

# GPS phase measurement cycle-slip detection based on a new wavelet function

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## ABSTRACT

Presently, cycle-slip detection is done between adjacent two points in many cycle-slip methods. Inherently, it is simple wavelet analysis. A new idea is put forward that the number of difference point can adjust by a parameter factor; we study this method to smooth raw data and detect cycle-slip with wavelet analysis. Taking CHAMP satellite data for example, we get some significant conclusions. It is showed that it is valid to detect cycle-slip in GPS phase measurement based on this wavelet function, and it is helpful to improve the precision of GPS data pre-processing and positioning.

**Keywords:** GPS phase measurement, wavelet analysis, cycle-slip detection, parameter factor

## 1. Introduction

In GPS data processing, cycle-slip is a mixed signal with characters of loss of lock occurrence between two epochs or lasting several minutes or more, we can judge the cycle-slip or outlier position and magnitude according to the adjacent epoch ambiguity or the ambiguity difference in two frequencies<sup>[1,2]</sup>. By far, there are many methods to detect and correct cycle-slip, there are: differential method<sup>[3]</sup>, Wide-Lane and pseudo-range combination<sup>[4]</sup>, ionospheric-free<sup>[4]</sup>, dynamic models, polynomial fitting<sup>[6]</sup>, the differential between the stations or the satellites, screen scanning, detecting and correcting cycle-slip by residual<sup>[5]</sup> and so on. All these methods can be considered to be done between adjacent two points. Inherently, it is a simple wavelet analysis. For example, Harr wavelet, which is the differential or derivation between adjacent two points, has good same steps. But sometimes it can not insure it's reliable with the difference of adjacent two points. Huang<sup>[7]</sup> detect cycle-slip by the Gauss function's one- and two-order differential as basic wavelet function, which is valid to the small cycle-slip detection based on the station-satellite double-difference, but invalid to the single-difference or un-difference. What is more, it needs double-frequency measurements, and do not determine the detail cycle-slip in every frequency. Guassian function, a 'window' function, is not good at perfect low-pass filter. JIA<sup>[8]</sup> analyzes the cycle-slip detection with high pass discrete wavelet transformation, its idea is detecting cycle-slip with wavelet coefficient, but Statistic test and judging methods are not solved. So, according to the cycle-slip properties, cycle-slip detection with the wavelet function constructed from the references<sup>[9]</sup> is discussed in this paper.

## 2. Cycle-slip properties

The cycle-slip is special problem in the carrier phase

measurement, we can correct the cycle-slip if we can determine the location and magnitude of cycle-slip precisely and renew its correct ones. All these work is named cycle-slip detection and correction<sup>[7]</sup>.

Considering phase measurement as time function, it can form a smooth curve without outlier and cycle-slip. Otherwise, the cycle-slip will break the smooth, and from this epoch, all the phases have the same slip till the next cycle-slip occurs.

With the different sampling intervals and observation status, the cycle-slip magnitude is from one to ten thousand cycles. Because of the influence of the noise, it is difficult to find the cycle-slip which is buried in the noise in the condition of the small sampling interval and adjacent epoch observation cycles.

How to eliminate the influence of the noise and display the small cycle-slip is the precondition to detect and correct the cycle-slip correctly and validly. The cycle-slip can consider as the signal's sudden jump in the view of the signals. We can analyze the phase observation by wavelet transformation to find the jump position, which can judge its magnitude through some methods.

## 3. Cycle-slip detecting observation

In GPS data processing, there are many measurements to detect the cycle-slip. In one station, we can choose single-frequency, double-frequency and combined phase and pseudo-range, Goad<sup>[9]</sup> constructed the ionospheric residuals models by a station's double-frequency, un-different measurement and detects the cycle-slip based on whether or not the ionospheric residual is singular. We can construct difference measurement to detect cycle-slip between stations or satellites, and detect cycle-slip according to the difference of different time stage but different error influence to detect cycle-slip. Two indicators ambiguity difference of double-frequency combined

by phase and pseudo-range and O-C residual in one epoch are selected to detect cycle-slip in this paper. We will give a simple introduction to the two indicators about the cycle-slip detection.

