Estimation of Ionospheric Delays in Dual Frequency Positioning
– Future Possibility of Using Pseudo Range Measurements –

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Abstract

The correct estimation of the ionospheric delays is very important for the precise kinematic positioning especially in case of the long baseline. In case of triple frequency system, the ionospheric delays can be estimated from the measurements, but, in case of dual frequency system, the situation is not so simple. The precision of those supplied by the external information source such as IONEX is not sufficient. The high frequency component is neglected, and the precision of the low frequency component is not sufficient for the long baseline positioning. On the other hand, the high frequency component can be estimated from the phase range measurements. If the low frequency components are estimated by using the external information source or pseudo range measurements, a more reasonable estimation of the ionospheric delays may be possible. It has already been discussed by the author that the estimation of the low frequency components by using the external information source is not sufficient but fairly effective. The estimation using the pseudo range measurements is discussed in the present paper. The accuracy is not sufficient at present because of the errors in the pseudo range measurements. It is clarified that the bias errors in the pseudo range measurements are responsible for the poor accuracy of the ionospheric delays. However, if the accuracy of the pseudo range measurements is improved in future, the method would become very promising.

Keywords: GPS, long baseline kinematic positioning, multiple frequency, dual frequency, ionospheric delay, pseudo range, phase range, IONEX, bias error in pseudo range, wide-lane combination, wide-lane ambiguity, HMW combination.

1. Introduction

The biggest problem in the high precision kinematic positioning is the determination of the initial phase ambiguity of L1 wave. It may be said that a complete solution is not obtained for this problem in case of the long baseline positioning.

The factors making the ambiguity determination difficult in case of long baseline kinematic positioning are the errors due to the satellite errors, the ionospheric delays and the tropospheric delays. The ionospheric delays may be the most important. An epoch making progress is expected after GPS Modernization and GALILEO take place in near future (Hatch (1996), Isshiki (2003a, b)).

The present GPS is a dual frequency system. The signals consist of L1 signal of $1.58GHz$ and L2 signal of $1.23GHz$. It may be impossible to solve the above-mentioned problem completely, since no methods can determine both the initial phase ambiguity and the ionospheric delay simultaneously. When GPS Modernization is realized and L5 wave of $1.18GHz$ is added to the present dual frequency system, the two independent geometry free combination are obtained for each satellite and receiver combination. Hence, the simultaneous determination of the initial phase ambiguity and the ionospheric delay become possible theoretically. So, if the determination of the wide-lane ambiguity by HMW (Hatch-Melbourne-Wübbena) combination is also used, L1 ambiguity may be determined without being assisted by any external sources and without being affected by the ionospheric delay.

In the present paper, a method for the correct estimation of the ionospheric delays in case of the dual frequency system is discussed. The high frequency component of the ionospheric delays can be estimated from the phase range measurements very correctly. If the low frequency components are estimated by using the external information source such as IONEX or pseudo range measurements, a reasonable estimation of the ionospheric delays may be possible. It has already been discussed by the author that the estimation of the low frequency components by using the external information source is not sufficient but fairly effective (Isshiki (2005)). The estimation using the pseudo range measurements is discussed in the present paper. Unfortunately, the accuracy is not sufficient at present because of the bias errors in the pseudo range measurements. However, if the accuracy of the pseudo range measurements is improved in future, the method would become very promising.

2. Multi-frequency Observation Equations and the Solution

Multiple frequency observations are essential for long baseline positioning. Let t refer to time and $P_{\kappa\alpha}^{i}(t)$ and $\Phi_{\kappa\alpha}^{i}(t)$ be the pseudo and the phase ranges of L_κ wave between the satellite i and the receiver α. $\rho_\alpha^i(t)$ is the true geometric range. $I_\alpha^i(t)$ and $T_\alpha^i(t)$ are the ionospheric and the tropospheric delays of L_1 wave. Then, the double difference observation equations are given as (Isshiki (2003a, b, c, 2004a, b, c))

$$P_{\kappa\alpha\beta}^{ij} = \rho_{\alpha\beta}^{ij} + \frac{f_1^2}{f_\kappa^2} I_{\alpha\beta}^{ij} + T_{\alpha\beta}^{ij} + e_{\kappa\alpha\beta}^{ij}, \tag{1a}$$

$$\Phi_{\kappa\alpha\beta}^{ij} = \rho_{\alpha\beta}^{ij} - \frac{f_1^2}{f_\kappa^2} I_{\alpha\beta}^{ij} + T_{\alpha\beta}^{ij} + \lambda_\kappa N_{\kappa\alpha\beta}^{ij} + \varepsilon_{\kappa\alpha\beta}^{ij}, \tag{1b}$$

where $e_{\kappa\alpha\beta}^{ij}$ and $\varepsilon_{\kappa\alpha\beta}^{ij}$ are observation errors of the pseudo and phase ranges, and

$$(\bullet)^{i}_{\alpha\beta} = (\bullet)^{i}_{\alpha} - (\bullet)^{i}_{\beta}, \tag{2a}$$

$$(\bullet)^{ij}_{\alpha} = (\bullet)^{i}_{\alpha} - (\bullet)^{j}_{\alpha}, \tag{2b}$$

$$(\bullet)^{ij}_{\alpha\beta} = (\bullet)^{i}_{\alpha\beta} - (\bullet)^{j}_{\alpha\beta} = (\bullet)^{ij}_{\alpha} - (\bullet)^{ij}_{\beta}. \tag{2c}$$

If the errors are neglected in the observation equations for L_κ and L_λ waves, $\rho^{ij}_{\alpha\beta}(t)+T^{ij}_{\alpha\beta}(t)$, $I^{ij}_{\alpha\beta}$, $N^{ij}_{\kappa\alpha\beta}$ and $N^{ij}_{\lambda\alpha\beta}$ are expressed in terms of $P^{ij}_{\kappa\alpha\beta}(t)$, $P^{ij}_{\lambda\alpha\beta}(t)$, $\Phi^{ij}_{\kappa\alpha\beta}(t)$ and $\Phi^{ij}_{\lambda\alpha\beta}(t)$.

