

# A New Approach for SINS Stationary Self-alignment Based on IMU Measurement

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## Abstract

For the poor observability of azimuth misalignment angle and east gyro drift rate of the traditional initial alignment, a brand-new SINS stationary fast self-alignment approach is proposed. By means of analyzing the characteristic of the strapdown inertial navigation system (SINS) stationary alignment seriously, the new approach takes full advantage of the specific force and angular velocity information given by inertial measurement unit (IMU) instead of the mechanization of SINS. Firstly, coarse alignment algorithm is presented. Secondly, a new fine alignment model for SINS stationary self-alignment is derived, and the observability of the model is analysed. Then, a modified Sage-Husa adaptive Kalman filter is introduced to estimate the misalignment angles. Finally, some computer simulation results illustrate the efficiency of the new approach and its advantages, such as higher alignment accuracy, shorter alignment time, more self-contained and less calculation.

**Keywords:** SINS; stationary self-alignment; IMU; modified Sage-Husa adaptive Kalman filter

## 1. Introduction

The object of the SINS self-alignment is to determine the direction cosine of the transformation from the body frame to the navigation frame, namely, the elements of initial attitude matrix, using the accelerometer and gyro outputs. As the alignment accuracy affects the accuracy of the navigation system directly, one of the most important requirements of SINS alignment is high alignment accuracy. In many practical applications, SINS alignment also requires high alignment speed and the capability of self-determination, especially for military applications.

At present, the alignment model given by Bar-Itzhack and Bermant with the observations of velocity, is widely used in general stationary SINS self-alignment, such as fast alignment proposed by JIANG CHENG FANG and DE JUN WAN, multiposition alignment presented by JANG GYU LEE, CHAN GOOK PARK and HEUNG WON PARK, while it is not completely observable. In this model the east gyro drift rate is only weakly coupled to the velocities which would serve as external information for the purpose of alignment. So it is hard to give more attention to alignment accuracy and alignment speed during stationary self-alignment using the traditional initial alignment methods.

In this paper, a new self-alignment approach based on the measurement of IMU is proposed for SINS on a stationary base. The approach needn't to carry out SINS navigation calculation, thereby, it breaks away from the model which is not completely observable. Meanwhile, it takes full advantage of specific force and angular velocity information which is the sensed output of IMU as well as the characteristic of SINS stationary alignment. In order to make sure the high accuracy and speed of alignment, a new alignment model which is completely observable, is established for the approach. On the other hand, for the sake

of self-alignment, IMU measurement is used as observations instead of velocities.

The main work of this paper is following: firstly, coarse alignment algorithm is presented. Secondly, a new fine alignment model for SINS stationary self-alignment is derived, and the observability of the model is analysed. Then, a modified Sage-Husa adaptive Kalman filter is introduced to estimate the misalignment angles. Finally, some computer simulation results illustrate the efficiency of the new approach and its advantages, such as higher alignment accuracy, shorter alignment time, more self-contained and less calculation.

## 2. Coarse Alignment

Normally, SINS initial alignment process is divided into two phases, i.e., coarse alignment and fine alignment. The purpose of coarse alignment is to provide a fairly good initial condition for the fine alignment processing. For SINS stationary alignment, the carrier is fixed to the Earth. And some characteristics of stationary alignment are concluded as follows:

- 1) As a stationary carrier ( $\mathbf{v}_{en} \equiv \mathbf{0}$ ), the acceleration in navigation frame (the local-level east, north and up frame) equals zero, that is

$$\mathbf{a}^n = \mathbf{0} \quad (1)$$

- 2) The angular velocity of the body frame with respect to the Earth-Fixed frame also equals zero, which can be written as

$$\boldsymbol{\omega}_{eb} = \mathbf{0} \quad (2)$$

According to the characteristics of SINS stationary alignment analyzed as above with the definition of specific

force ( $\mathbf{f} = \mathbf{a} - \mathbf{g}$ ), we have

$$\begin{aligned}\mathbf{f}_{ib}^n &= \mathbf{a}^n - \mathbf{g}^n \\ &= -\mathbf{g}^n \\ &= \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}\end{aligned}\quad (3)$$

and

$$\begin{aligned}\boldsymbol{\omega}_{ib}^n &= \boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{eb}^n \\ &= \boldsymbol{\omega}_{ie}^n \\ &= \begin{bmatrix} 0 \\ \omega_{ie} \cos \varphi \\ \omega_{ie} \sin \varphi \end{bmatrix}\end{aligned}\quad (4)$$

where  $g$  and  $\omega_{ie}$  represent the magnitude of gravity and Earth rate, respectively,  $\varphi$  is the local geographical latitude.

