# The Benefit of Ambiguity Resolution Using Triple Frequency

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## Abstract

Modernized GPS will have three frequencies modulated with three signals, which will be accessible to all users in the near future. This new frequency provides an opportunity to resolve the double differenced (DD) integer ambiguity very fast and with almost no baseline constraints.

In order to study the performance of triple frequency system for Ambiguity Resolution (AR) over the medium baseline under different ionospheric levels, the Klobuchar Model was implemented and used in our triple simulation to generate the ionospheric delay. Furthermore, the White-Gaussian noise applying to distance-dependent parameters was added to the DD ionospheric delay. For medium baseline (defined as here 20 to 40kms), success rates of AR has been pretty improved.

In this paper, the medium baseline AR strategies that take advantage of carrier phase measurement on the third frequency will be discussed.

Keywords: Modernization of GPS, Ambiguity resolution, triple-frequency

# **1. Introduction**

Precise Positioning with Global Navigation Satellite Systems (GNSS), such as the GPS and the future European Galileo, requires AR of carrier phase observations to their integer values. Various methods have been developed for fast and reliable AR over short baselines, but for medium baselines the needed time to fix ambiguities increases drastically due to the influence of various systematic effects, in particular the ionospheric error.

As part of the modernization of GPS, after December 2004, a third civil frequency at 1176.45 MHz is planed to operate on all new satellites. The main benefits for precise GPS positioning is that triple-frequency measurements will significantly help AR, and hence increase the reliability of precise positioning rather than positioning accuracy. During the past few years, a lot of research has focused on algorithm studies for AR using triple frequency. The study can be found in [4], which aimed to apply the Least-squares Ambiguity Decorrelation Adjustment (LAMBDA) to triple frequency system.

In the medium baseline, the DD ionospheric residual error may disturbs the convergence of float ambiguities and causes difficulties of AR. Therefore, in our procedure, the ionospheric delays are not modeled as completely unknown parameters. This model is referred to as the ionosphere-weighted model [7].

This paper starts with the data simulation of triple frequency. The ionospheric error model consisting the Klobuchar Model and the White-Gaussian noise are introduced in detail. Second, LAMBDA AR approach using the ionosphere-weighted model is described. Finally, we compare the success rate of AR in the case of medium baselines and the advantages of triple-frequency for the medium baseline are also discussed.

# 2. Triple Frequency Data Simulation

Triple frequency simulation data were made based on the ephemeris data, and the calculation time is 1st December 2004. Observation noise, multipath error for triple frequency simulation is generated using error parameters shown in Table 1. The procedure of generating Triple frequency simulation data is introduced in Figure 1.

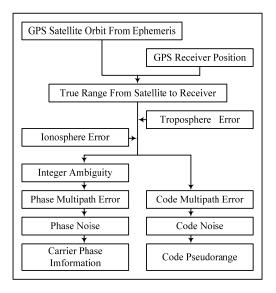


Figure 1. Triple frequencies data simulation

Error	Reference	Rorver	
Ionosphere Error	Klobuchar Model	Klobuchar Model	
Troposphere Error	Saastamoinen Model	Saastamoinen Model	
Code Noise	DLL Noise	DLL Noise	
Phase Noise	PLL Noise	PLL Noise	
Code Multipath Error	Ground	Ground	
Phase Multipath Error	Ground	Ground	

Table 1. Error parameters in the triple frequencies simulation

# 2.1 Receiver Noise

In the simulation, the code observation noise in L1, L2 and L5 signals can be generated as narrow correlator DLL (Delay Lock loop) thermal noise. Similar to code noise generation, the phase observation noise in L1, L2 and L5 can be generated as PLL (Phase Lock Loop) thermal noise. Table 2 shows the parameters used in our triple frequency noise simulation.

	PLL Bandwidth (Hz)	DLL Bandwidth (Hz)	chipping rate	Correlor chip (chip)	Integration time (ms)	DLL correlate factor
L1	5	0.5	293.05	0.1	20	0.5
L2	5	0.5	293.05	0.1	20	0.5
L5	5	0.5	29.305	1	20	0.5

Table 2. The parameter in the triple frequencies noise simulation

## 2.2 Multipath Error

In the simulation, both the reference and the rover station are only affected by the ground reflect multipath.

## 2.3 Ionospheric Error

In the simulation, to generate ionospheric error, the Klobuchar Model and the White-Gaussian noise are used. The Klobuchar Model is used to simulate the ionospheric delay, the White-Gaussian noise was applied as distance-dependent statistical parameter. The White-Gaussian noise was generated from the standard deviations actual DD ionospheric delay, and it was added to the DD ionospheric delay as shown in Table2-1.

Table 2-1. Standard deviations estimated from actual DD ionospheric delay

Baseline	20 km	30 km	40 km
Standard Deviations	00.3 (m)	0.045 (m)	0.7 (m)

## 2.4 Tropospheric Error

To generate tropospheric error, the Saastamoinen Model is used. The Saastamoinen Model is derived using gas laws and simplifying assumptions regarding changes in pressure, temperature, and humidity with altitude. In the simulation, therefore, mean value of observations in Tokyo were used for these parameters. Table 3 shows the parameters used. Table 2-2. The parameter in the Saastamoinen Model

Pressure (hPa)	Temperature (K)	Humidity (%)
1017.4	281.2	53

## 3. Implementation of Ionosphere-weighted model

#### 3.1 Extended Kalman Filter

The Kalman filter approach can be used to estimate the GPS unknown parameters. In this simulation, the ionosphere weighted model[6][7] is applied and unknown parameter vector x can be written as:

$$x = (r_r^T, \nabla \Delta N_{L1}^{0,1}, \nabla \Delta N_{L2}^{0,1}, \nabla \Delta N_{L5}^{0,1}, \nabla \Delta I_{L1}^{0,1})^T$$
(1)

where  $r_r^T$  is rover position coordinates and  $\nabla \Delta N^{0.1}$  is the integer ambiguity between 0 and 1 satellite pairs. Subscripts indicate the GPS frequency. Furthermore,  $\nabla \Delta I_{L1}^{0.1}$  is also modeled as the unknown parameter.