The initial phase ambiguity $N^{ij}_{W(\kappa,\lambda)\alpha\beta}$ of the wide-lane combination of L_κ and L_μ waves is obtained as

$$N^{ij}_{W(\kappa,\lambda)\alpha\beta} \equiv N^{ij}_{\kappa\alpha\beta} - N^{ij}_{\lambda\alpha\beta} = \frac{\Phi^{ij}_{\kappa\alpha\beta}(t)}{l_\kappa} - \frac{\Phi^{ij}_{\lambda\alpha\beta}(t)}{l_\lambda} - \frac{f_\kappa - f_\lambda}{f_\kappa + f_\lambda}\left(\frac{P^{ij}_{\kappa\alpha\beta}(t)}{l_\kappa} + \frac{P^{ij}_{\lambda\alpha\beta}(t)}{l_\lambda}\right), \tag{3}$$

where f_κ and l_κ are the frequency and wave length of L_κ wave. Equation (3) is HMW (Hatch-Melbourne-Wübbena) combination (Hatch (1982, 1996), Melbourne (1985), Wübbena (1985), Isshiki (2003b)). When $\kappa=1$ and $\lambda=2$ in equation (3), $f_1 \approx 1.58GHz$ and $f_2 \approx 1.23GHz$, then $(f_1-f_2)/(f_1+f_2) \approx 0.124$. So, the error introduced by the pseudo ranges is reduced to $1/10$. HMW combination is free from the ionospheric and tropospheric delays. So, the wide-lane ambiguities can be calculated precisely without being affected by the baseline length. The average over epochs can eliminate the random errors effectively.

The wide-lane combination $\Phi^{ij}_{W(\kappa,\lambda)\alpha\beta}(t)$ of $\Phi^{ij}_{\kappa\alpha\beta}(t)$ and $\Phi^{ij}_{\lambda\alpha\beta}(t)$ is given as

$$\begin{aligned}\Phi^{ij}_{W(\kappa,\lambda)\alpha\beta}(t) &\equiv \frac{l_{W(\kappa,\lambda)}}{l_\kappa}\Phi^{ij}_{\kappa\alpha\beta}(t) - \frac{l_{W(\kappa,\lambda)}}{l_\lambda}\Phi^{ij}_{\lambda\alpha\beta}(t) \\ &= \rho^{ij}_{\alpha\beta} - \left(\frac{l_{W(\kappa,\lambda)}}{\lambda_\kappa}\frac{f_1^2}{f_\kappa^2} - \frac{l_{W(\kappa,\lambda)}}{\lambda_\lambda}\frac{f_1^2}{f_\lambda^2}\right)I^{ij}_{\alpha\beta} \\ &\quad + l_{W\kappa\lambda}\left(N^{ij}_{\kappa\alpha\beta} - N^{ij}_{\lambda\alpha\beta}\right) + T^{ij}_{\alpha\beta} + \varepsilon^{ij}_{W(\kappa,\lambda)\alpha\beta} \\ &= \rho^{ij}_{\alpha\beta} + \frac{f_1^2}{f_\kappa f_\lambda}I^{ij}_{\alpha\beta} + l_{W(\kappa,\lambda)}N^{ij}_{W(\kappa,\lambda)\alpha\beta} + T^{ij}_{\alpha\beta} + \varepsilon^{ij}_{W(\kappa,\lambda)\alpha\beta},\end{aligned} \tag{4}$$

where the frequency $f_{W(\kappa,\lambda)}$ and wave length $l_{W(\kappa,\lambda)}$ of the wide-lane combination are defined as

$$\frac{f_{W(\kappa,\lambda)}}{c} = \frac{f_\kappa - f_\lambda}{c} = \frac{1}{l_\kappa} - \frac{1}{l_\lambda} = \frac{1}{l_{W(\kappa,\lambda)}}, \tag{5}$$

and $\varepsilon^{ij}_{W(\kappa,\lambda)\alpha\beta}$ is the error of $\Phi^{ij}_{W(\kappa,\lambda)\alpha\beta}(t)$. c is the light speed in vacuum.

In case of a triple frequency system like GALILEO and the future GPS, two independent wide-lane combinations can be formed. If an ionospheric-free combination is obtained by eliminating the ionospheric delay $I^{ij}_{\alpha\beta}$, an unambiguous observation equations may be obtained, since the widelane ambiguities are determine by HMW combinations. So, the coordinates may be determined without being affected by the ionospheric delays by solving the equations epoch by epoch (Hatch (1996)). However, if the difference between the frequencies is small, the error is magnified. The multi-path error, will invite rather big errors in the coordinates.

