Interactive Navigational Structures

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Abstract

Satellite systems for objects positioning appeared indispensable for performing basic tasks of maritime navigation. Navigation, understood as safe and effective conducting a vehicle from one point to another, within a specific physical-geographical environment. [Kopacz, Urbański, 1998]. However, the systems have not solved the problem of accessibility to reliable and highly accurate information about a position of an object, especially if surveyed toward on-shore navigational signs or in sea depth. And it's of considerable significance for many navigational tasks, carried out within the frameworks of special works performance and submarine navigation. In addition, positioning precisely the objects other than vessels, while executing hydrographical works, is not always possible with a use of any satellite system. Difficulties with GPS application show up also while positioning such off-lying dangers as wrecks, underwater and aquatic rocks also other natural and artificial obstacles. It is caused by impossibility of surveyors approaching directly any such object while its positioning. Moreover, determination of vessels positions mutually (mutual geometrical relations) by teams carrying out one common tasks at sea, demands applying the navigational techniques other than the satellite ones. Vessels' staying precisely on specified positions is of special importance in, among the others, the cases as follows:

- surveying vessels while carrying out bathymetric works, wire dragging;
- special tasks watercraft in course of carrying out scientific research, sea bottom exploration etc.

The problems are essential for maritime economy and the Country defence readiness. Resolving them requires applying not only the satellite navigation methods, but also the terrestrial ones.

The condition for implementation of the geo-navigation methods is at present the methods development – both: in aspects of their techniques and technologies as well as survey data evaluation. Now, the classical geo-navigation comprises procedures, which meet out-of-date accuracy standards. To enable meeting the present-day requirements, the methods should refer to well-recognised and still developed methods of contemporary geodesy. Moreover, in a time of computerization and automation of calculating, it is feasible to create also such software, which could be applied in the integrated navigational systems, allowing carrying out navigation, provided with combinatory systems as well as with the new positioning methods. Whereas, as regards data evaluation, there should be applied the most advanced achievements in that subject; first of all the newest, although theoretically well-recognised estimation methods, including M–estimation [Hampel et al. 1986; Wiśniewski 2005; Yang 1997; Yang et al. 1999]. Such approach to the problem consisting in positioning a vehicle in motion and solid objects under observation enables an opportunity of creating dynamic and interactive navigational structures.

The main subject of the theoretical suggested in this paper is the Interactive Navigational Structure. In this paper, the Structure will stand for the existing navigational signs systems, any observed solid objects and also vehicles, carrying out navigation (submarines inclusive), which, owing to mutual dependencies, (geometrical and physical) allow to determine coordinates of this new Structure's elements and to correct the already known coordinates of other elements.

Keywords: navigation, estimation, robust estimation, navigational systems

1. Interactive Navigational Structure. The Basic Assumptions

The main subject of the propositions suggested in this work is the *Interactive Navigational Structure* (hereinafter called *IANS*). In this paper, the Structure will stand for the existing navigational signs systems, any observed solid objects and also vehicles, carrying out navigation (submarines inclusive), which, owing to mutual dependencies, (geometrical and physical) allow to determine coordinates of this new Structure's elements and to correct the already known coordinates of other elements.

The suggested Interactive Navigational Structure - if considered from the geometrical point of view - is created by a set of points $\mathsf{Z}=\left\{Z_j : j=1,\ldots,n_z\right\}$ with coordinates given in a certain configuration (e.g. (X,Y)), also subsets of determined points $\mathsf{P}=\left\{P_i : i=1,\ldots,n_p\right\}$ and $\mathsf{R}=\left\{R_l : l=1,\ldots,n_R\right\}$. The Z set can be created by known in the navigation theory navigation systems. The determined points P are specific positions P_i of a watercraft in motion or positions of a group of crafts, which carry on a common navigation task (e.g. hydrographic survey sweeping, fighting vessels task force formation etc.). The R subset is created by the points, determined throughout P points, which, after fulfilling the settled requirements (especially within the accuracy scope) are to complement the set of adjustment Z.

Let's assume that within a certain navigational area and within a conventional stage (k) a set of points $\mathsf{Z}^{(k)}$ is available. For some reasons, the set is insufficient to carry on navigation within the next following stage ($k+1$). Basing on the $Z^{(k)}$ set, there area determined the specific $P^{(k)}$ positions, and throughout them, the coordinates of new $R^{(k)}$ points. Thus, within the (k+1) stage there is available a set of adjustment points $Z^{(k+1)} = \left\{ Z^{(k)}, R^{(k)} \right\}$. A chain of interactive navigational structure may have many links, unless at each of them the settled accuracy criteria are fulfilled. Generally, the relations between the elements of such chain are presented in Fig. 1.

