Development of an AOA Location Method Using Covariance Estimation

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Abstract

In last decades, several linearization methods for the AOA measurements have been proposed, for example, Gauss-Newton method and closed-form solution. Gauss-Newton method can achieve high accuracy, but the convergence of the iterative process is not always ensured if the initial guess is not accurate enough. Closed-form solution provides a non-iterative solution and it is less computational. It does not suffer from convergence problem, but estimation error is somewhat larger. This paper proposes a self-tuning weighted least square AOA algorithm that is a modified version of the conventional closed-form solution. In order to estimate the error covariance matrix as a weight, two-step estimation technique is used. Simulation results show that the proposed method has smaller positioning error compared to the existing methods.

Keywords: Angle of arrival, closed form solution, weighted least square

1. Introduction

Positioning methods in wireless communication systems are often classified into three categories; time of arrival (TOA), time difference of arrival (TDOA), and angle of arrival (AOA). In TOA method, distance between the user and sensor is determined from the measured one way propagation time of the signal traveling between them. In TDOA method, the difference in arrival times of a pair of sensors is measured. In AOA method, multi-element array antenna is required to measure the arrival angles of the signal from a user [1].

In this paper, we focus on AOA-based location method in a wireless communication network. The main advantage of AOA method is that it does not require highly accurate synchronization of sensors. On the other hand, sensors require regular calibration in order to compensate for temperature variation and antenna mismatches [2]. In 2D positioning, user position is defined at the intersection of two directional lines of azimuth. In practice, more than two sensors are commonly employed to reduce inaccuracies introduced by various error budgets, e.g., multi-path, antenna alignment error, thermal noise, and so on.

Because AOA measurements are nonlinear, linearization is often used to estimate the user position. Several linearization approaches for AOA measurements have been proposed, for example, Gauss-Newton (GN) method [3][4] and closed-form (CF) solution [5]. GN method can achieve high accuracy, but the convergence of the iterative process is not always ensured if the initial guess is not accurate enough. CF solution provides a non-iterative solution and it is less computational. It does not suffer from convergence problem, but the estimation error is somewhat larger.

This paper proposes a self-tuning weighted least square (STWLS) algorithm base on CF solution. To estimate the weighting matrix, two-step estimation technique is used in the proposed method. In section 2, the classical methods to estimate the position from the AOA measurements are briefly reviewed. Afterwards, we present the formulation of the proposed STWLS algorithm. Section 3 includes some simulations results useful to compare the proposed technique to the classical ones.

2. An AOA positioning algorithm using error covariance estimation technique

2.1 Measurement model

Let $\mathbf{x} = [x \;\; y]^T$ be the user position to be determined and the known coordinates of the i-th sensor be $\mathbf{x_i} = [x_i \;\; y_i]^T$, i=1, 2, …, m, where m is the total number of the sensors. AOA is obtained from the array antenna of a sensor as shown in Fig. 1. Angle α_i restricts the user location along a line called line of bearing (LOB) [2]. If there is a mismatch between the orientation of array antenna and the reference frame, offset angle should be compensated to get the azimuth that is given by

$$\alpha_{ri} = \alpha_i + \alpha_{off}^i \qquad (1)$$

where α_{off}^i is an offset of i-th sensor.

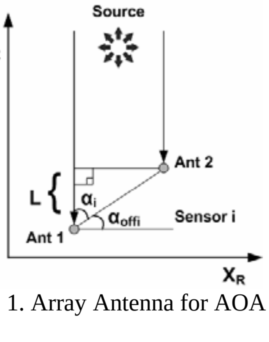


Fig. 1. Array Antenna for AOA

AOA azimuth has the following relationship in the reference frame. [3][4]

$$\tan(\alpha_{ri}) = \frac{y - y_i}{x - x_i} \tag{2}$$

Considering the measurement noise, AOA measurement obtained from the path difference of the array antenna can be represented as

$$d\cos(\alpha_{ri} + v_{\alpha i}) = L_i + v_{li} \tag{3}$$

where d is a distance between antennas and L_i is the measured path difference at antenna array. $v_{\alpha i}$ and v_{li} are measurement noises in angle α_{ri} and in LOB respectively. If v_{li} is assumed to be i.i.d. (independently and identically distributed) white Gaussian with variance of σ_a^2, AOA azimuth measurement and its noise are given by

$$\begin{aligned} f_i &= \alpha_{ri} + v_{\alpha i} \\ v_{\alpha i} &\sim N\left(0, \frac{\sigma_a^2}{d^2 \sin^2 \alpha_i}\right) \end{aligned} \tag{4}$$