For the wide-lane positioning in a dual frequency system like the present GPS, the ionospheric delay $I^{ij}_{\alpha\beta}$ has to be estimated in different ways. The varying component of $I^{ij}_{\alpha\beta}$ may be estimated precisely by the geometry-free combination $\Phi^{ij}_{G(\kappa,\lambda)\alpha\beta}(t)$ of $\Phi^{ij}_{\kappa\alpha\beta}(t)$ and $\Phi^{ij}_{\lambda\alpha\beta}(t)$:

$$\begin{aligned}\Phi^{ij}_{G(\kappa,\lambda)\alpha\beta}(t) &\equiv \Phi^{ij}_{\kappa\alpha\beta}(t) - \Phi^{ij}_{\lambda\alpha\beta}(t) \\ &= -\left(\frac{f_1^2}{f_\kappa^2} - \frac{f_1^2}{f_\lambda^2}\right)I^{ij}_{\alpha\beta}(t) + l_\kappa N^{ij}_{\kappa\alpha\beta} - l_\lambda N^{ij}_{\lambda\alpha\beta} + \varepsilon^{ij}_{G(\kappa,\lambda)\alpha\beta}(t),\end{aligned} \tag{6}$$

where $\varepsilon^{ij}_{G(\kappa,\lambda)\alpha\beta}$ is the error of $\Phi^{ij}_{G(\kappa,\lambda)\alpha\beta}(t)$. From equation (6), the variation of $I^{ij}_{\alpha\beta}$ is obtained as

$$\left(I^{ij}_{\alpha\beta}(t) - I^{ij}_{\alpha\beta}(0)\right) \approx -\frac{\left(\Phi^{ij}_{\kappa\alpha\beta}(t) - \Phi^{ij}_{\kappa\alpha\beta}(0)\right) - \left(\Phi^{ij}_{\lambda\alpha\beta}(t) - \Phi^{ij}_{\lambda\alpha\beta}(0)\right)}{(f_1/f_\kappa)^2 - (f_1/f_\lambda)^2}. \tag{7}$$

Now, $I^{ij}_{\alpha\beta}$ is written as

$$I^{ij}_{\alpha\beta}(t) = I^{ij}_{\alpha\beta}(0) + \left(I^{ij}_{\alpha\beta}(t) - I^{ij}_{\alpha\beta}(0)\right). \tag{8}$$

The first term on the right side of equation (8) must be estimated from the other information included in the observation or supplied from the external source such as IONEX (Global estimate of the ionospheric delays by JPL).

First, equation (8) is averaged over N epochs, and $I^{ij}_{\alpha\beta}(0)$ is written as

$$N \cdot I^{ij}_{\alpha\beta}(0) = \sum_{t=0}^{N-1} I^{ij}_{\alpha\beta}(t) - \sum_{t=0}^{N-1}\left(I^{ij}_{\alpha\beta}(t) - I^{ij}_{\alpha\beta}(0)\right). \tag{9}$$

If the error in the pseudo range is small, $I^{ij}_{\alpha\beta}(t)$ derived from the geometry-free combination of the pseudo ranges:

$$I^{ij}_{\alpha\beta}(t) \approx \frac{P^{ij}_{\kappa\alpha\beta}(t) - P^{ij}_{\lambda\alpha\beta}(t)}{(f_1/f_\kappa)^2 - (f_1/f_\lambda)^2} \tag{10}$$

may be used to estimate the first term on the right side of equation (9) as

$$\sum_{t=0}^{N-1} I^{ij}_{\alpha\beta}(t) \approx \sum_{t=0}^{N-1} \frac{P^{ij}_{\kappa\alpha\beta}(t) - P^{ij}_{\lambda\alpha\beta}(t)}{(f_1/f_\kappa)^2 - (f_1/f_\lambda)^2}. \tag{11}$$

Then, equations (7), (8), (9) and (11) gives

$$\begin{aligned}I^{ij}_{\alpha\beta}(t) &= \frac{1}{N}\sum_{t=0}^{N-1} \frac{P^{ij}_{\kappa\alpha\beta}(t) - P^{ij}_{\lambda\alpha\beta}(t)}{(f_1/f_\kappa)^2 - (f_1/f_\lambda)^2} \\ &\quad + \frac{1}{N}\sum_{t=0}^{N-1} \frac{\Phi^{ij}_{\kappa\alpha\beta}(t) - \Phi^{ij}_{\lambda\alpha\beta}(t)}{(f_1/f_\kappa)^2 - (f_1/f_\lambda)^2} - \frac{\Phi^{ij}_{\kappa\alpha\beta}(t) - \Phi^{ij}_{\lambda\alpha\beta}(t)}{(f_1/f_\kappa)^2 - (f_1/f_\lambda)^2}, \\ &\qquad t = 0,1,2,\cdots,N-1.\end{aligned} \tag{12}$$

In the present GPS system, $\kappa=1$ and $\lambda=2$, $f_1 \approx 1.58GHz$ and $f_2 \approx 1.23GHz$ and $\sigma\left[P^{ij}_{1\alpha\beta}(t)\right] = \sigma\left[P^{ij}_{2\alpha\beta}(t)\right] \approx 1m$. So, the error of $\sum_{t=0}^{N-1} I^{ij}_{\alpha\beta}(t)$ is estimated as $0.22m$ when $N_1 = 100$ (1 epoch = 30 sec). This result is rather disappointing.