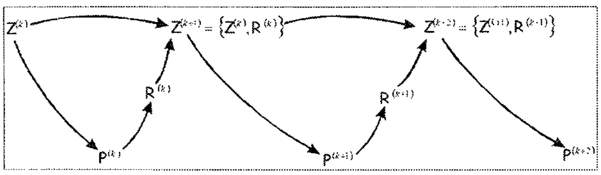

Figure 1. Conception of the Interactive Navigational Structure.

The sets and subsets discriminated above, are joined into a common observational arrangement by the geometrical quantities.

Let us assume that determination of a specific position P_i (element of P set) is carried out through bearings NR and distances d from the point P_i to some points of Z set (for example with a use of radar and gyroscope systems). The position P_i may also be determined, basing on the satellite navigation systems, as, for example, DGPS. Moreover, with expectation that from the position P_i there is to be carried out a bearing of point R and a distance thereto is measured, there is achieved an observational arrangement (Fig. 2) which stands for intrinsic, basic *IANS* element.

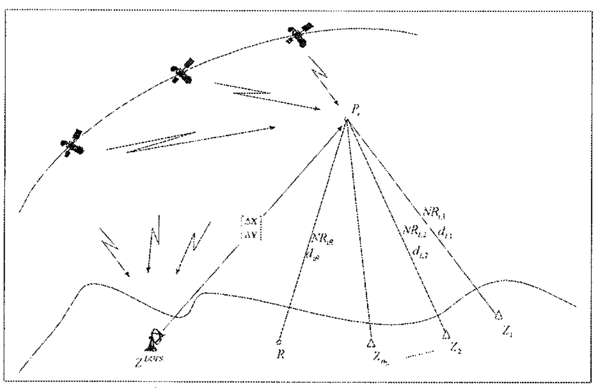

Figure 2. Basic *IANS* element.

The interactive character of a navigational structure including, first of all, an assumed possibility of „transferring" points R to Z set, is demanding creation of such observational arrangement, which, on one hand, meets the stipulation concerning *IANS* area development and on the other hand, enables carrying out the obtained determinations control. A singular *IANS* element fails to meet those requirements. In such an arrangement the points R are, at the utmost, uniquely determinable, thus unable to carry out reliable estimation of its positioning accuracy. The *IANS* element, as an intrinsic navigational structure, is also slightly robust to significant survey errors. A solution which considerably eliminates such sorts of inconvenience is connecting basic elements into the *IANS* chain. The observations to connect are in such case the observations, which refer to the R points' coordinates and the elements in two vessels arrangement, also mutual bearings and mutual distances. A fragment of the Interactive Navigational Structure chain is presented in Fig. 3.

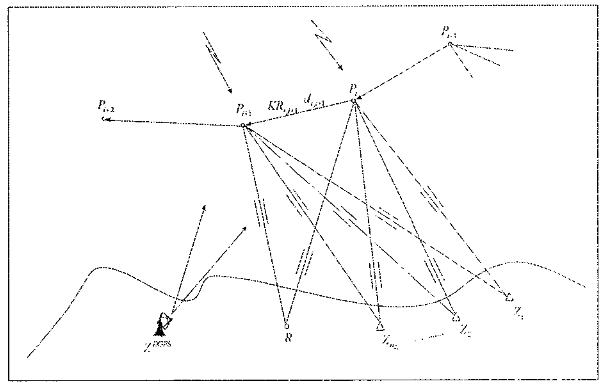

Figure 3. Interactive Navigational Structure Modulus

The Navigational Structure, presented in this study, is of interactive character and apart of the above, integrates different types of available information (bearings, distances, path vector elements, DGPS measurements). One should expect that only some variants of mutual connections of Z, P, R sets' elements will be used in practical application. It is also predicted, that even if any of the observations are practically executed, due to biasing thereof with major errors, they should also be rejected or attenuated applying any justified way. Thus, in general, let's accept that t functions, assuming values of $\langle 0;1 \rangle$ interval, are subordinated to the observations and coordinates of the points. The functions assume the extreme values when the observations (coordinates) are not taken into the commonly worked out observational arrangement $(t=0)$ or a part thereof is full $(t=1)$ [Czaplewski 2004]. However, if from some reason the observations are only damped, then $(0<t<1)$. The t functions, of such the general properties, in reference to the robust estimation principles, are called in the subject' literature the attenuation functions (e.g. [Hampel et al. 1986; Wiśniewski 1999, 2002,2005; Yang et al. 1999]). A particular case of the attenuation function t may be a such double value decisive function t, that [Czaplewski 2004]:

$$t(s) = \begin{cases} 1 & \text{if } s \text{ is acceptable} \\ 0 & \text{if } s \text{ is rejected} \end{cases}$$

where: s – is an optional element of the points or observations set.