### 3.1 wide-lane combination

Based on the references [2, 4, 10], the difference of ambiguity expressed as:

$$\begin{aligned} b_\delta &= b_1 - b_2 = \frac{1}{\lambda_\delta} (L_\delta - P_\delta) \\ &= \phi_1 - \phi_2 - \frac{(f_1 - f_2)}{c} \cdot \frac{f_1 P_1 + f_2 P_2}{f_1 + f_2} \end{aligned} \quad (1)$$

where,  $\phi_1, \phi_2$  are the phase of  $L_1, L_2$ ;  $f_1, f_2$  are the frequency of  $L_1, L_2$ ;  $\lambda_1 = c/f_1, \lambda_2 = c/f_2$  are the wavelength of the  $L_1, L_2$ ;  $b_1, b_2$  are the ambiguity of the  $L_1, L_2$ ;  $P_1, P_2$  are the pseudo-range of the  $L_1, L_2$ ;  $c$  is the light velocity;  $L_\delta$  is the distance in meter after wide-lane combination;  $\lambda_\delta = c/(f_1 - f_2) \approx 86.2cm$  is combined wavelength;  $b_\delta = b_1 - b_2$  is the difference of the double-frequency;  $P_\delta$  is the combined pseudo-range.

### 3.2 O-C residual

In GPS positioning and orbiting processing, thinking of the cycle-slip detection and correction, usually, we detect the cycle-slip by the O-C residual. The expression of the O-C residual is:

$$\begin{aligned} \Delta\rho_k &= \rho_{ok} - \rho_{ck} \\ &= \rho_{C3}^{(k)} - \left( \sqrt{(x^G - x_{C3})^2 + (y^G - y_{C3})^2 + (z^G - z_{C3})^2} + c \cdot \Delta t_{C3} + E \cdot B \right) \\ &= \frac{1}{\rho_c} [(x_G - \hat{x}_C) \Delta x_C + (y_G - \hat{y}_C) \Delta y_C + (z_G - \hat{z}_C) \Delta z_C] + c \cdot \Delta t_{C3} + E \cdot B \end{aligned} \quad (2)$$

Where,  $\rho_{ok}, \rho_{ck}$  are the observed measurement and the computed one between the satellite and the station at epoch  $k$ , respectively;  $\Delta\rho_k$  is the difference of the O-C residual

$$M(t) = \ddot{F}(t) = e^{-\frac{t^2}{2\delta^2}} \left( \frac{t \sin(\pi t)}{\pi \delta^4} + \frac{\sin(\pi t) - 2 \cos(\pi t)}{\delta^2} + \frac{2 \sin(\pi t) - \pi t \sin(\pi t) - 2 \cos(\pi t)}{t^2} \right) \quad (5)$$

### 4.3 Property of the wavelet function

between the satellite and the station at epoch  $k$ ;  $\bar{R}^G(x^G, y^G, z^G)$  is the coordination of GPS satellite;  $\bar{R}_C(x_C, y_C, z_C)$  is the coordination of the station, including the static or moving target on the ground and LEO satellite and so on. The subscript '3' is measurement after LC combination;  $\Delta t$  is clock correction.  $B$  is the ambiguity after combination;  $E$  is the ambiguity coefficient after LC combination. When the value is 1, it shows that the GPS signal is received in this epoch, and when the value is 0, it means that there is no signal is received in this epoch.

## 4. Wavelet function and its property

### 4.1 The principle and theorem

Shannon function has a good filtering properties (smoothing), and sensitive to the sudden jump data in the GPS data, but its attenuation speed to the extreme is slow and is poor local character. And Gaussian 'window' function can control the function's attenuation very well by the scaled factor, and poor to the low-pass filter. We get the two functions merits, supplement their defects each other and then construct the wavelet function, which is to meet the low-pass filter father-wavelet  $F(t)$  and the one-order mother-wavelet  $Z(t)$ , two-order mother-wavelet  $M(t)$  [1, 11].

### 4.2 wavelet function

According to the character of cycle-slip, and the advantage and disadvantage of the Shannon function and Gauss 'window' function, we detect cycle-slip of the GPS phase measurement with the wavelet we constructed in reference [1, 11].

Father wavelet:

$$F(t) = G(t) \cdot S(t) = e^{-\frac{t^2}{2\delta^2}} \cdot \frac{\sin(\pi t)}{\pi t} \quad (3)$$

One-order mother wavelet:

$$Z(t) = e^{-\frac{t^2}{2\delta^2}} \left[ \pi \cdot \frac{\cos(\pi t)}{\pi t} - \left( \frac{t}{\delta^2} + \frac{1}{t} \right) \cdot \frac{\sin(\pi t)}{\pi t} \right] \quad (4)$$

Two-order mother ranks wavelet:

In order to explain the constructed wavelet function, we introduce its properties as follows: integrity, orthogonal and

compactly supported.

