\ \vdots & \vdots & \vdots \\ h_x^m & h_y^m & h_z^m \end{bmatrix} \begin{bmatrix} \delta x_A \\ \delta y_A \\ \delta z_A \end{bmatrix} + \begin{bmatrix} w_A^1 \\ \vdots \\ w_A^m \end{bmatrix} \quad (2)$$

$$\delta r_A = H_A \delta u_A + w_A \quad (3)$$

Here  $\rho_0^i = \sqrt{(X^i - x_0)^2 + (Y^i - y_0)^2 + (Z^i - z_0)^2}$  is a computed range and  $(h_x^i = \frac{X^i - x_0}{\rho_0^i}, h_y^i = \frac{Y^i - y_0}{\rho_0^i}, h_z^i = \frac{Z^i - z_0}{\rho_0^i})$  is line-of-sight vector from a nominal point to a beacon. Since each range measurements is independent, covariance of measurement noise becomes diagonal matrix as shown in eq. (4).

$$Q_A = \text{cov}(w_A) = \text{diag}(\sigma_i^2) \quad (4)$$

Applying WLSQ (Weighted Least Squares Estimates) to equation (3), the position errors and its covariance can be found as in eq. (5) and (6). By adding the estimated position error  $\delta \hat{u}_A$  to initial nominal point  $u_0$ , the position can be fixed. These procedures are iterated to get a converged solution. If each beacon measures the range with the same accuracy,

$\sigma^2 = \sigma_1^2 = \dots = \sigma_m^2$  holds and eq. (6) becomes  $\text{cov}(\delta u_A) = \sigma^2 (H_A^T H_A)^{-1}$ .

$$\delta \hat{u}_A = (H_A^T Q_A^{-1} H_A)^{-1} H_A^T Q_A^{-1} \delta r_A \quad (5)$$

$$\text{cov}(\delta u_A) = (H_A^T Q_A^{-1} H_A)^{-1} \quad (6)$$

If the height of a node is fixed, using the nominal point  $u_0 = [x_0 \ y_0 \ z_F]^T$ , 2 dimensional position can be found using equation (7).

$$\begin{bmatrix} \rho_A^1 - \rho_0^1 \\ \vdots \\ \rho_A^m - \rho_0^m \end{bmatrix} = \begin{bmatrix} h_x^1 & h_y^1 \\ \vdots & \vdots \\ h_x^m & h_y^m \end{bmatrix} \begin{bmatrix} \delta x_A \\ \delta y_A \end{bmatrix} + \begin{bmatrix} w_A^1 \\ \vdots \\ w_A^m \end{bmatrix} \quad (7)$$

This method can be used to applications where the node height is fixed, such as a robot cleaner. And this method gives robust result because it is possible to find positions with two beacons only.

### 2.2.2 Modified Savarese Method

Eq. (8) can be obtained by ignoring the measurement errors  $w_A^i$  in equation (1), and squaring both sides to get  $(\rho_A^i)^2$ , and differencing with  $(\rho_A^m)^2$ .

$$\begin{aligned} & (\rho_A^i)^2 - (\rho_A^m)^2 - \{(X^i)^2 + (Y^i)^2 + (Z^i)^2\} + \{(X^m)^2 + (Y^m)^2 + (Z^m)^2\} \\ & = 2 \begin{bmatrix} X^m - X^i & Y^m - Y^i & Z^m - Z^i \end{bmatrix} \begin{bmatrix} x_A \\ y_A \\ z_A \end{bmatrix} \end{aligned} \quad (8)$$

Using  $\underline{B}^i = [X^i \ Y^i \ Z^i]^T$  and  $\underline{u} = [x_A \ y_A \ z_A]^T$ , eq. (8) becomes eq. (9).

$$(\rho_A^i)^2 - (\rho_A^m)^2 - (\underline{B}^i)^T \underline{B}^i + (\underline{B}^m)^T \underline{B}^m = 2(\underline{B}^m - \underline{B}^i)^T \underline{u} \quad (9)$$