Next, instead of the pseudo range, the ionospheric delay $I^{ij}_{IONX\alpha\beta}(t)$ supplied by IONEX is used. Then

$$\sum_{t=0}^{N-1} I^{ij}_{\alpha\beta}(t) \approx \sum_{t=0}^{N-1} I^{ij}_{IONX\alpha\beta}(t). \tag{13}$$

The error of this estimation may be about $0.1m$, not so bad. Then, equations (7), (8), (9) and (13) gives

$$I^{ij}_{\alpha\beta}(t) = \frac{1}{N}\sum_{t=0}^{N-1} I^{ij}_{IONX\alpha\beta}(t) + \frac{1}{N}\sum_{t=0}^{N-1} \frac{\Phi^{ij}_{\kappa\alpha\beta}(t) - \Phi^{ij}_{\lambda\alpha\beta}(t)}{(f_1/f_\kappa)^2 - (f_1/f_\lambda)^2}$$

$$-\frac{\Phi_{\kappa\alpha\beta}^{ij}(t)-\Phi_{\lambda\alpha\beta}^{ij}(t)}{(f_1/f_\kappa)^2-(f_1/f_\lambda)^2},\quad t=0,1,2,\cdots,N-1 \tag{14}$$

Similar equations for estimating the ionospheric delays such as equations (12) and (14) may also be obtained as follows (Isshiki (2006)). The geometry free combinations for pseudo and phase ranges between L_κ and L_λ waves are given as

$$P_{\kappa\alpha\beta}^{ij}(t)-P_{\lambda\alpha\beta}^{ij}(t)=\left(\frac{f_1^2}{f_\kappa^2}-\frac{f_1^2}{f_\lambda^2}\right)I_{\alpha\beta}^{ij}(t)+e_{G(\kappa,\lambda)\alpha\beta}^{ij}(t), \tag{15a}$$

$$\Phi_{\kappa\alpha\beta}^{ij}(t)-\Phi_{\lambda\alpha\beta}^{ij}(t)-l_\lambda N_{W(\kappa,\lambda)\alpha\beta}^{ij}$$
$$=-\left(\frac{f_1^2}{f_\kappa^2}-\frac{f_1^2}{f_\lambda^2}\right)I_{\alpha\beta}^{ij}(t)+(l_\kappa-l_\lambda)N_{\kappa\alpha\beta}^{ij}+\varepsilon_{G(\kappa,\lambda)\alpha\beta}^{ij}(t), \tag{15b}$$

where $N_{W(\kappa,\lambda)\alpha\beta}^{ij}$ is the wide-lane ambiguity defined by equation (3). The unknowns in equation (15) are $I_{\alpha\beta}^{ij}(0), I_{\alpha\beta}^{ij}(1),\cdots,I_{\alpha\beta}^{ij}(N-1)$ and $N_{\kappa\alpha\beta}^{ij}$. Then, a minimum value problem:

$$F(I_{\alpha\beta}^{ij}(0), I_{\alpha\beta}^{ij}(1),\cdots,I_{\alpha\beta}^{ij}(N-1), N_{\kappa\alpha\beta}^{ij})$$
$$\equiv\frac{\sigma_\varepsilon^2}{\sigma_e^2}\sum_{t=0}^{N-1}\left\{\left(\frac{f_1^2}{f_\kappa^2}-\frac{f_1^2}{f_\lambda^2}\right)I_{\alpha\beta}^{ij}(t)-\left[P_{\kappa\alpha\beta}^{ij}(t)-P_{\lambda\alpha\beta}^{ij}(t)\right]\right\}^2$$
$$+\sum_{t=0}^{N-1}\left\{-\left(\frac{f_1^2}{f_\kappa^2}-\frac{f_1^2}{f_\lambda^2}\right)I_{\alpha\beta}^{ij}(t)+(l_\kappa-l_\lambda)N_{\kappa\alpha\beta}^{ij}\right.$$
$$\left.-\left[\Phi_{\kappa\alpha\beta}^{ij}(t)-\Phi_{\lambda\alpha\beta}^{ij}(t)-l_\lambda N_{W(\kappa,\lambda)\alpha\beta}^{ij}\right]\right\}^2$$
$$=minimun \tag{16}$$

is introduced, where σ_e^2 and σ_ε^2 be the variances of $e_{G(\kappa,\lambda)\alpha\beta}^{ij}$ and $\varepsilon_{G(\kappa,\lambda)\alpha\beta}^{ij}$. The stationary conditions of this minimum value problem give

$$0=\frac{\partial F}{\partial I_{\alpha\beta}^{ij}(t)}$$
$$=\frac{2\sigma_\varepsilon^2}{\sigma_e^2}\left\{\left(\frac{f_1^2}{f_\kappa^2}-\frac{f_1^2}{f_\lambda^2}\right)I_{\alpha\beta}^{ij}(t)-\left[P_{\kappa\alpha\beta}^{ij}(t)-P_{\lambda\alpha\beta}^{ij}(t)\right]\right\}\left(\frac{f_1^2}{f_\kappa^2}-\frac{f_1^2}{f_\lambda^2}\right)$$
$$-2\left\{-\left(\frac{f_1^2}{f_\kappa^2}-\frac{f_1^2}{f_\lambda^2}\right)I_{\alpha\beta}^{ij}(t)+(l_\kappa-l_\lambda)N_{\kappa\alpha\beta}^{ij}\right.$$
$$\left.-\left[\Phi_{\kappa\alpha\beta}^{ij}(t)-\Phi_{\lambda\alpha\beta}^{ij}(t)-l_\lambda N_{W(\kappa,\lambda)\alpha\beta}^{ij}\right]\right\}\left(\frac{f_1^2}{f_\kappa^2}-\frac{f_1^2}{f_\lambda^2}\right), \tag{17a}$$