Description of the suggested Interactive Navigational Structure will simplify generalization of the decisive function t. The subsets of Z, Z^{DGPS}, P points and subsets of the observations discriminated above are the arguments for this generalization. Let's assume that O^Z is a subset of the observations carried out from the points P towards the adjustment points Z (aimed at determining positions P^Z of the points P), O^P is a subset of mutual observations between the points P (for example: the path vector elements), O^{DGPS} is the set of the observations DGPS (on the basis thereof there are determined alternative or intrinsic positions P^{DGPS} of P points), whereas O^R is a set of the observations carried out at the points P towards newly determined points R. It is assumed that generalization of the function t having s argument proceeding through the elements (subsets) of the following set:

$$\mathsf{S}=\left\{ \mathsf{O}^{Z}, \mathsf{O}^{P}, \mathsf{O}^{DGPS}, \mathsf{O}^{R}, \mathsf{Z}, \mathsf{Z}^{DGPS}, \mathsf{P} \right\},$$

with, in general, $\mathsf{P}=\left\{ \mathsf{P}^{Z}, \mathsf{P}^{DGPS} \right\}$, is a function

$$\mathsf{T}_{\mathsf{S}}(s)=\begin{cases} 1 & \text{if in the subset } s \text{ the acceptable elements does exist} \\ 0 & \text{if } s \text{ is an empty set or all its elements are unacceptable} \end{cases}$$

It is worth to add, that S is a set which corresponds to the complete (in accordance with the assumptions made as yet) structure; however it enables to select one of its variants, adjusted to a specific navigational task. An example thereof based on the adjustment points Z and observations O^{Z}, positions P^{Z} of the points P, is shown in the logical diagram presented in Fig. 4.

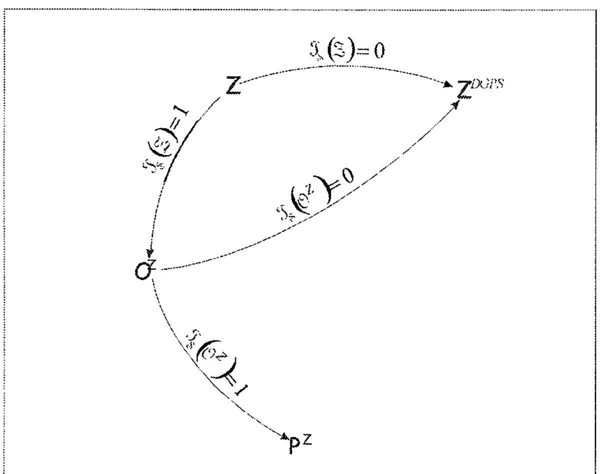

Figure 4. The selected variant of the decisive function application in *IANS*

In the example, a condition of determination of the position P^{Z} of the points P is acceptance for the sets Z and O^{Z}. The situation when $\mathsf{T}_{\mathsf{S}}(\mathsf{Z})=0$ or $\mathsf{T}_{\mathsf{S}}(\mathsf{O}^{Z})=0$, is forcing to choose an alternative way. According to the assumptions made in this work, the starting point for this path is a set of reference stations Z^{DGPS}. Its accessibility, also technical ability for execution of the set of observations O^{DGPS}, allows to develop *IANS*, to achieve the form presented in Fig. 5.

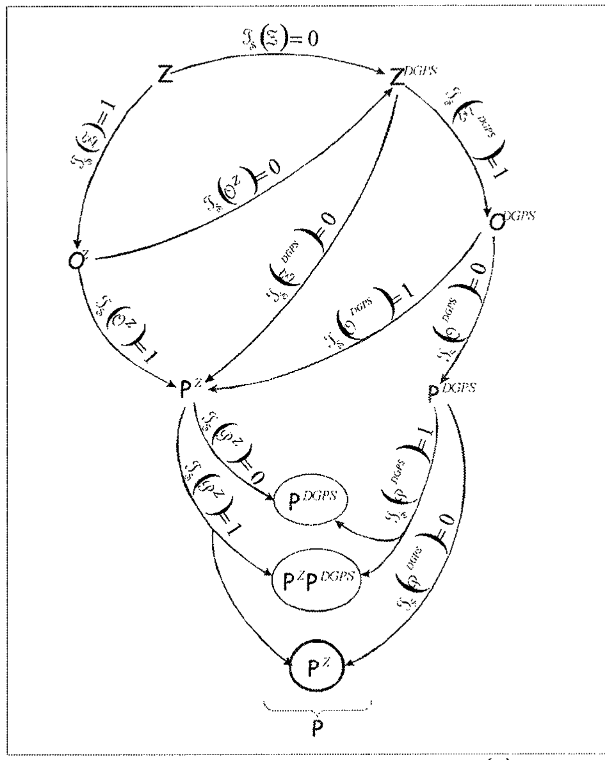

Figure 5. The possible *IANS* development in (k) stage

Let's presume at this moment, that according to the assumptions made before, *IANS* is still developed through newly determined points R. After fulfilment of the set up criteria, what in convention of the function T stands for meeting the condition $\mathsf{T}(\mathsf{R})=1$, the points, in the successive stage $(k+1)$, will become complementary for the previous (for k stage) set of the adjustment points. A diagram of such development, which at the same time is a supplement for the logical diagram presented in Fig. 5., is shown in Fig. 6.