2.2 Gauss-Newton method and Closed-Form solution

The linearized equation for AOA measurements used in GN method is written as [3]

$$\begin{aligned} &\delta\mathbf{f} = \mathbf{H_a}\delta\mathbf{x_a} + \mathbf{w_a} \\ &\delta\mathbf{f} = \begin{bmatrix} f_1 - \alpha_{o1} \\ \vdots \\ f_m - \alpha_{om} \end{bmatrix} \quad \mathbf{H_a} = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \vdots & \vdots \\ \frac{\partial f_m}{\partial x} & \frac{\partial f_m}{\partial y} \end{bmatrix} \\ &\delta\mathbf{x_a} = \begin{bmatrix} x - x_o \\ y - y_o \end{bmatrix} \quad \mathbf{w_a} = \begin{bmatrix} v_{\alpha 1} \\ \vdots \\ v_{\alpha m} \end{bmatrix} \end{aligned} \tag{5}$$

where (x_o, y_o) is a nominal user position and α_{oi} is nominal azimuth angle between the i-th sensor and nominal user position. The position estimate can be obtained using weighted least squares (WLS) that is given by

$$\begin{aligned} &\delta\hat{\mathbf{x}}_\mathbf{a} = (\mathbf{H_a^T} Q_a^{-1} \mathbf{H_a})^{-1} \mathbf{H_a^T} Q_a^{-1} \delta\mathbf{f} \\ &\hat{\mathbf{x}}_\mathbf{a} = \mathbf{x}_o + \delta\hat{\mathbf{x}}_\mathbf{a} \\ &Q_a = \frac{\sigma_a^2}{d^2} \begin{bmatrix} 1/\sin^2(\alpha_{r1}) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/\sin^2(\alpha_{rm}) \end{bmatrix} \end{aligned} \tag{6}$$

GN method is an iterative method. It starts with an initial guess and improves the estimate at each step by determining the local WLS solution. An initial guess close to the true solution is needed to avoid local minima. Selection of such a nominal point is not simple in practice. Moreover, convergence of the iterative process is not assured. It is also computationally intensive since WLS computation is required in each iteration.

In the CF solution, the linearization departs from the rearrangement of Eq. (2) that is given by [5]

$$\begin{aligned} &\tan f_i = \frac{\sin f_i}{\cos f_i} = \frac{y - y_i}{x - x_i} \\ &x \sin f_i - y \cos f_i = x_i \sin f_i - y_i \cos f_i \end{aligned} \tag{7}$$

Eq. (7) leads to the following matrix-vector notation.

$$\begin{aligned} \mathbf{h} &\equiv \begin{bmatrix} x_1 \sin f_1 - y_1 \cos f_1 \\ \vdots \\ x_m \sin f_m - y_m \cos f_m \end{bmatrix} \\ &= \begin{bmatrix} \sin f_1 & -\cos f_1 \\ \vdots & \vdots \\ \sin f_m & \cos f_m \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \equiv \mathbf{G_a x_a} \end{aligned} \tag{8}$$

Using least square (LS), position estimate is given by

$$\hat{\mathbf{x}}_\mathbf{a} = (\mathbf{G_a^T G_a})^{-1} \mathbf{G_a^T h} \tag{9}$$

2.3 Self-tuning Weighted Least Square

Although the CF solution is non-iterative and does not suffer from the convergence problem, its estimation error is somewhat larger because the characteristics of noise in Eq. (8) is not carefully considered. This paper proposes a modified version of the conventional CF solution using STWLS. Since the covariance matrix of pseudo-measurement noise in Eq. (8) is affected by unknown user position, two-step estimation technique is used to estimate the weighting matrix first. This is why the proposed method is named “self-tuning” WLS.

Precisely speaking, together with noise, Eq. (8) should be written as

$$\frac{y - y_i}{x - x_i} = \tan(f_i - v_{\alpha i}) = \frac{\tan f_i - \tan v_{\alpha i}}{1 + \tan f_i \tan v_{\alpha i}} \tag{10}$$