Eq. (10) holds for the  $m$  beacons, and can be represented with simplified form in eq. (11).

$$\begin{bmatrix} (\rho_A^1)^2 - (\rho_A^m)^2 - (\underline{B}^1)^T \underline{B}^1 + (\underline{B}^m)^T \underline{B}^m \\ (\rho_A^2)^2 - (\rho_A^m)^2 - (\underline{B}^2)^T \underline{B}^2 + (\underline{B}^m)^T \underline{B}^m \\ \vdots \\ (\rho_A^{m-1})^2 - (\rho_A^m)^2 - (\underline{B}^{m-1})^T \underline{B}^{m-1} + (\underline{B}^m)^T \underline{B}^m \end{bmatrix} = 2 \begin{bmatrix} (\underline{B}^m - \underline{B}^1)^T \\ (\underline{B}^m - \underline{B}^2)^T \\ \vdots \\ (\underline{B}^m - \underline{B}^{m-1})^T \end{bmatrix} \underline{u} \quad (10)$$

$$\underline{\nabla \rho} = \underline{\nabla B} \underline{u} \quad (11)$$

Using WLSQ, the position and its covariance can be found as in eq. (12) and (13) where the element of covariance matrix is  $\sigma_{Si}^2 = \text{cov}(2(\rho_i y_i - \rho_m w_m) + (w_i^2 - w_m^2)) = 2\sigma^2(3\sigma^2 + 4\rho_i^2 + 4\rho_m^2)$ .

$$\hat{u} = (\underline{\nabla B}^T Q_S^{-1} \underline{\nabla B})^{-1} \underline{\nabla B}^T Q_S^{-1} \underline{\nabla \rho} \quad (12)$$

$$Q_S = \text{diag}(\sigma_{Si}^2) \quad (13)$$

A position is calculated without considering weight in the original Savarese method but in this paper it is included as in eq. (12) because the measurement noise increases in proportion to range. When the beacons are installed on the same height, the last column of matrix  $\underline{\nabla B}$  becomes 0 so that the inverse of

$(\nabla B^T \nabla B)^{-1}$  can not be found. To solve this singularity problem, the beacon should be installed on the different heights but sometimes it gives limitation to installation. This paper presented an alternate positioning method using eq. (14) instead of eq. (8). The height  $z_A$  can be found using equation (15) using the fixed  $(x_A, y_A)$ . More precise height can be obtained by averaging the results from all beacons.

$$(\rho_A^i)^2 - (\rho_A^m)^2 - \{(X^i)^2 + (Y^i)^2\} + \{(X^m)^2 + (Y^m)^2\} = 2 \begin{bmatrix} X^m - X^i & Y^m - Y^i \end{bmatrix} \begin{bmatrix} x_A \\ y_A \end{bmatrix} \quad (14)$$

$$(Z^i - z_A)^2 = (\rho_A^i)^2 - (X^i - x_A)^2 - (Y^i - y_A)^2 \quad (15)$$

### 3. System Implementation

Figure 5 illustrates the hardware structure of a node and a beacon. The hardware consists of a RF part, an ultrasound circuit and a microcontroller. A RF part is made with Chipcon's CC2420 chip and an ultrasound circuit with AT/R40-10P of KOMAN TECHNICS. An Atmel's AVR ATmega128 is used as a microprocessor. A beacon makes the 40 kHz frequency pulse to drive ultrasonic transducer using PWM (Pulse Width Modulation) controller in ATmega128. An ultrasound receiver is in a beacon and an ultrasound transmitter is in a node. To synchronize between RF and ultrasound, the SFD signal from CC2420 chip is used. SFD signal is generated when a data frame is transmitted and received using ZigBee RF communication [19], so by using the SFD signal to transmit ultrasound, synchronization can be done. At a beacon, the activation of SFD means the arrival of RF signal. By starting a timer when the SFD comes and stopping a timer when an ultrasound arrives, the time interval between RF and ultrasound can be measured.