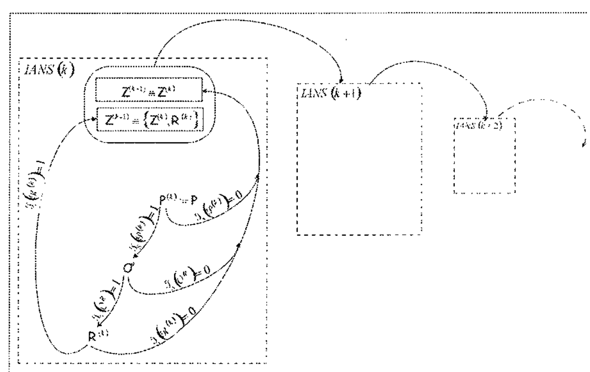

Figure 6. Possible *IANS* development in $(k+1)$ stage

2. Basic Element of *IANS* - the Adjustment Task and its Solution

Let us assume, that at the $P_i \in \mathsf{P}$ position of the vessel, starting to create *IANS*, the sets of points $\mathsf{Z}_i \subset \mathsf{Z}$ and reference stations of DGPS system $\mathsf{Z}_i^{DGPS} \subset \mathsf{Z}^{DGPS}$ are available. The bearings and distances to the points Z_i, and additionally to the points $R_1,\ldots,R_{n_{R_i}}$, which form the set $\mathsf{R}_i \subset \mathsf{R}$, are carried out at the point P_i. After the settled criteria are fulfilled, the above mentioned points shall become complementary for the previously formed set Z, supporting the further process of navigation. Let us also assume, that on the basis of the set Z_i^{DGPS} and the results of O^{DGPS} DGPS survey (however with the set's structure details omitted), the coordinates $X_{P_i}^{DGPS}, Y_{P_i}^{DGPS}$ of the point P_i are known as well. The undertaken, elementary navigational situation is displayed in Fig. 2.

By joining the functional models and the statistical model which was described in [Czaplewski 2004] and the function of target $\Phi^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)}, \hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)}\right)$, the following adjustment assignment can be obtained:

$$\left.\begin{array}{l}
\left.\begin{array}{l}
\mathbf{V}_{x_i} = \mathbf{A}_{P_i}\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)} + \mathbf{A}_R^{(i)}\hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)} + \mathbf{L}_{x_i} \\
\mathbf{V}_{\mathbf{X}_i^{DGPS}} = \hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)} \qquad + \mathbf{X}_{P_i}^0 - \mathbf{X}_{P_i}^{DGPS}
\end{array}\right\} \textit{functional models} \\
\left.\begin{array}{l}
\mathbf{C}_{x_i} = \sigma_0^2\mathbf{Q}_{x_i} = \sigma_0^2\mathbf{P}_{x_i}^{-1} \\
\mathbf{C}_{\mathbf{X}_{P_i}^{DGPS}} = \sigma_0^2\mathbf{Q}_{\mathbf{X}_{P_i}^{DGPS}} = \sigma_0^2\mathbf{P}_{\mathbf{X}_{P_i}^{DGPS}}^{-1}
\end{array}\right\} \rightarrow \left.\begin{array}{l}
\tilde{\tilde{\mathbf{C}}}_{x_i} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}_{x_i}^{-1} \\
\tilde{\tilde{\mathbf{C}}}_{\mathbf{X}_{P_i}^{DGPS}} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_{P_i}^{DGPS}}^{-1}
\end{array}\right\} \textit{statistic models} \\
\min_{\Omega}\Phi^{D-R}\left(\mathbf{d}_{\mathbf{X}_{P_i}}^{(i)}, \mathbf{d}_{\mathbf{X}_{R_i}}^{(i)}\right) = \Phi^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)}, \hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)}\right) = \mathbf{V}_{x_i}^T\tilde{\tilde{\mathbf{P}}}_{x_i}\mathbf{V}_{x_i} + \mathbf{V}_{\mathbf{X}_{P_i}^{DGPS}}^T\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_{P_i}^{DGPS}}\mathbf{V}_{\mathbf{X}_{P_i}^{DGPS}} \\
\Omega = \left(\mathbf{d}_{\mathbf{X}_{P_i}}^{(i)}, \mathbf{d}_{\mathbf{X}_{R_i}}^{(i)}\right)
\end{array}\right\} \quad (1)$$

with the equivalent covariance matrixes:

$$\tilde{\tilde{\mathbf{C}}}_{x_i} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}_{x_i}^{-1}, \quad \tilde{\tilde{\mathbf{C}}}_{\mathbf{X}_{P_i}^{DGPS}} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_{P_i}^{DGPS}}^{-1}$$

with the following designations:

$$\mathbf{V}_i = \begin{bmatrix} \mathbf{V}_{x_i} \\ \mathbf{V}_{\mathbf{X}_i^{DGPS}} \end{bmatrix}, \quad \mathbf{A}_i^{(i)} = \begin{bmatrix} \mathbf{A}_{P_i} & \mathbf{A}_R^{(i)} \\ \mathbf{I}_{(2)} & \mathbf{0} \end{bmatrix}, \quad \mathbf{L}_i = \begin{bmatrix} \mathbf{L}_{x_i} \\ \mathbf{X}_{P_i}^0 - \mathbf{X}_{P_i}^{DGPS} \end{bmatrix},$$

$$\hat{\mathbf{d}}_{\mathbf{X}_i} = \begin{bmatrix} \hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)} \\ \hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)} \end{bmatrix},$$

$$\tilde{\tilde{\mathbf{C}}}_i = Diag\left(\tilde{\tilde{\mathbf{C}}}_{x_i}, \tilde{\tilde{\mathbf{C}}}_{\mathbf{X}_{P_i}^{DGPS}}\right), \quad \tilde{\tilde{\mathbf{P}}}_i = Diag\left(\tilde{\tilde{\mathbf{P}}}_{x_i}, \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_{P_i}^{DGPS}}\right)$$

$$\tilde{\tilde{\mathbf{P}}}_{x_i} = \mathbf{T}(x_i)\hat{\mathbf{P}}_{x_i}\mathbf{T}(x_i) = \mathbf{T}(x_i)\mathbf{T}(x_i)\hat{\mathbf{P}}_{x_i} = \mathbf{T}(x_i)\hat{\mathbf{P}}_{x_i} =$$
$$= \mathbf{T}(x_i)\mathbf{T}(\mathbf{V}_{x_i})\mathbf{P}_{x_i} = \tilde{\mathbf{T}}(x_i, \mathbf{V}_{x_i})\mathbf{P}_{x_i}$$

$$\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}} = \tilde{\mathbf{T}}_{sqr}(\mathbf{X}_i^{DGPS}, \mathbf{V}_{\mathbf{X}_i^{DGPS}})\mathbf{P}_{\mathbf{X}_i^{DGPS}}\tilde{\mathbf{T}}_{sqr}(\mathbf{X}_i^{DGPS}, \mathbf{V}_{\mathbf{X}_i^{DGPS}})$$

the assignment (1) can also be presented in the form as follows:

$$\left.\begin{array}{l}
\mathbf{V}_i = \mathbf{A}_i\hat{\mathbf{d}}_{\mathbf{X}_i} + \mathbf{L}_i \\
\tilde{\tilde{\mathbf{C}}}_i = \sigma_0^2\tilde{\tilde{\mathbf{P}}}_i^{-1} \\
\min_{\mathbf{d}_{\mathbf{X}_i}}\Phi^{D-R}\left(\mathbf{d}_{\mathbf{X}_i}\right) = \Phi^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_i}\right) = \mathbf{V}_i^T\tilde{\tilde{\mathbf{P}}}_i\mathbf{V}_i
\end{array}\right\} \quad (2)$$

The classic form of the above allows for presenting (without unnecessary derivations) the following solution (e.g.[Baran 1999; Wiśniewski 2000, 2004,2005]:

$$\hat{\mathbf{d}}_{\mathbf{X}_i} = -\left(\mathbf{A}_i^T\tilde{\tilde{\mathbf{P}}}_i\mathbf{A}_i\right)^{-}\mathbf{A}_i^T\tilde{\tilde{\mathbf{P}}}_i\mathbf{L}_i \quad (3)$$

Moreover, in case a *row* $\left(\mathbf{A}_i^T\tilde{\tilde{\mathbf{P}}}_i\mathbf{A}_i\right) = r_i$ (as in classic solutions), then $\left(\mathbf{A}_i^T\tilde{\tilde{\mathbf{P}}}_i\mathbf{A}_i\right)^{-} = \left(\mathbf{A}_i^T\tilde{\tilde{\mathbf{P}}}_i\mathbf{A}_i\right)^{-1}$. Instead, the variance coefficient estimator can be determined applying the formula [Wiśniewski 1999, 2002; Yang 1997]:

$$\hat{\sigma}_0^2 = \frac{1}{f_i}\mathbf{V}_i^T\tilde{\tilde{\mathbf{P}}}_i\mathbf{V}_i \quad (4)$$