Eq. (10) can be written as a matrix form that is given by

$$\begin{aligned} \mathbf{h} &\equiv \begin{bmatrix} x_1 \sin f_1 - y_1 \cos f_1 \\ \vdots \\ x_m \sin f_m - y_m \cos f_m \end{bmatrix} \\ &= \begin{bmatrix} \sin f_1 & -\cos f_1 \\ \vdots & \vdots \\ \sin f_m & -\cos f_m \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \\ &+ \begin{bmatrix} -\tan v_{\alpha 1}[(y - y_1)\sin f_1 + (x - x_1)\cos f_1] \\ \vdots \\ -\tan v_{\alpha m}[(y - y_m)\sin f_m + (x - x_m)\cos f_m] \end{bmatrix} \\ &\equiv \mathbf{G_a x_a} + \mathbf{n_{\alpha r}} \end{aligned} \tag{11}$$

Rewrite the component of pseudo-measurement noise as

$$n_{\alpha ri} \equiv \tan v_{\alpha i} \sin v_{\alpha i} (K_{xi} \sin \alpha_{ri} - K_{yi} \cos \alpha_{ri}) \\ - \sin v_{\alpha i} (K_{xi} \cos \alpha_{ri} + K_{yi} \sin \alpha_{ri}) \quad (12)$$

where $K_{xi} = x - x_i$, $K_{yi} = y - y_i$. If $v_{\alpha i}$ is sufficiently small, Eq. (12) is approximated to

$$n_{\alpha ri} \cong -v_{\alpha i} (K_{xi} \cos \alpha_{ri} + K_{yi} \sin \alpha_{ri}) \quad (13)$$

Hence, from Eq. (4) and (13), pseudo-measurement noise is approximated to i.i.d. white Gaussian with zero mean and the covariance matrix that is given by

$$\mathrm{cov}(\mathbf{n}_{\alpha \mathbf{r}}) = \Psi = \begin{bmatrix} A_1^2 \sigma_a^2 & 0 & 0 \\ 0 & \ddots & \vdots \\ 0 & \cdots & A_m^2 \sigma_a^2 \end{bmatrix} \quad (14)$$

where

$$A_i = (x - x_i) \cos \alpha_{ri} + (y - y_i) \sin \alpha_{ri} .$$

The error covariance matrix Ψ in Eq. (14) will be used as a weighting matrix of WLS. However, user position x, y and azimuth angle α_{ri} in Eq. (14) are unknown. In the proposed method, CF solution is executed first to get the estimate $\hat{\mathbf{x}}_{\mathbf{a}} = [\hat{x},\ \hat{y}]^{\mathbf{T}}$ used in determining Ψ. α_{ri} will be replaced with azimuth measurement f_i. Then, the estimate of covariance matrix of pseudo-measurement noise can be obtained from

$$\hat{\Psi} = \begin{bmatrix} \hat{A}_1^2 \sigma_a^2 & 0 & 0 \\ 0 & \ddots & \vdots \\ 0 & \cdots & \hat{A}_1^2 \sigma_a^2 \end{bmatrix} \quad (15)$$

where

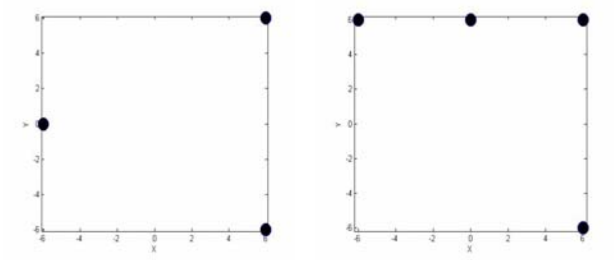
$$\hat{A}_i = (\hat{x} - x_i) \cos f_i + (\hat{y} - y_i) \sin f_i .$$

The final position estimate of the proposed method is solved using WLS that is given by

$$\hat{\mathbf{x}}_{\mathbf{a}}' = (\mathbf{G}_{\mathbf{a}}^{\mathbf{T}} \hat{\Psi}^{-1} \mathbf{G}_{\mathbf{a}})^{-1} \mathbf{G}_{\mathbf{a}}^{\mathbf{T}} \hat{\Psi}^{-1} \mathbf{h} \quad (16)$$

3. Simulation results

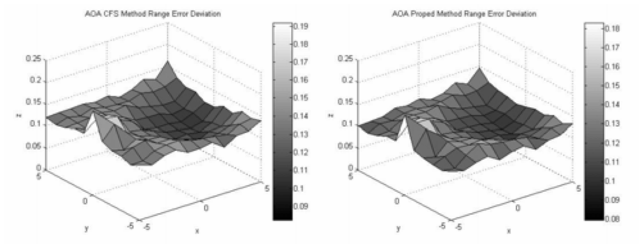
In computer simulations, the size of work space is assumed to be 12 by 12m. Two cases of sensor geometry shown in Fig. 2 are analyzed. The sensors are located at (-6, 0), (6, 6), (6, -6) for 3-sensor case and (-6, 6), (6, 6), (6, -6), (0, 6) for 4-sensor. σ_a^2 in Eq. (4) is set to be 10^{-4}m. The position estimate is analyzed at every grid point, and the distance between two adjacent grid points is 1m in x- and y-axis direction. At each grid point, 100 trials are repeated for Monte Carlo simulation to get the standard deviation of the estimation error and error distribution. In GN method, the iteration is terminated when the estimate converges within 10^{-2}m.