Ultrasound receiver uses an envelop detector circuit with an experimentally determined threshold. Since all ultrasound receivers have a same fixed amplification gain and corresponding threshold, detection delay increases as the range grows. A calibration is possible by controlling the gain or changing threshold in compliance with range or adjusting weight in a positioning process. To transmit ultrasound to wide ranges, the ultrasonic sensor with wide beam patterns is preferred. At this paper, the ultrasonic sensor with 100 deg beam pattern is used.

Figure 6 shows the flow chart of a node and a beacon. A node enables 40 kHz PWM signal and transmits ZigBee signal to all beacons by setting as a broadcasting packet. And an ultrasound transmitter is triggered by SFD signal which happens at the start of ZigBee packet transmission. A node has to wait until an ultrasound flies to the farthest beacon before a node sends another packet. After that, a node requests beacons to send back the measurement sequentially, and each beacons transmit a packet which contains the range measurements as a response. A node re-tries the request 3 times when a node does not receive a reply from the corresponding beacon within a certain period of time. During the packet receiving process, the RSSI measurement from the CC2420 is also measured.

There are two 8-bit counters and two 16-bit counters in ATmega128 [20] and one of 16-bit counters is used to measure the time difference of the RF and ultrasound. A 7.3728 MHz is used as microprocessor clock and it is divided by 8 to be a timer clock. Therefore, the resolution of the timer is 1.08 us, or equivalently the range measurement resolution is 0.367 mm and the maximum range becomes 24.174 m when the speed of ultrasound is 340 m/s.

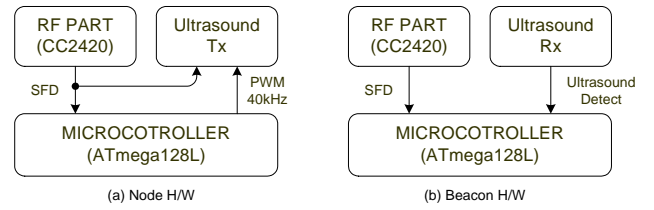


Figure 5. Hardware structure of Node and beacon.

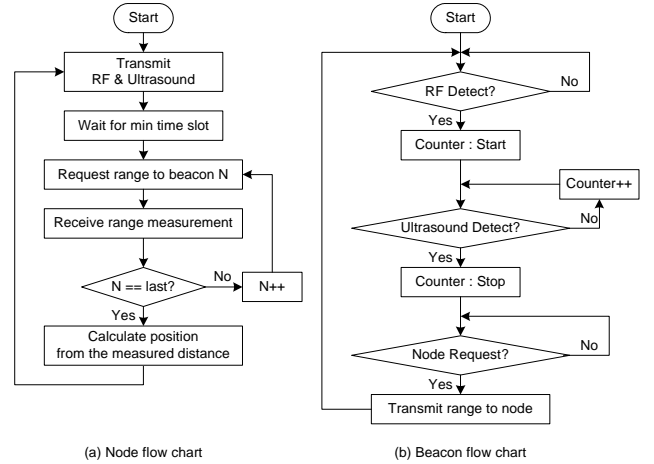


Figure 6. Flow chart of Node and Beacon.

A node calculates a position using the gathered range measurements. A node can use a linearization method or a modified Savarese method as a positioning algorithm. If a node has the ranges measurement from more than 3 beacons, 3 dimensional positions can be found. On the other hand, if a node has the ranges measurement from only 2 beacons, 2 dimensional positions can be found.

## 4. Experimental results

The test bed for experiments is configured as in Figure 2. The width and depth of test bed is 3 m and height is 2.8 m. Four beacons are installed at the corners of the structure. The heights of all beacons are the same. Both static and dynamic experiments are performed. For static experiments, 500 measurements at 49 reference points whose positions are known. The reference points are on the bottom of test bed with 40 cm grid. The positions are found using both linearization method and modified Savarese method. At each reference point RMS (Root Mean Squares) errors are computed to evaluate the accuracy. RMS (3D) is the RMS computed using 3 dimensional positions, while RMS (2D) is computed using 2 dimensional positions. For dynamic experiments, a node is installed on a rotating chair to evaluate the repeatability of accuracy. To compare the performance of the positioning methods, the range measurements are gathered and stored in a PC, and the positions are determined with stored measurement in a post processing manner. Because the maximum range in the test bed is limited to 30 m, a node can send an ultrasound at every 88 ms, it corresponds to 11 Hz positioning rate.