$$0=\frac{\partial F}{\partial N_{\kappa\alpha\beta}^{ij}}=2\sum_{t=0}^{N-1}\left\{-\left(\frac{f_1^2}{f_\kappa^2}-\frac{f_1^2}{f_\lambda^2}\right)I_{\alpha\beta}^{ij}(t)+(l_\kappa-l_\lambda)N_{\kappa\alpha\beta}^{ij}\right.$$
$$\left.-\left[\Phi_{\kappa\alpha\beta}^{ij}(t)-\Phi_{\lambda\alpha\beta}^{ij}(t)-l_\lambda N_{W(\kappa,\lambda)\alpha\beta}^{ij}\right]\right\}(l_\kappa-l_\lambda). \tag{17b}$$

From equation (17), equation (12) is obtained, when σ_ε^2 is much less than σ_e^2.

If an equation:

$$I_{IONX\alpha\beta}^{ij}(t)=I_{\alpha\beta}^{ij}(t)+\varepsilon_{IONX\alpha\beta}^{ij}. \tag{18}$$

is used instead of equation (15a), equation (14) is derived, when σ_ε^2 is much less than σ_e^2.

By the way, the errors of the pseudo ranges are important for estimating the errors involved in the ionospheric delays $I_{\alpha\beta}^{ij}$ estimated by using the pseudo and phase ranges. The errors of the pseudo ranges are estimated as follows. The three geometry free combinations are obtained from equation (1) as

$$P_{\kappa\alpha\beta}^{ij}(t)-\Phi_{\kappa\alpha\beta}^{ij}(t)=2\frac{f_1^2}{f_\kappa^2}I_{\alpha\beta}^{ij}(t)-l_\kappa N_{\kappa\alpha\beta}^{ij}+e_{\kappa\alpha\beta}^{ij}(t)-\varepsilon_{\kappa\alpha\beta}^{ij}(t), \tag{19a}$$

$$P_{\lambda\alpha\beta}^{ij}(t)-\Phi_{\lambda\alpha\beta}^{ij}(t)=2\frac{f_1^2}{f_\lambda^2}I_{\alpha\beta}^{ij}(t)-l_\lambda N_{\lambda\alpha\beta}^{ij}+e_{\lambda\alpha\beta}^{ij}(t)-\varepsilon_{\lambda\alpha\beta}^{ij}(t), \tag{19b}$$

$$\Phi_{\kappa\alpha\beta}^{ij}(t)-\Phi_{\lambda\alpha\beta}^{ij}(t)$$
$$=-\left(\frac{f_1^2}{f_\kappa^2}-\frac{f_1^2}{f_\lambda^2}\right)I_{\alpha\beta}^{ij}(t)+l_\kappa N_{\kappa\alpha\beta}^{ij}-l_\lambda N_{\lambda\alpha\beta}^{ij}+\varepsilon_{\kappa\alpha\beta}^{ij}(t)-\varepsilon_{\lambda\alpha\beta}^{ij}(t). \tag{19c}$$

If $\varepsilon_{\kappa\alpha\beta}^{ij}(t)$ and $\varepsilon_{\lambda\alpha\beta}^{ij}(t)$ are neglected and the ambiguities are known, the unknowns in equation (19) are $I_{\alpha\beta}^{ij}(t)$, $e_{\kappa\alpha\beta}^{ij}(t)$ and $e_{\lambda\alpha\beta}^{ij}(t)$. Then, $e_{\kappa\alpha\beta}^{ij}(t)$ and $e_{\lambda\alpha\beta}^{ij}(t)$ are given by solving equations (19) as

$$e_{\kappa\alpha\beta}^{ij}(t)\approx P_{\kappa\alpha\beta}^{ij}(t)+\frac{\left(1+(f_\kappa/f_\lambda)^2\right)\Phi_{\kappa\alpha\beta}^{ij}(t)-2\Phi_{\lambda\alpha\beta}^{ij}(t)}{1-(f_\kappa/f_\lambda)^2}$$
$$-\frac{\left(1+(f_\kappa/f_\lambda)^2\right)l_\kappa N_{\kappa\alpha\beta}^{ij}-2l_\lambda N_{\lambda\alpha\beta}^{ij}}{1-(f_\kappa/f_\lambda)^2}, \tag{20a}$$

$$e_{\lambda\alpha\beta}^{ij}(t)\approx P_{\lambda\alpha\beta}^{ij}(t)+\frac{2(f_\kappa/f_\lambda)^2\Phi_{\kappa\alpha\beta}^{ij}(t)-\left(1+(f_\kappa/f_\lambda)^2\right)\Phi_{\lambda\alpha\beta}^{ij}(t)}{1-(f_\kappa/f_\lambda)^2}$$
$$-\frac{2(f_\kappa/f_\lambda)^2 l_\kappa N_{\kappa\alpha\beta}^{ij}-\left(1+(f_\kappa/f_\lambda)^2\right)l_\lambda N_{\lambda\alpha\beta}^{ij}}{1-(f_\kappa/f_\lambda)^2}. \tag{20b}$$

3. Numerical Results

In the following numerical calculations, observation data of fixed stations shown in Table 1 are used. These data were observed by GEONET of GSI (Geographical Survey Institute, Japan) on December 6, 2002 and downloaded from the homepage of GSI. The ionosphere was rather active in 2002. The two pseudo range signals and the two phase signals of L_1 and L_2 waves of GPS are given there. The epoch is 30 seconds, and the data between 00:00:00 UT and 02:00:00 UT are used for the calculations. The coordinates (x,y,z) of the stations and the baseline length dr between the stations shown in Table 1 are very precise ones downloaded from the same site. In the following calculations, the precise orbits of the satellites are used, and the tropospheric delays are estimated by Saastamoinen model.

