Direct Calculation For Large Deformation

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Abstract

The paper proposes a condition that should be satisfied when using the combination with different carrier phase observations to get the high precision deformation value. If the condition is satisfied, on the basis of DC algorithm, when the deformation is relatively large (0.7m), high precision deformation value can be obtained.

Keywords: GPS, Deformation monitoring, Integer Ambiguity.

1. Introduction

According to the deformation characteristics of the monitored objects, there are three different monitoring modes in GPS deformation monitoring[6] ,namely periodicity repeating surveying, immovable continuous GPS station array, and the real time dynamic monitoring. The first two modes are suitable for the slow deformation, and the static relative positioning is adopted to process the data. The third mode usually fits rapid deformation or slow deformation with breaking deformation, and the OTF method is mainly adopted to process the data. In classical static relative positioning approaches , integer ambiguities are estimated based on carrier phase measurements and calculated by an adjustment procedure which also computes other unknown parameters. It needs not only the equations to compute the float carrier phase solution but also to search and fix the float solution. So it takes a long time to resolute integer ambiguities. These methods exploit a few observations of past epochs and must guarantee to trace the satellites during observations and have no cycle slips.

Therefore a new method of integer ambiguity resolution——DC (Direction Calculate) is proposed according to the characteristics of GPS monitoring network[1] . When the displacement of monitoring points is less than 0.16m using L1 carrier phase, the algorithm can solve the ambiguities in real time. It need not search or fix them. Sometimes the deformation is relatively large, more than 0.2 meter and even more than 0.5 meter, the algorithm should be developed further. So on the basis of paper [1], this paper introduces a new method to get the high precision deformation value using the combination with different carrier phase. When the deformation is less than 0.7 meter, high precision deformation value can be obtained. The method is also applied to general GPS short-baseline relative positioning.

2. Principle of Direct Calculation

In GPS deformation monitoring network, the distance of the baseline is short, generally from several hundred meters to several kilometers, then the direction cosines of base station and monitored station to the same satellite can be considered as the same, so the common errors, such as clock off-set of the satellite, errors of ephemeredes, the delay errors of the atmosphere etc. can be eliminated or reduced well. The position of the satellites can be calculated according to broadcast ephemeris and precise ephemeris after delete unhealthy satellites from satellite ephemeredes.

Assuming at epoch t, let j denotes the (known) reference satellite, k denotes the other satellite, λ denotes the wavelength and denoting the points by 1 and 2, the single-difference equation is given by Eq.(1.1) and Eq.(1.2). The concept of DGPS is given by Fig.1

$$(\Delta N_{1-2}^{j}+\Delta\phi_{1-2}^{j})\ \lambda=\Delta\rho_{1-2}^{j}+c\times(V_{t_1^R}-V_{t_2^R}) \tag{1.1}$$

$$(\Delta N_{1-2}^{k}+\Delta\phi_{1-2}^{k})\ \lambda=\Delta\rho_{1-2}^{k}+c\times(V_{t_1^R}-V_{t_2^R}) \tag{1.2}$$

The difference of the above two equations are:

$$(\Delta N_{1-2}^{k}-\Delta N_{1-2}^{j}+\Delta\phi_{1-2}^{k}-\Delta\phi_{1-2}^{j})\ \lambda=\Delta\rho_{1-2}^{k}-\Delta\rho_{1-2}^{j} \tag{1.3}$$

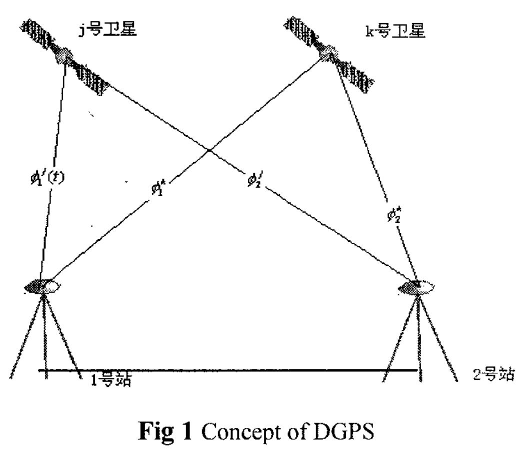

Fig 1 Concept of DGPS

Using additionally the shorthand notations

$$\nabla\Delta N_{1-2}^{kj}=\Delta N_{1-2}^{k}-\Delta N_{1-2}^{j} \tag{1.4}$$

$$\nabla\Delta\phi_{1-2}^{kj}=\Delta\phi_{1-2}^{k}-\Delta\phi_{1-2}^{j} \tag{1.5}$$