### 4.1 Static experiment

With the stored range measurements, the positions of a node are determined by using a linearization and a modified Savarese method. In the experiments, range measurements from more than

3 beacons are always available.

When using the 2D linearization method, many wrong positions are found because of the divergent problem which is sensitive to the initial guess. However, 3D linearization method always gives the correct solutions with initial point as (0, 0, 0). The positioning results with the 3D linearization method are shown in Figure 7. Red o is a true position; blue x is an estimated position. RMS (3D) errors of the positions are shown in Figure 8. In Figure 8, the maximum RMS (3D) error is 0.0424 m at (3, 3), the minimum RMS (3D) error is 0.0136 m at (4, 2), the average RMS (3D) error is 0.0248 m with standard deviation 0.0018 m. Total RMS (3D) error is 1.2129 m. Here (n,m) means the location of node. For example (1,1) means the lower left location and (7,7) means upper right node in Figure 7.

The Savarese method can not determine the position because all beacons are installed on the same height, while the modified Savarese method provides position. The positioning results with the modified Savarese is similar to those of the linearization method: the maximum RMS (3D) error is 0.0533 m at (6, 7), the minimum RMS (3D) error is 0.0122 m at (3, 6), the average RMS (3D) error is 0.0252 m with standard deviation 0.0029 m, and total RMS (3D) error is 1.2355 m.

Table 1 summarized the mean and standard deviation of RMS (2D) errors and the RMS (3D) errors. Table shows the linearization method gives slightly better performance. The squaring error in the modified Savarese method is supposed as a reason.

Table 1. RMS errors according to two positioning algorithms.

RMS error[m]		Algorithm	
		3D Linearization	Modified Savarese
2D	mean	0.0192	0.0200
	std	0.0015	0.0024
	sum	0.9403	0.9815
3D	mean	0.0248	0.0252
	std	0.0018	0.0029
	sum	1.2129	1.2355

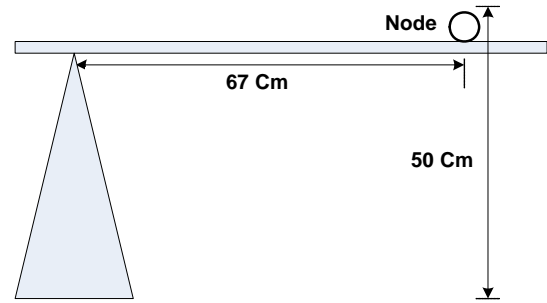


Figure 9. The rotary machine for a dynamic experiment.

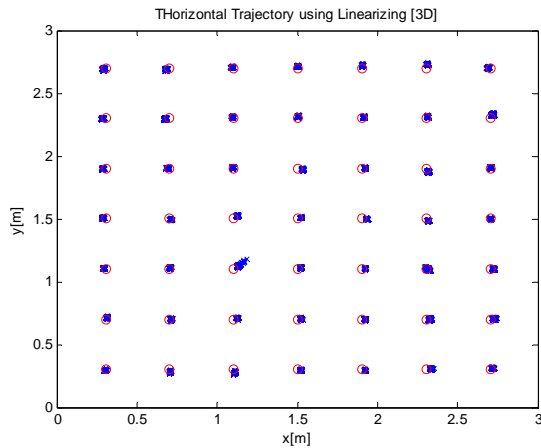


Figure 7. Horizontal Trajectory using the 3D linearization method.