The process of resolving the task is of an iterative character. A start-step of the process is the classic adjustment applying the least squares method with the decisive weights matrix a priori

$$\mathbf{P}_i^{(0)} = Diag\left(\tilde{\mathbf{P}}_{x_i}, \tilde{\mathbf{P}}_{\mathbf{X}_i^{DGPS}}\right) = \tilde{\mathbf{P}}_i \quad (5)$$

every following step

$$\mathbf{P}^{(l+1)} = Diag\left(\mathbf{P}_{x_i}^{(l+1)}, \mathbf{P}_{\mathbf{X}_i^{DGPS}}^{(l+1)}\right)$$

where:

$$\mathbf{P}_{x_i}^{(l+1)} = \tilde{\mathbf{T}}_{sqr}\left(x_i, \mathbf{V}_{x_i}^{(l)}\right)\mathbf{P}_{x_i}^{(l)}\tilde{\mathbf{T}}_{sqr}\left(x_i, \mathbf{V}_{x_i}^{(l)}\right)$$

$$\mathbf{P}_{\mathbf{X}_i^{DGPS}}^{(l+1)} = \tilde{\mathbf{T}}_{sqr}\left(\mathbf{X}_{P_i}^{DGPS}, \mathbf{V}_{\mathbf{X}_{P_i}^{DGPS}}^{(l)}\right)\mathbf{P}_{\mathbf{X}_{P_i}^{DGPS}}^{(l)}\tilde{\mathbf{T}}_{sqr}\left(\mathbf{X}_{P_i}^{DGPS}, \mathbf{V}_{\mathbf{X}_{P_i}^{DGPS}}^{(l)}\right)$$

In the iterative process of the adjustment task solution, the decisive weights matrix is a priori $\tilde{\mathbf{P}}_i$, converted into the decisive – equivalent form $\tilde{\tilde{\mathbf{P}}}_i$. Each stepwise weights matrix $\mathbf{P}^{(l+1)}$ refers to the increments vector $\mathbf{d}_{\mathbf{X}_i}^{(l+1)}$ and the corrections vector $\mathbf{V}_i^{(l+1)}$. Therefore, in essence, resolving the adjustment task consists in forming sequences.

$$\left(\mathbf{P}_i^{(0)} = \tilde{\mathbf{P}}_i\right) \rightarrow \mathbf{P}_i^{(l+1)} \xrightarrow{l=0,1,\ldots\ldots} \tilde{\tilde{\mathbf{P}}}_i$$
$$\mathbf{d}_{\mathbf{X}_i}^{(l)} \xrightarrow{l=0,1,\ldots\ldots} \hat{\mathbf{d}}_{\mathbf{X}_i}$$
$$\mathbf{V}_i^{(l)} \xrightarrow{l=0,1,\ldots\ldots} \mathbf{V}_i$$

The problem, substantial for practical solving the task (1), is selecting the standardized corrections $\bar{\upsilon} = \frac{\upsilon}{\hat{\sigma}_\upsilon}$. The estimator $\hat{\sigma}_\upsilon$ of the standard deviation σ_υ of the correction υ is also a root of the respective diagonal element of the estimator $\hat{\mathbf{C}}_{\mathbf{V}_i}$ of the corrections vector $\mathbf{V}_i$ covariance matrix $\mathbf{C}_{\mathbf{V}_i}$, it means $\hat{\sigma}_{\upsilon_j} = \sqrt{\left[\hat{\mathbf{C}}_{\mathbf{V}_i}\right]_{jj}}$. Anyhow, as:

$$\hat{\mathbf{C}}_{\mathbf{V}_i} = \hat{\sigma}_0^2\left\{\tilde{\tilde{\mathbf{P}}}_i - \mathbf{A}_i\left(\mathbf{A}_i^T\tilde{\tilde{\mathbf{P}}}_i\mathbf{A}_i\right)^{-1}\mathbf{A}_i^T\right\} \quad (6)$$

so in every $l-th$ iterative step:

$$\sigma_{\upsilon_j}^{(l)} = \sigma_0^{(l)}\sqrt{\left[\tilde{\tilde{\mathbf{P}}}_i^{(l)} - \mathbf{A}_i\left(\mathbf{A}_i^T\tilde{\tilde{\mathbf{P}}}_i^{(l)}\mathbf{A}_i\right)^{-1}\mathbf{A}_i^T\right]_{jj}} \quad (7)$$

where: $\sigma_0^{(l)} \xrightarrow{l=0,1,\dots} \hat{\sigma}_0$

3. Development of the *IANS* Chain - the Adjustment Task and its Solution

An intrinsic *IANS* element for $i=1$ is the one which enables adjusting only the proper position P_i. The points which are included in R_i can be in this case determined uniquely. Therefore let's assume such a navigational situation, in which after dislocation of a vessel to the position P_{i+1} there are carried out surveys towards the adjustment points $\mathsf{Z}_{i+1} \subset \mathsf{Z}$, also DGPS surveys based on the reference stations $\mathsf{Z}_{i+1}^{DGPS} \subset \mathsf{Z}$, surveys to the points $\mathsf{R}_i \subset \mathsf{R}$, determined before and surveys to the new (for the P_{i+1} positions) points $\mathsf{R}_{i+1} \subset \mathsf{R}$ as well. We also assume that the route elements are known (a course, distance travelled) – Fig. 3. Using models which was describing in [Czaplewski 2004] the authors propose following robust-decision adjustment task:

$$\left.\begin{array}{l}
\mathbf{V}_{x_{i+1}} = \mathbf{A}_{P_{i+1}}\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{P_{i+1}}} + \mathbf{A}_{R_{i+1}}\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{R_{i+1}}} + \mathbf{A}^{(i+1)}_{R_i}\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{R_i}} + \mathbf{A}^{(i+1)}_{P_i}\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{P_i}} + \mathbf{L}_{x_{i+1}} \\
\mathbf{V}_{\mathbf{X}^{DGPS}_{i+1}} = \hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{P_{i+1}}} + \mathbf{X}^0_{P_{i+1}} - \mathbf{X}^{DGPS}_{P_{i+1}} \\
\mathbf{V}_{\hat{\mathbf{X}}_{P_i}} = \hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{P_i}} - \hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}} \\
\mathbf{V}_{\hat{\mathbf{X}}_{R_i}} = \hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{R_i}} - \hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}} \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\mathbf{C}_{x_{i+1}} = \sigma_0^2\mathbf{P}^{-1}_{x_{i+1}} \quad \rightarrow \tilde{\tilde{\mathbf{C}}}_{x_{i+1}} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}_{x_{i+1}} \\
\mathbf{C}_{\mathbf{X}^{DGPS}_{i+1}} = \sigma_0^2\mathbf{P}^{-1}_{\mathbf{X}^{DGPS}_{i+1}} \quad \rightarrow \tilde{\tilde{\mathbf{C}}}_{\mathbf{X}^{DGPS}_{i+1}} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}_{\mathbf{X}^{DGPS}_{i+1}} \\
\mathbf{C}_{\hat{\mathbf{X}}^{(i)}_{P_i}} = \sigma_0^2\mathbf{P}^{-1}_{\hat{\mathbf{X}}^{(i)}_{P_i}} \quad \rightarrow \tilde{\tilde{\mathbf{C}}}_{\hat{\mathbf{X}}^{(i)}_{P_i}} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}_{\hat{\mathbf{X}}^{(i)}_{P_i}} \\
\mathbf{C}_{\hat{\mathbf{X}}^{(i)}_{R_i}} = \sigma_0^2\mathbf{P}^{-1}_{\hat{\mathbf{X}}^{(i)}_{R_i}} \quad \rightarrow \tilde{\tilde{\mathbf{C}}}_{\hat{\mathbf{X}}^{(i)}_{R_i}} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}_{\hat{\mathbf{X}}^{(i)}_{R_i}} \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\min\limits_{\mathbf{d}_{\mathbf{X}_{i+1}}} \Phi^{D-R}\left(\mathbf{d}_{\mathbf{X}_{i+1}}\right) = \Phi^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right) = \\
= \Phi_x^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right) + \Phi_{DGPS}^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right) + \Phi_{P_i}^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right) + \Phi_{R_i}^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right)
\end{array}\right\} \quad (8)$$

with equivalent covariance matrixes $\tilde{\tilde{\mathbf{C}}} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}$ which substitute the original matrixes $\mathbf{C} = \sigma_0^2\mathbf{P}^{-1}$. By introducing the designations:

$$\mathbf{V}_{i+1} = \begin{bmatrix} \mathbf{V}_{x_{i+1}} \\ \mathbf{V}_{\mathbf{X}^{DGPS}_{i+1}} \\ \mathbf{V}_{\hat{\mathbf{X}}_{P_i}} \\ \mathbf{V}_{\hat{\mathbf{X}}_{R_i}} \end{bmatrix}, \quad \mathbf{A}_{i+1} = \begin{bmatrix} \mathbf{A}_{P_{i+1}} & \mathbf{A}_{R_{i+1}} & \mathbf{A}^{(i+1)}_{R_i} & \mathbf{A}^{(i+1)}_{P_i} \\ \mathbf{I}_{(2)} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_{(2)} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{(2n_{R_i})} & \mathbf{0} \end{bmatrix},$$

$$\mathbf{L}_i = \begin{bmatrix} \mathbf{L}_{x_{i+1}} \\ \mathbf{X}^0_{P_{i+1}} - \mathbf{X}^{DGPS}_{P_{i+1}} \\ -\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}} \\ -\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}} \end{bmatrix}$$