1. One-, two-order mother wavelet is the continuous wavelet. The equal step sampling is not need in signal wavelet transformation, which is superiority to the digital filter.

2. The integrity and orthogonal of this wavelet function is not need to discuss because the mother wavelet's series expansion is not consider here.

3. Compactly supported, Shannon wavelet' compactly supported length can be infinite, but the convergent velocity can controlled by scaled factor. So it is enough to consider the range as [-10, 10] or [-20, 20] in this wavelet function.

4. Father- wavelet F(t) has a good smoothing and compressing effect, and can controls the smoothing and compressing degree by scaled factor  $\delta$ .

5. Wavelet transformation can bring marginal effects, the larger scaled factor  $\delta$ , the heavier marginal effect, what is more,

the longer compact supported will bring heavier marginal effect.

6. One-order mother-wavelet Z(t) and two-order mother-wavelet has a good effect to detect the signal singular, if the scaled factor  $\delta$  is fixed, we can show the singular of raw signal according to the amplitude.

## 5. The wavelet analysis of the GPS data

In order to analyze this wavelet function to GPS data processing, taking GPS measurement of 7-16-2001 for example, we analyze the residuals of  $b_\delta = b_1 - b_2$ , O-C, the sampling interval is 10s.

### 5.1 The wavelet analysis to the ambiguity difference of the same epoch

#### 5.1.1 Mother-wavelet transformation

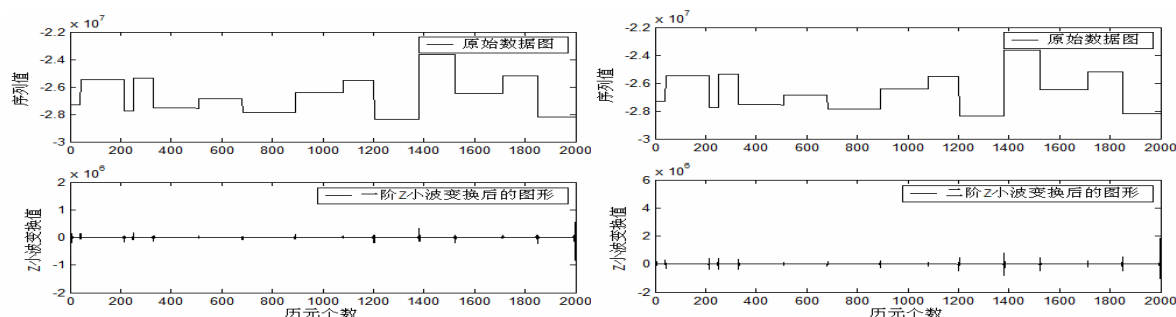


Fig.1 analyzing data received from PRN30 satellite with Z (left) and M (right) wavelet