The observable vector for epoch k can be written as:

$$z_{k} = (\nabla \Delta \phi_{L1}^{0,1} \cdot \lambda_{L1}, \nabla \Delta \phi_{L2}^{0,1} \cdot \lambda_{L2}, \nabla \Delta \phi_{L5}^{0,1} \cdot \lambda_{L5},$$

$$\nabla \Delta P_{L1}^{0,1}, \nabla \Delta P_{L2}^{0,1}, \nabla \Delta P_{L5}^{0,1})^{T}$$
(2)

where  $\nabla \Delta \phi^{0,1} \cdot \lambda$  and  $\nabla \Delta P^{0,1}$  are the DD phase observable and the DD pseudorange observable, respectively.

Hence, the observation equations, (3) and (4), shown in the following measurement model are explicitly expressed, not linearly combined.

$$\nabla \Delta \phi_{L1}^{l,m} \cdot \lambda_{L1} = \nabla \Delta \rho_{k}^{l,m} + \nabla \Delta N_{L1}^{l,m} \cdot \lambda_{L1} - \nabla \Delta I_{k}^{l,m}$$

$$\nabla \Delta \phi_{L2}^{l,m} \cdot \lambda_{L2} = \nabla \Delta \rho_{k}^{l,m} + \nabla \Delta N_{L2}^{l,m} \cdot \lambda_{L2} - \gamma \cdot \nabla \Delta I_{k}^{l,m}$$

$$\nabla \Delta \phi_{L5}^{l,m} \cdot \lambda_{L5} = \nabla \Delta \rho_{k}^{l,m} + \nabla \Delta N_{L5}^{l,m} \cdot \lambda_{L5} - \beta \cdot \nabla \Delta I_{k}^{l,m}$$

$$\nabla \Delta P_{L1}^{l,m} = \nabla \Delta \rho_{k}^{l,m} + \nabla \Delta I_{k}^{l,m}$$

$$\nabla \Delta P_{L2}^{l,m} = \nabla \Delta \rho_{k}^{l,m} + \gamma \cdot \nabla \Delta I_{k}^{l,m}$$

$$\Delta \Delta P_{L5}^{l,m} = \nabla \Delta \rho_{k}^{l,m} + \beta \cdot \nabla \Delta I_{k}^{l,m}$$

$$(4)$$

where;

 $\nabla \Delta \rho_k^{l,m}$ : DD geometrical distance for satellite pair l and m  $\gamma$  :  $(f_k / f_k)^2$ 

$$\beta \qquad : \qquad (f_1/f_2)$$

Through linearization and expressed design matrix form:

$$H = \begin{bmatrix} e^{l,m^{T}}, \lambda_{L1}, 0, 0, -1 \\ e^{l,m^{T}}, 0, \lambda_{L2}, 0, -\gamma \\ e^{l,m^{T}}, 0, 0, \lambda_{L5}, -\beta \\ e^{l,m^{T}}, 0, 0, 0, 0, 1 \\ e^{l,m^{T}}, 0, 0, 0, 0, \gamma \\ e^{l,m^{T}}, 0, 0, 0, 0, \beta \end{bmatrix}$$
(5)

where  $e^{l,m} = e_0^{\ l} - e_0^{\ m}$  and stochastic measurement noise are assumed that phase and pseudo-range are  $\sigma_{\Phi} = 3mm$  and  $\sigma_p = 30cm$ , respectively.

Stochastic system noise can be written as:

$$Q = diag(0,0,0,0,0,0,\sigma_I^2)$$
(6)

where the ionospheric system noise,  $\sigma_I$ , can be modeled using random walk.

# 3.2 LAMBDA method

In this simulation, in order to obtain the integer ambiguity, LAMBDA method is implemented. This integer search method has developed by the Delft geodetic computing centre[3]. The float ambiguities and their covariance are readily available from the Kalman filter, mentioned as previous section. In the implementation of ratio test, the most commonly used criterion, we set to three as the threshold value.

# 4. Results

The impact of the medium baseline AR performance by using triple frequency is investigated here using LAMBDA method. The procedure of Ambiguity resolution is introduced in Figure 4-1.

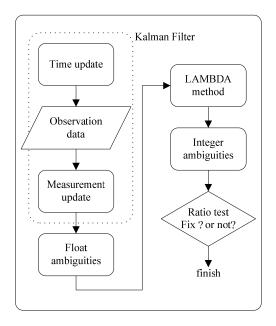


Figure 4-1. Ambiguity resolution

Figure 4-2 shows the float solution of L1 using dual frequency and triple frequency by kalman filter. And Table 4 shows success rates of AR in different cases of medium baseline (defined as here 20 to 40kms).

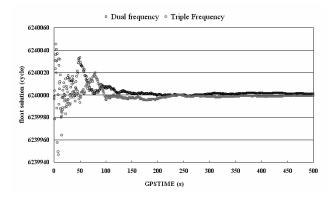


Figure 4-2. float solution of L1 by kalman filter

Table 4. Ambiguity Success Rate in medium baseline

Pressure (hPa)	Temperature (K)	Humidity (%)
1017.4	281.2	53

## 5. Conclusion

This paper investigated success rate AR for medium baseline using triple frequency. The results has been shown that the success rate of triple frequency system is greater than that dual frequency system. In the case of triple frequency, the time to convergence performance has been improved.

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