For the SINS, the accelerometer and gyro output can be expressed respectively as

$$\mathbf{f}_{ib}^b = \mathbf{C}_n^b \mathbf{f}_{ib}^n$$

$$\boldsymbol{\omega}_{ib}^b = \mathbf{C}_n^b \boldsymbol{\omega}_{ib}^n$$

They also can be written as

$$\mathbf{f}_{ib}^n = \mathbf{C}_b^n \mathbf{f}_{ib}^b \quad (5)$$

$$\boldsymbol{\omega}_{ib}^n = \mathbf{C}_b^n \boldsymbol{\omega}_{ib}^b \quad (6)$$

The vector cross-product based on  $\mathbf{f}_{ib}^n$  and  $\boldsymbol{\omega}_{ib}^n$  is given by

$$\begin{aligned}\mathbf{f}_{ib}^n \times \boldsymbol{\omega}_{ib}^n &= [\mathbf{f}_{ib}^n \times] \boldsymbol{\omega}_{ib}^n \\ &= \mathbf{C}_b^n [\mathbf{f}_{ib}^b \times] \mathbf{C}_n^b \boldsymbol{\omega}_{ib}^n \\ &= \mathbf{C}_b^n [\mathbf{f}_{ib}^b \times] \boldsymbol{\omega}_{ib}^b \\ &= \mathbf{C}_b^n (\mathbf{f}_{ib}^b \times \boldsymbol{\omega}_{ib}^b)\end{aligned}\quad (7)$$

where  $[\mathbf{f}_{ib}^n \times]$  and  $[\mathbf{f}_{ib}^b \times]$  denote the skew-symmetric matrix of  $\mathbf{f}_{ib}^n$  and  $\mathbf{f}_{ib}^b$  respectively.

Then, combine (5), (6) and (7) into one matrix equation to obtain

$$\begin{bmatrix} \mathbf{f}_{ib}^n & \boldsymbol{\omega}_{ib}^n & \mathbf{f}_{ib}^n \times \boldsymbol{\omega}_{ib}^n \end{bmatrix} = \mathbf{C}_b^n \begin{bmatrix} \mathbf{f}_{ib}^b & \boldsymbol{\omega}_{ib}^b & \mathbf{f}_{ib}^b \times \boldsymbol{\omega}_{ib}^b \end{bmatrix}$$

The transformation matrix can be expressed as

$$\mathbf{C}_b^n = \begin{bmatrix} \mathbf{f}_{ib}^n & \boldsymbol{\omega}_{ib}^n & \mathbf{f}_{ib}^n \times \boldsymbol{\omega}_{ib}^n \end{bmatrix} \begin{bmatrix} \mathbf{f}_{ib}^b & \boldsymbol{\omega}_{ib}^b & \mathbf{f}_{ib}^b \times \boldsymbol{\omega}_{ib}^b \end{bmatrix}^{-1} \quad (8)$$

Equation (8) shows that the output of the accelerometers and gyros of a stationary SINS can be used to determine the attitude matrix directly.

### 3. Fine Alignment

Coarse alignment is based on an idealization in which there are no accelerometer and gyro errors. But both accelerometers and gyros output data have errors in reality, especially the gyros may have large rate biases. So the attitude matrix given by coarse alignment may have some error. Now, using the notation  $\hat{\mathbf{C}}_b^n$  to denote the attitude matrix given by coarse alignment, and  $\mathbf{C}_b^n$  represent the ideal attitude matrix, the relationship between  $\hat{\mathbf{C}}_b^n$  and  $\mathbf{C}_b^n$  can be described as

$$\hat{\mathbf{C}}_b^n = \mathbf{C}_b^n + \delta \mathbf{C}_b^n \quad (9)$$

$\delta \mathbf{C}_b^n$  is caused by errors in the orientation of the body frame with respect to the navigation frame. In terms of small misalignment angles,  $\delta \mathbf{C}_b^n$  may be represented in the equivalent form of a skew-symmetric matrix

$$\delta \mathbf{C}_b^n = [\boldsymbol{\phi} \times] \mathbf{C}_b^n \quad (10)$$

where  $\boldsymbol{\phi}$  denote the vector of misalignment angles, namely,

$$\boldsymbol{\phi} = [\phi_E \quad \phi_N \quad \phi_U]^T.$$

#### 3.1 System Equation

The mission of fine alignment is to obtain more accurate attitude matrix based on coarse alignment, that is, to estimate precise misalignment angles. In SINS stationary alignment, the misalignment angles remain constant, while no SINS navigation calculation is carried out. Then, the differential equations of misalignment angles can be written as

$$\begin{cases} \dot{\phi}_E = 0 \\ \dot{\phi}_N = 0 \\ \dot{\phi}_U = 0 \end{cases} \quad (11)$$