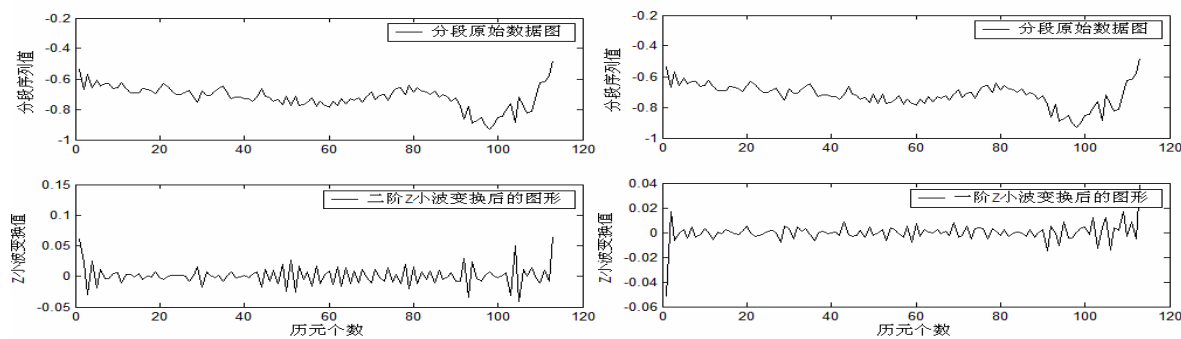


Fig.2 analyzing data no cycle-slip with Z (left) and M (right) wavelet

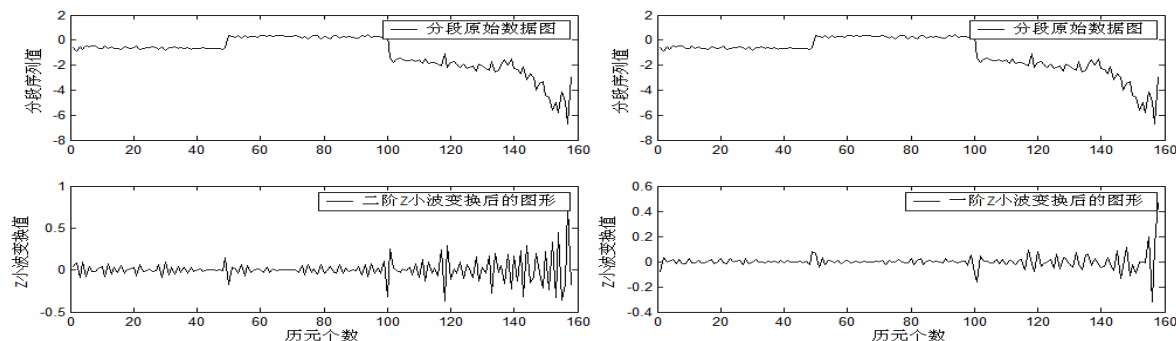


Fig.3 analyzing data after adding one cycle-slip with Z (left) and M (right) wavelet

From figure 1-3, we can conclude that there are different marginal effects because of the different scaled factor of  $\delta$ ; we should consider marginal effect with wavelet transformation in GPS data processing. What is more, from the figure 1, we can conclude that we can detect the prominent cycle-slip completely with this wavelet function; from the figure 2, the GPS data sequence is a stationary signal without cycle-slip after wavelet transformation; from figure 3, we add a cycle-slip in a GPS data sequence factitiously, we can find a prominent sudden jump after wavelet transformation.

### 5.1.2 Smoothing and compress effect

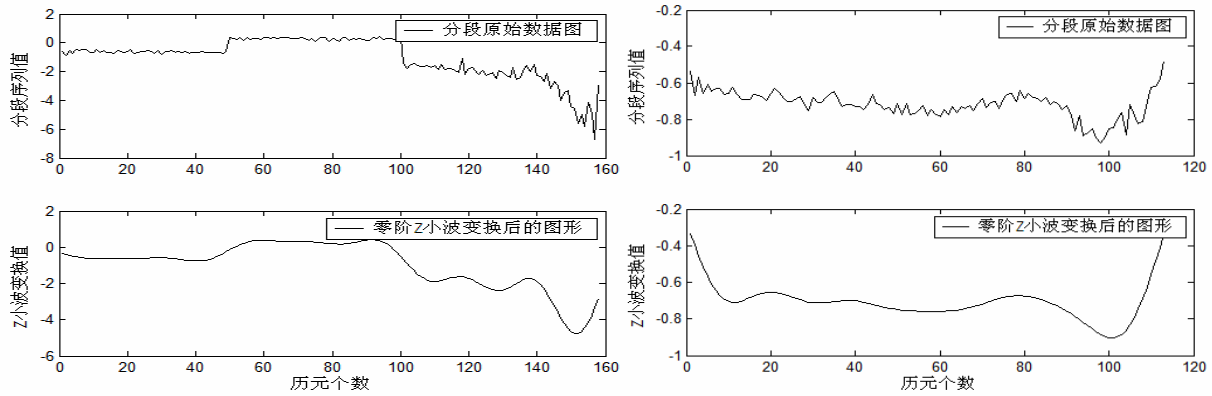


Fig.4 Data smoothing and compress with father wavelet  $F(t)$

Because Shannon function has good smoothing and filtering, father wavelet  $F(t)$  constructed by Shannon function and Gauss 'window' function has good smoothing and compress effect, which can controlled by scaled factor  $\delta$ . From Figure 4, it is showed that the smoothed data can reflect the raw signal's nature, which can explain that father wavelet  $F(t)$  has good smoothing and compress effect to GPS data processing.