In Table 2, the wide-lane ambiguities of L_1 and L_2 signals calculated by equation (3), that is, HMW combinations are shown. It is verified that the wide-lane ambiguities is determined quite successfully irrespective of the baseline length by HMW combinations.

In Figure 1, the double differenced ionospheric delays between Sapporo and Eniwa estimated by phase and pseudo ranges, that is, equation (12) are compared with the correct ones and the ones estimated by phase ranges and IONEX, that is, equation (14). The correct ones are obtained by using the correct $L_{W(1,2)}$ ambiguities calculated by HMW combinations, L_1 ambiguities calculated by the static positioning of the ionosphere free combinations of L_1 and L_2 waves and the geometry free combinations of the same waves. The similar results between Sapporo and Ichikawa are shown in Figure 2. The accuracies of the estimations using the pseudo ranges are the lowest, since the errors included in the pseudo ranges are poor.

Equation (20) gives a means to calculate the errors e_1 and e_2 in the pseudo ranges. The results are shown in table 3 for Sapporo-Eniwa baseline and in table 4 for Sapporo-Ichikawa baseline. As shown in Tables 3c and 4c, the differences of averages or biases $\bar{e}_1$ and $\bar{e}_2$ are very big for some combinations of the satellites. For example, in case of Table 3c, the differences $\bar{e}_1 - \bar{e}_2$ for ((5)-(14)) and ((30)-(14)) are much bigger than those for ((6)-(14)) and ((25)-(14)). And this corresponds to the big errors in Figures 1(a) and 1(d). Similar relationship holds for Sapporo-Ichikawa baseline. So, the bias components in the errors of the pseudo range are responsible for the poor accuracy of the ionospheric delays.

Table 3a. The errors e_1 included in the pseudo ranges P_1 (Sppr-Enwa: 02.12.06, 00:00:00UT-02:00:00).

	((5)-(14))	((6)-(14))	((25)-(14))	((30)-(14))
Average	0.078451	0.053028	0.10726	0.100272
Std dev	0.495064	0.512458	0.409651	0.379293

Table 3b. The errors e_2 included in the pseudo ranges P_2 (Sppr-Enwa: 02.12.06, 00:00:00UT-02:00:00).

	((5)-(14))	((6)-(14))	((25)-(14))	((30)-(14))
Average	-0.02514	0.046158	0.10252	-0.05403
Std dev	0.695531	0.798407	0.634925	0.489992

Table 3c. The averages or biases $\bar{e}_1$ and $\bar{e}_2$ (Sppr-Enwa: 02.12.06, 00:00:00UT-02:00:00).

	((5)-(14))	((6)-(14))	((25)-(14))	((30)-(14))
$\bar{e}_1$	0.078451	0.053028	0.10726	0.100272
$\bar{e}_2$	-0.02514	0.046158	0.10252	-0.05403
$\bar{e}_1 - \bar{e}_2$	0.103596	0.00687	0.00474	0.154299

Table 4a. The errors e_1 included in the pseudo ranges P_1 (Sppr-Ichk: 02.12.06, 00:00:00UT-02:00:00).

	((5)-(14))	((6)-(14))	((25)-(14))	((30)-(14))
Average	-0.01877	0.063468	0.310909	-0.00529
Std dev	0.469117	0.43593	0.525301	0.470828

Table 4b. The errors e_2 included in the pseudo ranges P_2 (Sppr-Ichk: 02.12.06, 00:00:00UT-02:00:00).

	((5)-(14))	((6)-(14))	((25)-(14))	((30)-(14))
Average	-0.0517	0.024594	-0.36598	-0.09567
Std dev	0.91217	0.837574	0.852099	1.154072

Table 4c. The averages or biases $\bar{e}_1$ and $\bar{e}_2$ (Sppr-Ichk: 02.12.06, 00:00:00UT-02:00:00).

	((5)-(14))	((6)-(14))	((25)-(14))	((30)-(14))
$\bar{e}_1$	-0.01466	0.072276	0.318551	0.00427
$\bar{e}_2$	-0.0517	0.024594	-0.36598	-0.09567
$\bar{e}_1 - \bar{e}_2$	0.037038	0.047681	0.684536	0.099938

However, they may be improved in future, when the errors in the pseudo ranges are decreased in the future GPS receiver technology and in the future GNSS system, namely, modernized GPS and GALILEO etc.

In Figure 3, the baseline length between Sapporo and Eniwa calculated by the wide-lane combinations using the ionospheric delays estimated by phase and pseudo ranges (blue line) is compared with the correct one using the ionosphere free combinations (magenta line) and the one calculated by the wide-lane combinations using the ionospheric delays estimated by phase ranges and IONEX (yellow line). The similar result on the baseline length between Sapporo and Ichikawa is shown in Figure 4. The accuracy of the baseline lengths calculated by the wide-lane combinations using the ionospheric delays estimated by phase and pseudo ranges is rather poor because of the big errors involved in pseudo ranges.