$$\nabla\Delta\rho_{1-2}^{kj}=\Delta\rho_{1-2}^{k}-\Delta\rho_{1-2}^{j} \tag{1.6}$$

and substituting (1.4), (1.5) and (1.6) into (1.3) gives

$$\nabla\Delta N_{1-2}^{kj} = \nabla\Delta\rho_{1-2}^{kj} / \lambda - \nabla\Delta\phi_{1-2}^{kj} \quad (1.7)$$

where the term $\nabla\Delta N_{1-2}^{kj}$ denotes the (double-) difference of the phase ambiguities between two epochs, $\nabla\Delta\phi_{1-2}^{kj}$ denotes the (double-) difference of the measured carrier phase expressed in cycles between two epochs and $\nabla\Delta\rho_{1-2}^{kj}$ denotes the (double-) difference of geometric range between two epochs.

In this mathematical model, when the accurate coordinates of the monitoring point and satellites are known at each epoch, the integer ambiguities can be calculated directly. Using L1 carrier phase, when the displacement of the monitoring station is less than 0.1648 meter, the impact value of the integer ambiguities is less than half cycle. Eq.(1.7) is the DC algorithm.

$$\Delta d = \sqrt{dx_p^2 + dy_p^2 + dz_p^2} = dx_p\sqrt{3} \le \frac{\sqrt{3}}{2}\lambda_{L_1} = 0.09515\sqrt{3} = 0.1648m \quad (1.8)$$

3. Condition of Direct Calculation to get the high precision deformation

When the displacement is large to 0.7 me, if we hope that the impact value of the integer ambiguities is less than half-cycle, applying Eq.(1.7), the wavelength of the carrier phase is:

$$\lambda \ge 2\Delta d / \sqrt{3} \quad (1.9)$$

substituting the displacement of 0.7m into （1.9）gives

$$\lambda \ge 2 \times 0.7 / \sqrt{3} \ge 0.8083m \quad (1.10)$$

Different phase combinations are considered as different phase observables with different wavelengths and accuracy [2]. Assuming φ_1 and φ_2 are two carrier phase .The linear combination of them is defined by

$$\varphi_{n,m} = n\varphi_1 + m\varphi_2 \quad (1.11)$$

where n and m are arbitrary numbers. The substitution of the relations $\varphi_i = f_i t$ for the corresponding frequencies f_1 and f_2 gives

$$\varphi = nf_1 t + mf_2 t = ft \quad (1.12)$$

Therefore,

$$f = nf_1 + mf_2 \quad (1.13)$$

is the frequency and

$$\lambda = \frac{c}{f} = \frac{c}{nf_1 + mf_2} \quad (1.14)$$

is the wavelength of the linear combination. Choosing proper n and m can yield the corresponding wavelength which is more than 0.8083m. Then compute the integer ambiguities according to the Eq.(1.7) can guarantee that the impact value of the integer ambiguities is less than half cycle.

The advantage of a linear combination with integer numbers is that the integer nature of the ambiguities is preserved and can be computed easily. Considering a certain noise level for phases, the noise level increases for the linear combination. So solving the deformation on the basis of L1 carrier phase will have relatively low noise. To get the integer ambiguities of L_1 and L_2, another linear combination φ_s whose wavelength is more than 0.8083m is needed. Then compute the integer ambiguities according to the Eq.(1.15).