Besides to determine misalignment angles in the alignment, we also need to estimate the biases of accelerometers and gyros which would be used to compensate the output of IMU during navigation. Here, the biases of accelerometers and gyros are considered as some noise processes which are consist of first order Gauss-Markov noise and Gaussian white noise. Thus, the IMU error model is written as

$$\begin{cases} \hat{f}_{ibx}^b = f_{ibx}^b + \delta f_{ibx}^b = f_{ibx}^b + \nabla_x + w_{ax} \\ \hat{f}_{iby}^b = f_{iby}^b + \delta f_{iby}^b = f_{iby}^b + \nabla_y + w_{ay} \\ \hat{f}_{ibz}^b = f_{ibz}^b + \delta f_{ibz}^b = f_{ibz}^b + \nabla_z + w_{az} \\ \hat{\omega}_{ibx}^b = \omega_{ibx}^b + \delta \omega_{ibx}^b = \omega_{ibx}^b + \varepsilon_x + w_{gx} \\ \hat{\omega}_{iby}^b = \omega_{iby}^b + \delta \omega_{iby}^b = \omega_{iby}^b + \varepsilon_y + w_{gy} \\ \hat{\omega}_{ibz}^b = \omega_{ibz}^b + \delta \omega_{ibz}^b = \omega_{ibz}^b + \varepsilon_z + w_{gz} \end{cases} \quad (12)$$

where

$$\begin{cases} \dot{\nabla}_x = -\beta_{\nabla_x} \nabla_x + w_{\nabla_x} \\ \dot{\nabla}_y = -\beta_{\nabla_y} \nabla_y + w_{\nabla_y} \\ \dot{\nabla}_z = -\beta_{\nabla_z} \nabla_z + w_{\nabla_z} \\ \dot{\varepsilon}_x = -\beta_{\varepsilon_x} \varepsilon_x + w_{\varepsilon_x} \\ \dot{\varepsilon}_y = -\beta_{\varepsilon_y} \varepsilon_y + w_{\varepsilon_y} \\ \dot{\varepsilon}_z = -\beta_{\varepsilon_z} \varepsilon_z + w_{\varepsilon_z} \end{cases} \quad (13)$$

Now, combine (11) and (13), system equation can be written as

$$\dot{X} = FX + W \quad (14)$$

where

$$X = [\phi_E \quad \phi_N \quad \phi_U \quad \nabla_x \quad \nabla_y \quad \nabla_z \quad \varepsilon_x \quad \varepsilon_y \quad \varepsilon_z]^T$$

$$F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\beta_{\nabla_x} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\beta_{\nabla_y} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\beta_{\nabla_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\beta_{\varepsilon_x} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_{\varepsilon_y} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_{\varepsilon_z} \end{bmatrix}$$

$$W = [0 \quad 0 \quad 0 \quad w_{\nabla_x} \quad w_{\nabla_y} \quad w_{\nabla_z} \quad w_{\varepsilon_x} \quad w_{\varepsilon_y} \quad w_{\varepsilon_z}]^T$$

### 3.2 Measurement Equation

Taking care of the error of accelerometers, (5) can be written as

$$\begin{aligned} f_{ib}^n &= C_b^n f_{ib}^b \\ &= (\hat{C}_b^n - \delta C_b^n)(\hat{f}_{ib}^b - \delta f_{ib}^b) \end{aligned}$$

$$= \hat{C}_b^n \hat{f}_{ib}^b - \delta C_b^n \hat{f}_{ib}^b - \hat{C}_b^n \delta f_{ib}^b + \delta C_b^n \delta f_{ib}^b$$

Ignoring the high-order terms, we have

$$\begin{aligned} &= \hat{C}_b^n \hat{f}_{ib}^b - [\phi \times] C_b^n \hat{f}_{ib}^b - \hat{C}_b^n (\nabla + w_a) \\ &= \hat{C}_b^n \hat{f}_{ib}^b - [\phi \times] \hat{f}_{ib}^n - \hat{C}_b^n \nabla - \hat{C}_b^n w_a \\ &= \hat{C}_b^n \hat{f}_{ib}^b + [\hat{f}_{ib}^n \times] \phi - \hat{C}_b^n \nabla - \hat{C}_b^n w_a \end{aligned}$$