### 5.1.3 Other wavelet transformation

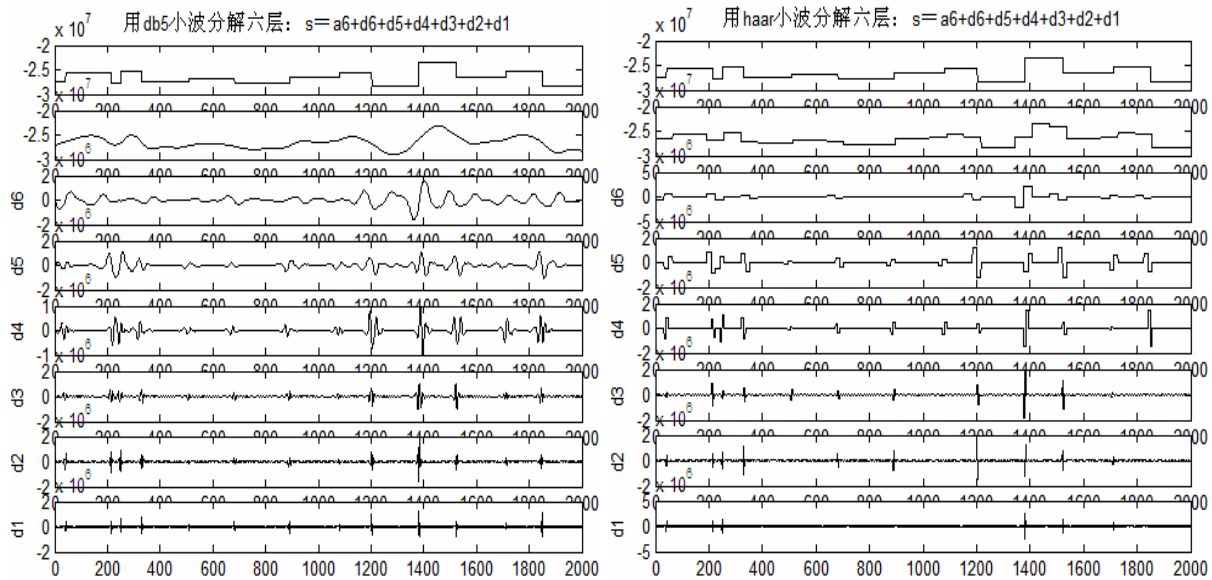


Fig.5 Data wavelet transformation with Daubechies and Haar wavelet

From Fig.5, it is showed that according to analyzing the GPS data, Daubechies and Haar wavelet function can detect the different singular by frequency layer and cannot get all frequencies singular in one transformation. From the right Fig.5, we can conclude that it cannot reflect the small cycle-slip in d1 layer. From the left Fig.5, we can conclude that it is not prominent to detect the singular point. So, Daubechies and Haar wavelet function are not good at smoothing and locality.

## 5.2 Wavelet analysis of O-C residual

### 5.2.1. No cycle-slip Sequence

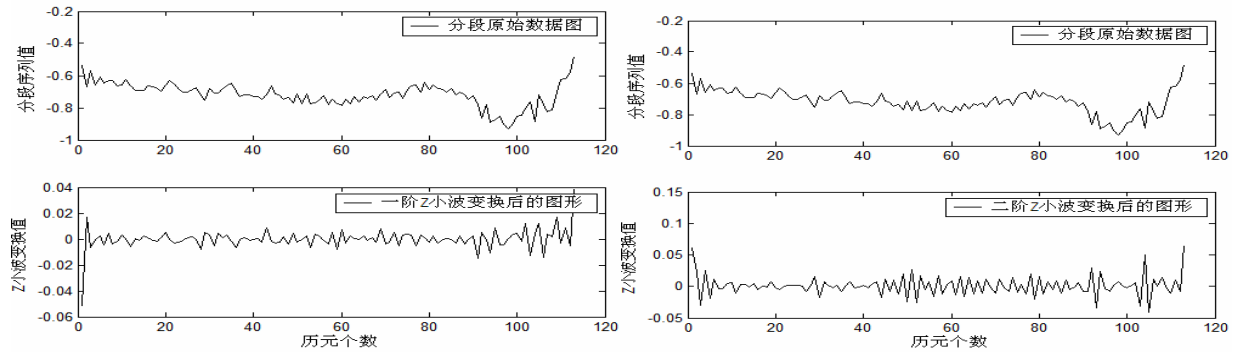


Fig.6 analyzing data no cycle-slip with Z (left) and M (right) wavelet

From Fig.6, it is showed that: (1), GPS data has no singular point after mother wavelet transformation if there is no cycle-slip; (2), the different scaled factor  $\delta$  and compact supported length will bring up different marginal effect. Thereof, we should think about the scaled factor  $\delta$  and step  $h$  when transform GPS data with wavelet function.