With the linear combination integer ambiguities N_w and N_s, the he integer ambiguities of L_1 and L_2 can be obtained from

$$\begin{cases} N_w = n_1 N_1 + m_1 N_2 \\ N_s = n_2 N_1 + m_2 N_2 \end{cases} \quad (1.15)$$

Considering

$$\begin{pmatrix} N_1 \\ N_2 \end{pmatrix} = \begin{pmatrix} n_1 & m_1 \\ n_2 & m_2 \end{pmatrix}^{-1} \begin{pmatrix} N_w \\ N_s \end{pmatrix} = \frac{1}{\begin{vmatrix} n_1 & m_1 \\ n_2 & m_2 \end{vmatrix}} \begin{pmatrix} m_2 & -m_1 \\ -n_2 & n_1 \end{pmatrix} \begin{pmatrix} N_w \\ N_s \end{pmatrix} \quad (1.16)$$

where N_w, N_s, n_1, m_1, n_2 and m_2 are all integer numbers. To be sure that the integer nature of the ambiguities N_1 and N_2 is preserved, the condition

$$\begin{vmatrix} n_1 & m_1 \\ n_2 & m_2 \end{vmatrix} = 1 \quad (1.17)$$

must be fulfilled and Eq.(1.17) can be rewritten as

$$n_1 m_2 - n_2 m_1 = 1 \quad (1.18)$$

Eq.(1.18) is the condition that should be fulfilled to get the integer ambiguities N_1 and N_2 from Eq.(1.15).

The wide lane and super wide lane result from the choice

$$n_1 = 1 \quad m_1 = -1 \quad n_2 = -3 \quad m_2 = 4 \quad (1.19)$$

And they fulfill the Eq.(1.15)

$$n_1 m_2 - n_2 m_1 = 1 \times 4 - (-3) \times (-1) = 1 \quad (1.20)$$

Moreover the noise for wide lane and super wide lane is relatively small in all linear combination. The wavelength of wide lane is $\lambda_w = 86.19cm$ and that of super wide lane is $\lambda_s = 162.81cm$. Both of them fulfill the Eq.(1.10). Therefore applying wide lane and super wide lane to compute the integer ambiguities of L1 and L2 can not only preserve integer nature of the ambiguities N_1 and , but also fix the integer ambiguities easily.

4. Application of Direct Calculation in general GPS short-baseline ambiguity resolution

The determination of integer ambiguity in short-baseline in relative positioning with carrier phase is the most common problem in GPS application. If Direct Calculation can solve this problem, it will be a great progress in integer ambiguities resolution. It is known that relative positioning with pseudorange double difference need not determine the integer ambiguities and can calculate the baseline quickly. But the precision of a pseudorange derived from code measurements is low. However the precision of relative positioning with phase double difference is high. The disadvantage of it is that it has concerned with a complicated integer ambiguity resolution. The Direct Calculation algorithm has both the two advantages. The deformation can be calculated according to the following steps:

1) Using the code pseudoranges to calculate the approximate coordinate of the unknown station. Make sure that the accuracy of the coordinates calculated is better than 0.7 m.
2) Compute the integer ambiguity N_w of wide lane and the integer ambiguity N_S of super wide lane according to Eq.(1.7). with the initial coordinate of the unknown station.
3) Apply Eq.(1.13) to compute the integer ambiguity N_1 and N_2 of L_1 and L_2 and substitute the calculated integer ambiguity N_1 to the following equation

$$V = a\delta X + b\delta Y + c\delta Z - L \tag{1.21}$$

where: a, b and c are the direction cosine from station to satellite; $[\delta X \quad \delta Y \quad \delta Z]^T$ is the deformation information ; L is the constant term.

4) Exploit the L_1 carrier phase observables to get the deformation of the unknown station.

$$\hat{X} = (B^T B)^{-1} B^T L \tag{1.22}$$

Where $X = [\delta X \quad \delta Y \quad \delta Z]^T$ is the deformation information; L is the constant term.

5. Test and Analyses

An experiment was carried out with TRIMBLE 5700 GPS receiver. Six satellites were tracked for two hours at sample rates of fifteen seconds. TGO 5.1 soft calculated the coordinates of the monitoring points the following values in meters (-2794583.3077 , 4649775.1620 , 3342969.6442) , (-2793370.6767, 4649979.2830 , 3343690.6012) . Using double-differences with satellite 4 as reference satellite, the ambiguities yielded by DC in cycles are listed in Table.4.1. The deformations computed and the real deformations are compared in Table .4.2.