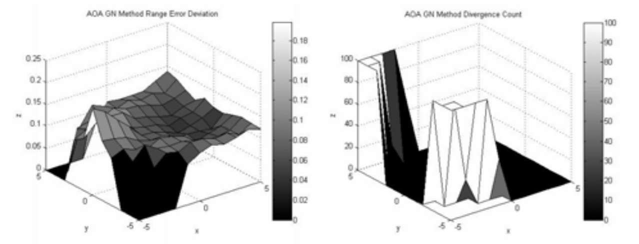
(a) 3-sensor (b) 4-sensor

Fig. 2. Sensor geometry

Three kinds of AOA location methods are compared; GN method, SF solution, and the proposed STWLS. Fig. 3 compares the standard deviation of estimation error for 3-sensor case. Some numerical results are given in Table 1 for detailed comparison. In GN method, trials diverge when the user is located around (- 5, -5) and (-5, 5) as shown in Fig. 3(d). Although GN method shows the best performance at 40% of work space, it shows worse performance or even diverges at the other region. It means that GN method is sensitive to sensor geometry. Compared to CF solution, the proposed method shows better performance at all the grid points, and the average of error reduction is about 8%.



(a) CF solution (b) Proposed STWLS



(c) GN method (d) Number of trials diverged in GN method.

Fig. 3. Standard deviation for 3-sensor case

Table 1. Standard deviation for 3-sensor case

User position	CF Solution	Proposed method	GN method	User position	CF Solution	Proposed method	GN method
(-5,-5)	0.133	0.118	-	(0, 5)	0.126	0.114	0.106
(-5,0)	0.190	0.182	0.175	(3,-3)	0.109	0.100	0.108
(-5,5)	0.116	0.097	-	(3, 0)	0.091	0.090	0.084
(-3,-3)	0.120	0.101	0.119	(3, 3)	0.103	0.097	0.108
(-3,0)	0.141	0.133	0.126	(5,-5)	0.144	0.130	0.135
(-3,3)	0.123	0.118	0.113	(5, 0)	0.110	0.105	0.112
(0,-5)	0.131	0.123	0.113	(5, 5)	0.145	0.134	0.151
(0, 0)	0.095	0.092	0.095				

Fig. 4 shows the error distribution of the position estimate for

3-sensor case. Size of error ellipse of the CF solution is the biggest compared to other methods. Ellipse size of the proposed method is similar to that of the GN method. Note that the proposed method is non-iterative and do not suffer from the convergence problem.

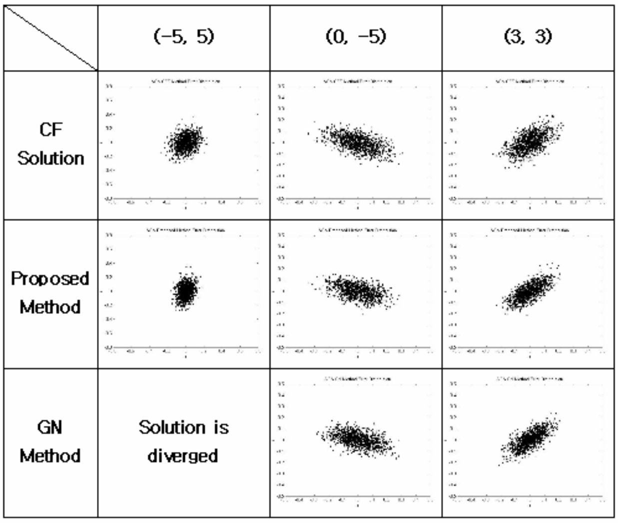
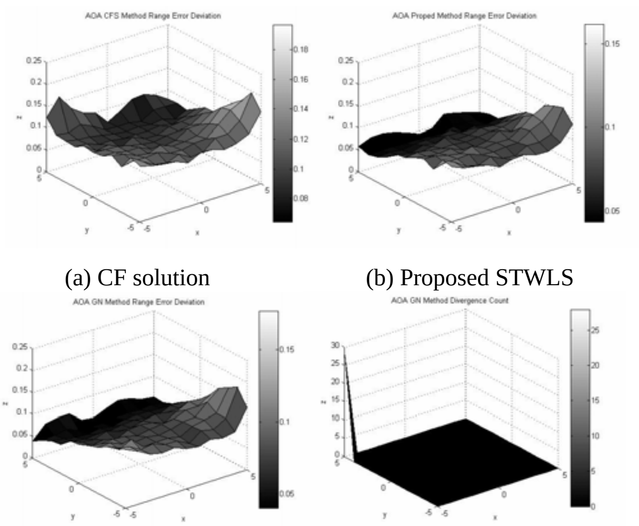