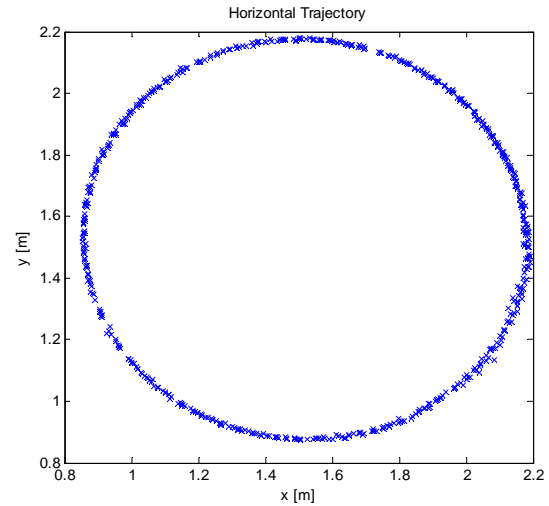


Figure 10. Repeatability test results (angular rate 4.6cm/s).

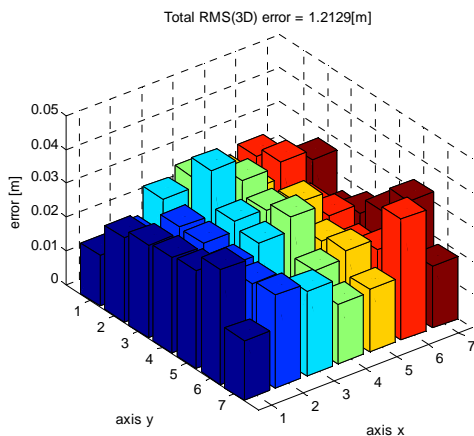


Figure 8. RMS (3D) errors using the linearization method.

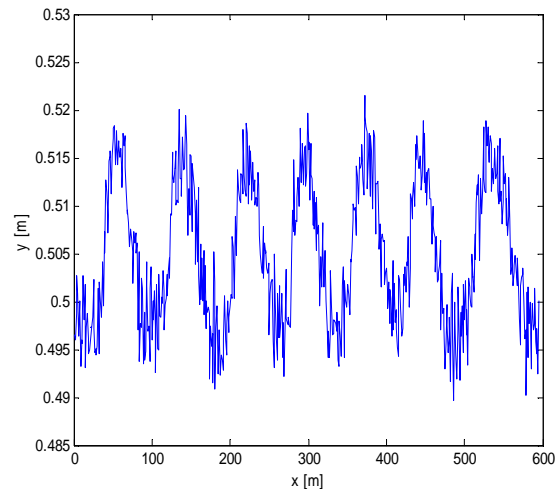


Figure 11. The Height.

## 4.2 Dynamic experimental results

Dynamic experiments are performed with the rotating chair as in Figure 9. A rod is installed on the chair, a node is placed on the rod with a radius 67 cm and a height 50 cm. The rotating center of chair is in the middle of the test bed of Figure 2. The node on the rotating chair turns 7 times during 91seconds. 3 dimensional positions are determined using the linearization method. Figure 10 confirms the repeatability of accuracy. The 7 trajectories are within 2.5 cm boundary as predicted in the static test. The positioning results, blue x do not show clear circle because chair is turned by human hand. Especially the small position jumps in the lower right corner of Figure 10 are results of irregularity of bearings. The determined height of the node is shown in Figure 11. The height oscillates from 49.5 cm to 51.5 cm as the node turns, it also shows the repeatability.

## 5. Conclusion

In this paper, an efficient positioning method using ZigBee and ultrasound is proposed and evaluated using real experiments. The proposed method can solve the problems such as: the difficulty of scheduling and synchronization of beacons, lack of flexibility in adding and removing of beacons, and low-rate positioning rate. In the proposed method a node transmits ZigBee and ultrasound simultaneous to beacons, and beacons find the range with the arrival time difference of two signals, and send back the range measurements to a node by ZigBee communication protocol. Finally, a node determines its position with the received ranges to keep the privacy. Furthermore, new positioning algorithms to avoid the installation limitation that the heights of beacons are differ. The proposed methods are evaluated in the test bed which has 4 beacons and 1 node. The static test results show that less than 2 cm RMS (2D) error can be achieved with a positioning rate of 11Hz. The 2D linearization method suffers the divergent problem and the unmodified Savarese method shows singularity problem when the height of all beacons is same as expected. The modified Savarese method gives slightly worse result than the linearization method because of squaring loss. The proposed method is expected to be easily adopted in many WSN applications such as smart homes, health cares, ITS, environmental monitoring and robots.