We have designed ten tests in which the displacement changes from 0.1608m to 0.6160m.In each tests, compute the deformations at each epoch according to DC. It can be seen from Table.4.1 that integer ambiguities computed from different displacement is all correct and from Table.4.2 the averages deformation of three directions of all epochs computed from different base stations well match the real deformation. The maximum difference from the designed deformation is only 1.9mm and the minimum difference from the designed deformation is 0.3mm. It indicates that the deformation information solved by DC is reliable and correct.

Compared with other methods, DC in this paper has the following characteristics. Firstly, since the first period precise baseline vectors between base station and monitoring station are used as initial condition in GPS deformation monitoring, the ambiguity resolution can become easier. It need not to search the integer ambiguities. Secondly, adopting the single epoch algorithm, the troublesome problem of detecting and repairing cycle slips is avoided, and DC can be applied in static and dynamic deformation monitoring. Finally, when the initial coordinates whose accuracy is less than 0.6m, DC can determine the integer ambiguities correctly in short-baseline data processing and can also obtain the high precision deformation value.

Acknowledgement

The research project is funded by the National Natural Science Foundation of China (40474003) and Special Project Fund of Taishan Scholars of Shandong Province (TSXZ0502).

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Table 4.1 The float ambiguities calculated by the DC algorithm

Deformation (m)	4-8 (L1 accurate cycles 69) (L2 accurate cycles 99)	4-11 (L1 accurate cycles 8) (L2 accurate cycles 86)	4-20 (L1 accurate cycles 58) (L2 accurate cycles 186)	4-28 (L1 accurate cycles 12) (L2 accurate cycles 16)

	wide line	super wide lane	N1	N2	wide line	super wide lane	N1	N2	wide line	super wide lane	N1	N2	wide line	super wide lane	N1	N2
0.1608	-30	189	69	99	-78	320	8	86	-128	570	58	186	-4	28	12	16
0.1778	-30	189	69	99	-78	320	8	86	-128	570	58	186	-4	28	12	16
0.1990	-30	189	69	99	-78	320	8	86	-128	570	58	186	-4	28	12	16
0.3000	-30	189	69	99	-78	320	8	86	-128	570	58	186	-4	28	12	16
0.3674	-30	189	69	99	-78	320	8	86	-128	570	58	186	-4	28	12	16
0.4011	-30	189	69	99	-78	320	8	86	-128	570	58	186	-4	28	12	16
0.4500	-30	189	69	99	-78	320	8	86	-128	570	58	186	-4	28	12	16
0.5000	-30	189	69	99	-78	320	8	86	-128	570	58	186	-4	28	12	16
0.5196	-30	189	69	99	-78	320	8	86	-128	570	58	186	-4	28	12	16
0.6160	-30	189	69	99	-78	320	8	86	-128	570	58	186	-4	28	12	16

Table 4.2 Comparison between deformations computed and the real deformations

Displacement(m)	Real Deformation			Deformation computed			Comparison		
	dX(m)	dY(m)	dZ(m)	dX(m)	dY(m)	dZ(m)	dX(m)	dY(m)	dZ(m)
0.1608	0.1150	-0.1050	-0.0400	0.1150	-0.1050	-0.0400	0.0000	0.0000	0.1608
0.1778	0.1400	0.0450	-0.1000	0.1400	0.0450	-0.1000	0.0000	0.0000	0.1778
0.1990	0.1400	-0.1000	0.1000	0.1400	-0.1000	0.1000	0.0000	0.0000	0.0000
0.3000	0.2000	-0.2000	-0.1000	0.1985	-0.1981	-0.0985	-0.0015	0.0019	0.0015
0.3674	0.3000	-0.1500	-0.1500	0.2991	-0.1496	-0.1497	-0.0009	0.0004	0.0003
0.4011	0.3000	-0.2200	-0.1500	0.2991	-0.2197	-0.1497	-0.0009	0.0003	0.0003
0.4500	0.3000	-0.1500	-0.3000	0.2991	-0.1497	-0.2997	-0.0009	0.0003	0.0003
0.5000	0.4000	0.0000	0.3000	0.3991	0.0005	0.3004	-0.0009	0.0005	0.0004
0.5196	0.5000	-0.1000	-0.1000	0.4991	-0.0996	-0.0997	-0.0009	0.0004	0.0003
0.6160	0.5500	-0.1200	-0.2500	0.5490	-0.1196	-0.2497	-0.0010	0.0004	0.0003