### 5.2.2. Cycle-slip sequence

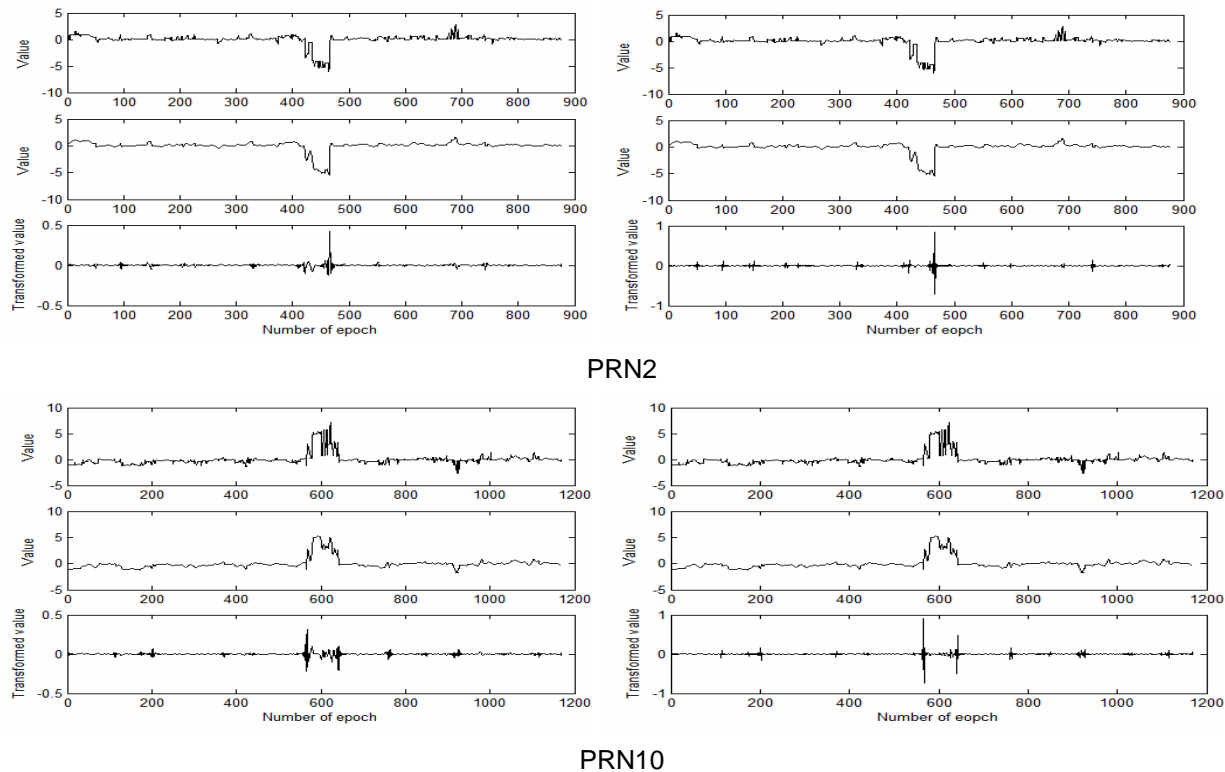


Fig.7 Analyzing O-C residual with Z (left) and M (right) wavelet

From Fig.7, we can conclude that, father wavelet has a good smoothing and compress effect to the observation after the O-C residual smoothing, and the one-order mother wavelet  $Z(t)$  and two-order mother wavelet  $M(t)$  can detect the remainder wavelet cycle-slip very well.

## 6. Conclusion

Presently, cycle-slip detection is done between adjacent two

points in many cycle-slip methods. Inherently, it is simple wavelet analysis. A new idea is put forward that the number of

difference point can adjust by a parameter factor; we get the merits of the Shannon function and Gauss function to construct a new wavelet function, which is applied into cycle-slip detection of CHAMP GPS phase measurement. We get some helpful conclusions: father wavelet has a good smoothing and compress effect to GPS data; one-order and two-order mother wavelet are valid to detect the cycle-slip, but different parameter factor and compactly supported length can bring marginal effect, compared to Daubechies and Haar function, it has character of filtering and locality. Based on the wavelet function, it is valid and reliably to improve the cycle-slip detection, it will helpful to improve the precision of pre-processing data and position.

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