Fig. 4. Error distribution for 3-sensor

Fig. 5 shows the standard deviation of the estimation error for 4-sensor case and some numerical results are also given in Table 2. In this case, GN method diverges only when the user is location at (-5, 5) as shown in Fig. 4(d). GN method shows the best performance at 67% of work space including 2 grid points that GN method shows the same performance with the proposed method. Compared to 3-sensor case, this means that the performance of GN method is improved fast as the number of sensors becomes large. The proposed method shows better performance than CF solution at all the grid points. The average of error reduction is about 17%. Furthermore, the proposed method shows best performance at 47% of work space. As the number of sensors becomes large, the performance improvement of the proposed method is almost same to that of GN method. Note that there still exists the convergence problem in GN method.



(a) CF solution (b) Proposed STWLS

(c) GN method (d) Number of trials diverged in GN method.

Fig. 5. Estimated standard deviation for 3-sensor

Table 2. Estimated standard deviation for 4-sensor

User position	CF Solution	Proposed method	GN method	User position	CF Solution	Proposed method	GN method
(-5,-5)	0.125	0.123	0.110	(0, 5)	0.076	0.042	0.041
(-5,0)	0.102	0.091	0.098	(3,-3)	0.123	0.111	0.118
(-5,5)	0.109	0.059	-	(3, 0)	0.099	0.090	0.082
(-3,-3)	0.119	0.119	0.102	(3, 3)	0.065	0.055	0.055
(-3,0)	0.092	0.079	0.085	(5,-5)	0.180	0.136	0.122
(-3,3)	0.074	0.054	0.053	(5, 0)	0.113	0.100	0.100
(0,-5)	0.106	0.096	0.088	(5, 5)	0.091	0.049	0.050
(0, 0)	0.086	0.086	0.085				

Fig. 5 shows the error distribution of the position estimate for 4-sensor case. Size of error ellipsoid of the CF solution is the biggest and ellipsoid size of the proposed method is similar to that of the GN method if converged. Comparing Fig. 4 and Fig. 6, it is reconfirmed that the performance of STWLS is improved fast as the number of sensors increases.

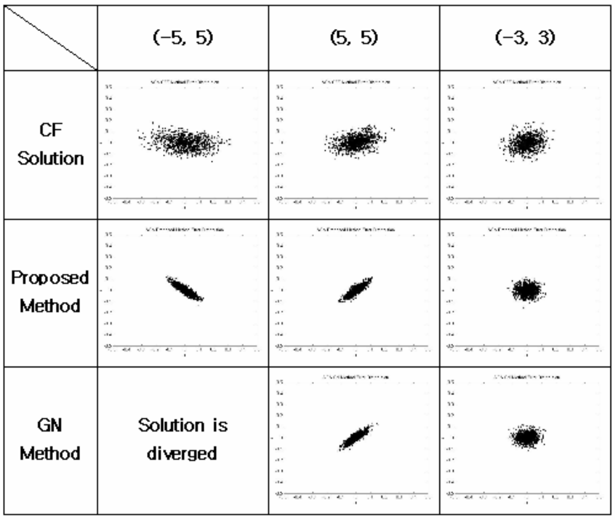


Fig. 6. Error distribution for 4-sensor

4. Conclusion

This paper presents a new AOA location algorithm that is the modified version of classical CF solution. In order to improve the accuracy, two-step STWLS is employed in the proposed method. Simulation results show that the performance of the proposed method is about 7~15% better than that of CF solution. The proposed method shows similar performance to the GN method if it converges. Furthermore, as the number of sensors increases, the estimation error of the proposed method is reduced as fast as that of GN method. The most remarkable feature of the proposed method is that it does not suffer from convergence problems, providing an accurate position estimate. Thus, the proposed method can be a good alternative solution for AOA-based positioning for wireless communication systems.

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