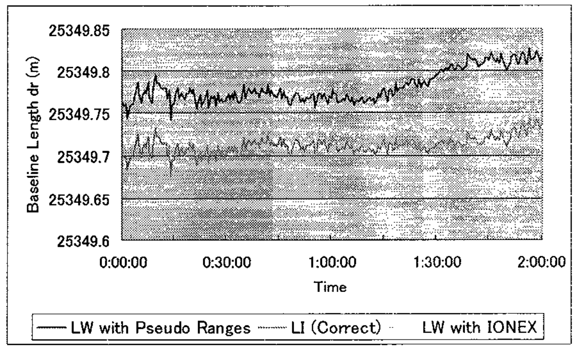

Figure 3. Baseline length between Sapporo and Eniwa (baseline length: about 25km) calculated by the wide-lane combinations using the ionospheric delays estimated by phase and pseudo ranges (blue line) compared with the correct one using the ionosphere free combinations (magenta line) and the one using the ionospheric delays estimated by phase ranges and IONEX (yellow line).

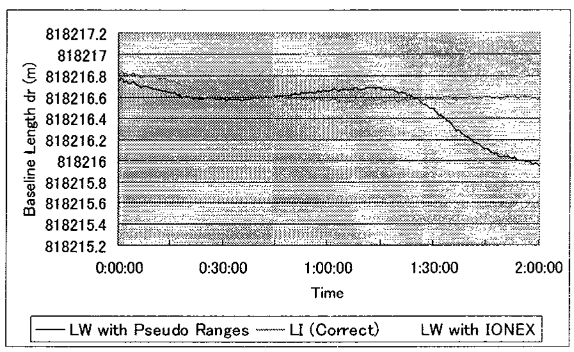

Figure 4. Baseline length between Sapporo and Ichikawa (baseline length: about 800km) calculated by the wide-lane combinations using the ionospheric delays estimated by phase and pseudo ranges (blue line) compared with the correct one using the ionosphere free combinations (magenta line) and the one using the ionospheric delays estimated by phase ranges and IONEX (yellow line).

4. Conclusions

In the present paper, the ionospheric delays are estimated under the present dual frequency system by using both the pseude and phase ranges. Unfortunately, the accuracy is rather poor because of the big errors involved in pseudo ranges. It is clarified that the bias components in the errors of the pseudo range are responsible for the poor accuracy of the ionospheric delays. However, if the accuracy of the pseudo range measurements is improved in future GPS receiver technology and in future GNSS systems such as GPS Modernization and GALILEO, the method would become very promising.

The baseline lengths between Sapporo and Eniwa (about 25km) and Sapporo and Ichikawa (about 800km) are calculated by the wide-lane combinations using the ionospheric delays based on the phase and pseudo ranges and are compared with the correct results and the results estimated by phase ranges and IONEX (Isshiki (2004c, 2005, 2006)). Both for the short and long baselines, the accuracy of the baselines obtained by using the ionospheric delays based on the phase and pseudo ranges is poor.

If the more precise ionospheric estimation becomes available in future, the precision of the wide-lane positioning may be increased further, and the wide-lane positioning discussed in the present and previous papers may become one of the precise and convenient positioning methods.

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Table 1. Station Coordinates (x, y, z) and baseline length dr (downloaded from the homepage of GEONET: 02.12.06).

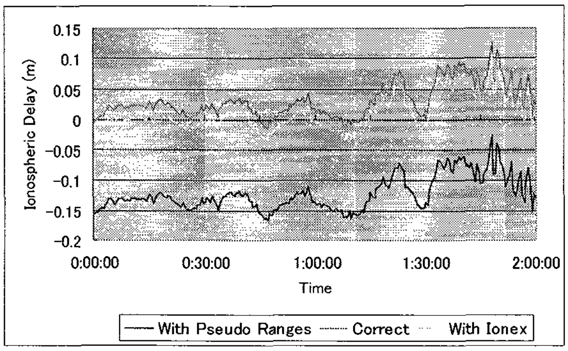
Stn Name	Stn.ID	Abrib.	x (m)	y (m)	z (m)	dr (m)
Sapporo	950128	Sppr	-3647449.8988	2923169.2544	4325315.3884	0
Eniwa	960522	Enwa	-3667125.8966	2908882.6546	4318149.1057	25349.7028
Ichikawa	93023	Ichk	-3967874.2402	3340981.7058	3699025.1252	818216.6565

Table 2a. Wide-lane ambiguities calculated by HMW combinations (Sppr-Enwa: 02.12.06, 00:00:00UT-02:00:00).

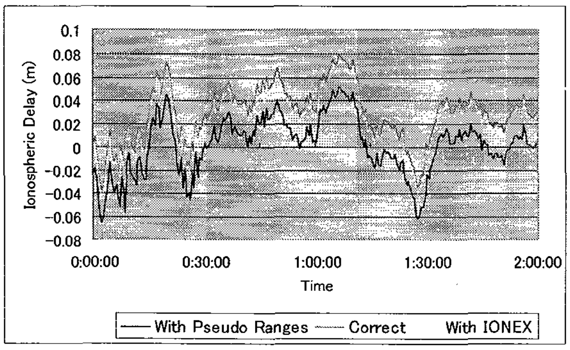
	((05)-(14))	((06)-(14))	((25)-(14))	((30)-(14))
Float (Mean)	2200137.932	-4995375.995	-5336259.061	-562506.032
Fix	2200138.000	-4995376.000	-5336259.000	-562506.000
Std. Dev.	0.031	0.033	0.028	0.023

Table 2b. Wide-lane ambiguities calculated by HMW combinations (Sppr-Ichk: 02.12.06, 00:00:00UT-02:00:00).