Define specific force measurement as

$$Z_f = f_{ib}^n - \hat{C}_b^n \hat{f}_{ib}^b = [\hat{f}_{ib}^n \times] \phi - \hat{C}_b^n \nabla - \hat{C}_b^n w_a \quad (15)$$

In the same way, Taking care of the error of gyro, (6) can be written as

$$\begin{aligned} \omega_{ib}^n &= C_b^n \omega_{ib}^b \\ &= (\hat{C}_b^n - \delta C_b^n)(\hat{\omega}_{ib}^b - \delta \omega_{ib}^b) \\ &= \hat{C}_b^n \hat{\omega}_{ib}^b - \delta C_b^n \hat{\omega}_{ib}^b - \hat{C}_b^n \delta \omega_{ib}^b + \delta C_b^n \delta \omega_{ib}^b \end{aligned}$$

Ignoring the high-order terms, we have

$$\begin{aligned} &= \hat{C}_b^n \hat{\omega}_{ib}^b - [\phi \times] C_b^n \hat{\omega}_{ib}^b - \hat{C}_b^n (\varepsilon + w_g) \\ &= \hat{C}_b^n \hat{\omega}_{ib}^b - [\phi \times] \hat{\omega}_{ib}^n - \hat{C}_b^n \varepsilon - \hat{C}_b^n w_g \\ &= \hat{C}_b^n \hat{\omega}_{ib}^b + [\hat{\omega}_{ib}^n \times] \phi - \hat{C}_b^n \varepsilon - \hat{C}_b^n w_g \end{aligned}$$

Define angular velocity measurement as

$$Z_\omega = \omega_{ib}^n - \hat{C}_b^n \hat{\omega}_{ib}^b = [\hat{\omega}_{ib}^n \times] \phi - \hat{C}_b^n \varepsilon - \hat{C}_b^n w_g \quad (16)$$

Then, combine (15) and (16), measurement equation can be written as

$$Z = HX + V \quad (17)$$

where

$$Z = \begin{bmatrix} Z_f \\ Z_\omega \end{bmatrix} = \begin{bmatrix} f_{ib}^n - \hat{C}_b^n \hat{f}_{ib}^b \\ \omega_{ib}^n - \hat{C}_b^n \hat{\omega}_{ib}^b \end{bmatrix}$$

$$H = \begin{bmatrix} [\hat{f}_{ib}^n \times] & -\hat{C}_b^n & \mathbf{O}_{3 \times 3} \\ [\hat{\omega}_{ib}^n \times] & \mathbf{O}_{3 \times 3} & -\hat{C}_b^n \end{bmatrix}$$

$$V = \begin{bmatrix} w_a \\ w_g \end{bmatrix}$$

### 3.3 Observability Analysis

The accuracy and speed of alignment is decided by the performance of filter, which is decided by the observability of

model. So the observability analysis of model must be performed before filter can commence. The SINS stationary alignment model established in this paper, is a linear time-invariant system whose observability can be obtained by the analysis of the observable matrix. The rank of the observable matrix is

$$\text{rank}(\mathbf{Q}) = \text{rank} \begin{pmatrix} \mathbf{H} \\ \mathbf{HF} \\ \mathbf{HF}^2 \\ \dots \\ \mathbf{HF}^{n-1} \end{pmatrix} = 9$$

It shows that the model is complete observable. And the filter is capable of estimating the state with good performance, therefore, the model will lead to a high accuracy and speed of alignment.

### 3.4 Modified Sage-Husa Adaptive Kalman Filter

Sage-Husa adaptive Kalman filter algorithm proposed by Sage A P and Husa G W, is a filter algorithm which can estimate system noise and measurement noise online in real-time. However, the algorithm could run well under the unknown prior statistical characteristics circumstance. There are some problems with the algorithm, such as, 1) stability and astringency of measurement noise is poor, which affect stability of state estimation and filter result directly, 2) system noise and measurement noise can't be obtained accuracy at the same time, 3) The minus operation would make the matrix of system noisy estimation and the matrix of measure noisy estimation lose half positive or positive, which will make the filter diverge.