## Reference

1. *IEEE Standards 802 Part 15.4: Wireless Medium Access Control (MAC) and Physical Layer (PHY) Specifications for Low-Rate Wireless Personal Area Networks (LR-WPANS)*, (IEEE Inc., 2003).
2. Konrad Lorincz and Matt Welsh, "Motetrack: A robust, decentralized approach to rf-based location tracking," *In Proceedings of the International Workshop on Location- and Context-Awareness (Loca 2005)*, 2005.
3. D. Lymberopoulos, Q. Lindsey and A. Savvides, "An Empirical Analysis of Radio Signal Strength Variability in IEEE 802.15.4 Networks using Monopole Antennas," *ENALAB Technical Report*, 2005.05.01.
4. *CC2431 Datasheet (rev 1.01)*. Chipcon.
5. Masashi Sugano, Tomonori Kawazoe, Yoshikazu Ohta, and Masayuki Murata, "Indoor Localization System Using RSSI Measurement of Wireless Sensor Network Based on ZigBee Standard," *In The IASTED International Conference on Wireless Sensor Networks (WSN 2006)* , Banff (Canada), July 2006.
6. K. Vandenbussche. Fine-grained indoor localization using wireless sensor nodes. Master's thesis, Delft University of Technology, August 2005.
7. A. Savvides, C. Han, and M. Strivastava, "Dynamic finegrained localization in ad-hoc networks of sensors," *In 7th annual international conference on Mobile computing and networking*, pages 166--179, Rome, Italy, 2001.
8. P. Bahl and V. N. Padmanabhan, "RADAR: An in-building RF-based user location and tracking system," *In Proc. IEEE Infocom*, pages 775--784, Tel-Aviv, Israel, April 2000.
9. R. Want, A. Hopper, V. Falcao and J. Gibbons, "The Active Badge Location System," *In ACM Transactions on Information Systems*, 10(1):91-102, January 1992.
10. *Ubisense homepage*. <http://www.ubisense.net/>
11. Nissanka Bodhi Priyantha, "The Cricket Indoor Location System," *In PhD Thesis, Massachusetts Institute of Technology*, June 2005.
12. Su Yong Kim, "Position Estimation and Performance Evaluation of a Mobile Robot Using Ultrasonic Pseudo-Satellites," *In Master Thesis, Busan National University*, February 2005.
13. Han Soo Kim, "A Study on Range Expansion of Ultrasonic Local Positioning System," *In Master Thesis, Busan National University*, February 2006.
14. Dong-Hwal Lee, Jong-Jin Park, Su-Yong Kim, Young-Song Mun, Man-Hhyung Lee, "A Study on the Application of U-SAT System for the Indoor Positioning Technology of Ubiquitous Computing," *In Journal of Control, Automation, and Systems Engineering*, Vol. 12, No. 9, September, 2006.
15. Soohyung Cho, Lee Sanghak, "Design of Low Powered Indoor Positioning System based on Sensor Network," *In Korea Electronics Technology Institute Intelligent IT System Research Center*.
16. Elliot Kaplan, *Understanding GPS Principles and applications* (Artech House, 1996).
17. C. Savarese, K. Langendoen, J. Rabaey, "Robust positioning algorithms for distributed ad-hoc wireless sensor networks," *In USENIX Technical Annual Conference*, Monterey, CA, 2002, pp. 317-328.
18. M.-G. Di Benedetto and G. Giancola, *Understanding Ultra Wide Band Radio Fundamentals* (Prentice Hall, 2004)
19. *CC2420 Preliminary Datasheet (rev 1.2)*, Chipcon AS, 2004.
20. *ATmega128 datasheet*, Atmel.