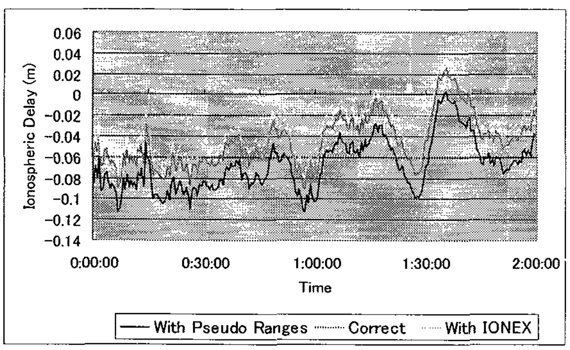
	((05)-(14))	((06)-(14))	((25)-(14))	((30)-(14))
Float (Mean)	3945315.988	-8337131.946	-7584052.919	-181954.946
Fix	3945316.000	-8337132.000	-7584053.000	-181955.000
Std. Dev.	0.032	0.031	0.030	0.033



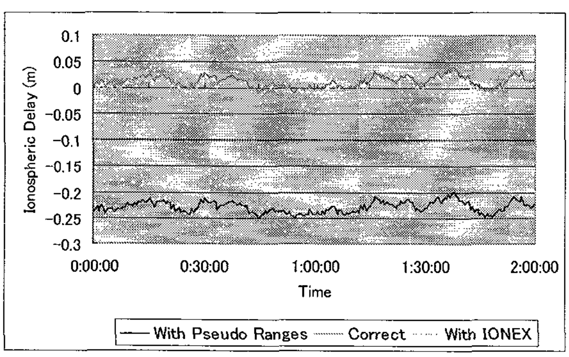
(a) Difference between satellites 5 and 14.



(b) Difference between satellites 6 and 14.

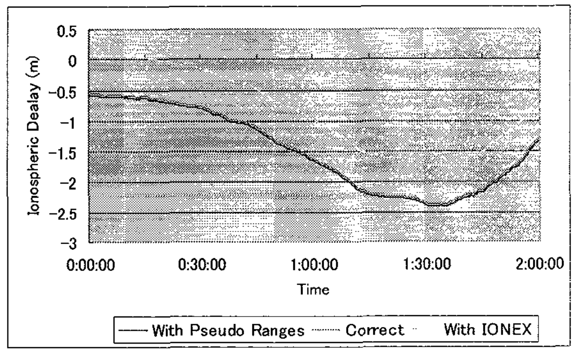


(c) Difference between satellites 25 and 14.

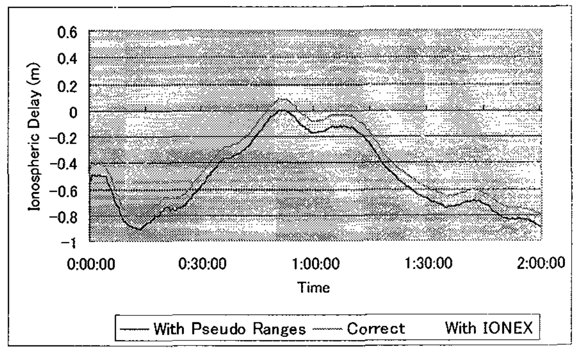


(d) Difference between satellites 30 and 14.

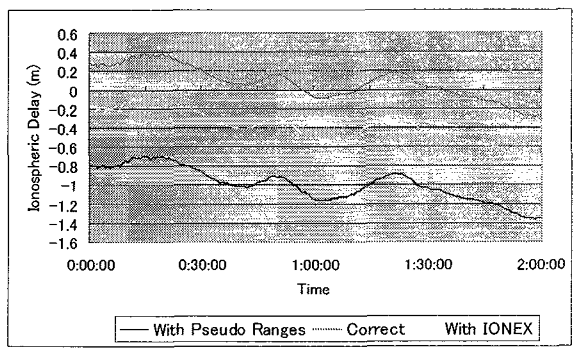
Figure 1. Double differenced ionospheric delays between Sapporo and Eniwa (baseline length: about 25km) estimated by phase and pseudo ranges (blue line) compared with the correct ones (magenta line) and the ones estimated by phase ranges and IONEX (yellow line).



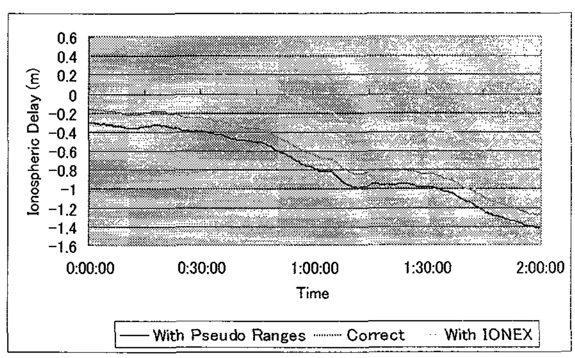
(a) Difference between satellites 5 and 14.



(b) Difference between satellites 6 and 14.



(c) Difference between satellites 25 and 14.



(d) Difference between satellites 30 and 14.

Figure 2. Double differenced ionospheric delays between Sapporo and Ichikawa (baseline length: about 800km) estimated by phase and pseudo ranges (blue line) compared with the correct ones (magenta line) and the ones estimated by phase ranges and IONEX (yellow line).