$$\tilde{\tilde{\mathbf{C}}}_{i+1} = Diag\left(\tilde{\tilde{\mathbf{C}}}_{x_{i+1}}, \tilde{\tilde{\mathbf{C}}}_{\mathbf{X}^{DGPS}_{i+1}}, \tilde{\tilde{\mathbf{C}}}_{\hat{\mathbf{X}}^{(i)}_{P_i}}, \tilde{\tilde{\mathbf{C}}}_{\hat{\mathbf{X}}^{(i)}_{R_i}}\right),$$

$$\tilde{\tilde{\mathbf{P}}}_{i+1} = Diag\left(\tilde{\tilde{\mathbf{P}}}_{x_{i+1}}, \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}^{DGPS}_{i+1}}, \tilde{\tilde{\mathbf{P}}}_{\hat{\mathbf{X}}^{(i)}_{P_i}}, \tilde{\tilde{\mathbf{P}}}_{\hat{\mathbf{X}}^{(i)}_{R_i}}\right)$$

the task (8) can be presented in the following form

$$\left.\begin{array}{l}
\mathbf{V}_{i+1} = \mathbf{A}_{i+1}\hat{\mathbf{d}}_{\mathbf{X}_{i+1}} + \mathbf{L}_{i+1} \\
\tilde{\tilde{\mathbf{C}}}_{i+1} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}_{i+1} \\
\min\limits_{\mathbf{d}_{\mathbf{X}_{i+1}}} \Phi^{D-R}\left(\mathbf{d}_{\mathbf{X}_{i+1}}\right) = \Phi^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right)
\end{array}\right\} \quad (9)$$

Its solution is the estimator as follows:

$$\hat{\mathbf{d}}_{\mathbf{X}_{i+1}} = -\left(\mathbf{A}^T_{i+1}\tilde{\tilde{\mathbf{P}}}_{i+1}\mathbf{A}_{i+1}\right)^{-1}\mathbf{A}^T_{i+1}\tilde{\tilde{\mathbf{P}}}_{i+1}\mathbf{L}_{i+1} \quad (10)$$

Moreover, (if $\left(\mathbf{A}^T_{i+1}\tilde{\tilde{\mathbf{P}}}^{-1}_{i+1}\mathbf{A}_{i+1}\right)^{-1}$ does exist)

$$\hat{\mathbf{C}}_{\hat{\mathbf{X}}_{i+1}} = \hat{\mathbf{C}}_{\hat{\mathbf{d}}\mathbf{X}_{i+1}} = \hat{\sigma}_0^2\left(\mathbf{A}^T_{i+1}\tilde{\tilde{\mathbf{P}}}_{i+1}\mathbf{A}_{i+1}\right)^{-1} \quad (11)$$

and

$$\hat{\mathbf{X}}_{i+1} = \mathbf{X}^0_{i+1} + \hat{\mathbf{d}}_{\mathbf{X}_{i+1}} = \begin{bmatrix} \mathbf{X}^0_{P_{i+1}} \\ \mathbf{X}^0_{R_{i+1}} \\ \mathbf{X}^0_{R_i} \\ \mathbf{X}^0_{P_i} \end{bmatrix} + \begin{bmatrix} \hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{P_{i+1}}} \\ \hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{R_{i+1}}} \\ \hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{R_i}} \\ \hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{P_i}} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{X}}^{(i+1)}_{P_{i+1}} \\ \hat{\mathbf{X}}^{(i+1)}_{R_{i+1}} \\ \hat{\mathbf{X}}^{(i+1)}_{R_i} \\ \hat{\mathbf{X}}^{(i+1)}_{P_i} \end{bmatrix}$$

4. Conclusion

- The most essential outcome of theoretical studies presented in this work, are the suggestions related to the technology of producing and working out the results of observations, carried out in the Interactive Navigational Structures. Implementing such a Structure into navigation practice will enable supporting the positioning process by taking use of objects, which until present, in classic navigation, have been omitted due to a lack of information about their coordinates. Establishing and dynamic developing the Interactive Navigational Structures is of special importance, in case the available positioning systems appear insufficient

- *IANS* can be based on various systems and navigational observations, including the satellite GPS systems. Selection of an observational model, accommodated to any current navigational situation, is simplified owing to the decisive functions, recommended in [Czaplewski 2004]. Applying the above functions, in conjunction with the functions of attenuation, has resulted in making more efficient the process of estimation, robust for out-lying observations.

- Due to diversity of the observations sets and the "by stages" way of working them out, the estimation is of sequential character.

- There is a possibility to use the described structures in submarine navigation. A lack of any classic navigational systems in sea depth has been forcing to seek new solutions, as the Interactive Navigational Structure is. The above solutions may also be extensively employed in radar navigation. Radar observations are often biased with gross errors, caused by radar echo generation technique. The robust estimation, if applied in the version presented in this work, may significantly improve final determinations' standard.

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