In order to solve the problems analyzed as above, a modified Sage-Husa adaptive Kalman filter algorithm is proposed as follow

$$\hat{\mathbf{X}}_k = \hat{\mathbf{X}}_{k|k-1} + \mathbf{K}_k \boldsymbol{\varepsilon}_k \quad (18)$$

$$\boldsymbol{\varepsilon}_k = \mathbf{Z}_k - \mathbf{H}_k \mathbf{X}_{k|k-1} \quad (19)$$

$$\hat{\mathbf{X}}_{k|k-1} = \boldsymbol{\Phi}_{k,k-1} \hat{\mathbf{X}}_{k-1} + \hat{\mathbf{q}}_{k-1} \quad (20)$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T [\mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_{k-1}]^{-1} \quad (21)$$

$$\mathbf{P}_{k|k-1} = \boldsymbol{\Phi}_{k,k-1} \mathbf{P}_{k-1} \boldsymbol{\Phi}_{k,k-1}^T + \hat{\mathbf{Q}}_{k-1} \quad (22)$$

$$\mathbf{P}_k = [\mathbf{I} - \mathbf{K}_k \mathbf{H}_k] \mathbf{P}_{k|k-1} \quad (23)$$

$$\hat{\mathbf{q}}_k = \frac{1}{k} [(k-1)\hat{\mathbf{q}}_{k-1} + \hat{\mathbf{X}}_k - \boldsymbol{\Phi}_{k,k-1} \hat{\mathbf{X}}_{k-1}] \quad (24)$$

$$\hat{\mathbf{Q}}_k = \frac{1}{k} [(k-1)\hat{\mathbf{Q}}_{k-1} + \mathbf{K}_k \boldsymbol{\varepsilon}_k \boldsymbol{\varepsilon}_k^T \mathbf{K}_k^T] \quad (25)$$

## 4. Simulation

During the simulation, parameters of a medium accuracy IMU are used. Its details are shown as following.

Initial misalignment angles	1°
Accelerometers white noise biases	100 μg
Accelerometers first order markov biases	100 μg
Accelerometers first order markov time constant	3600 s
Gyros white noise drift	0.01° h <sup>-1</sup>
Gyros first order markov drift	0.01° h <sup>-1</sup>
Gyros first order markov time constant	3600 s

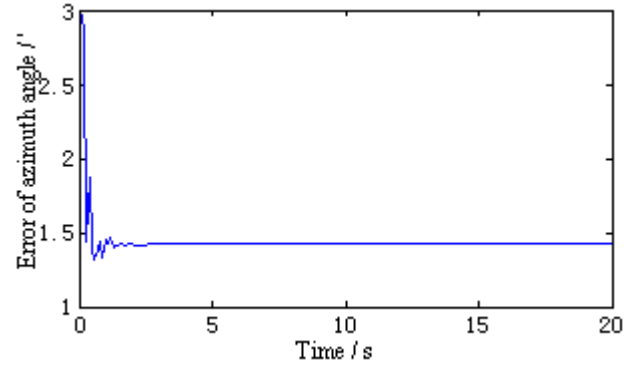


Figure 1 Error of azimuth angle

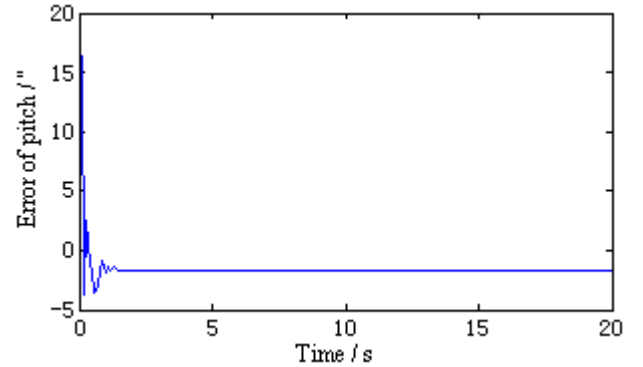


Figure 2 Error of pitch

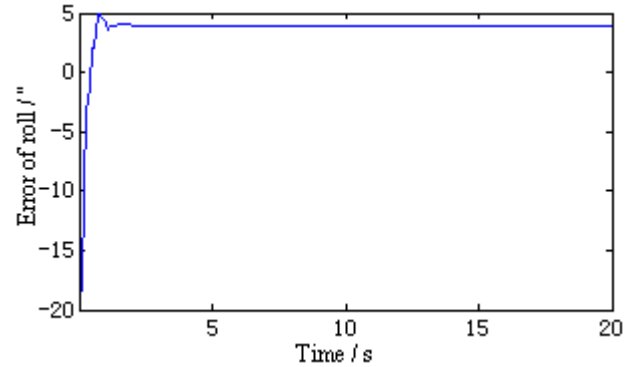


Figure 3 Error of roll

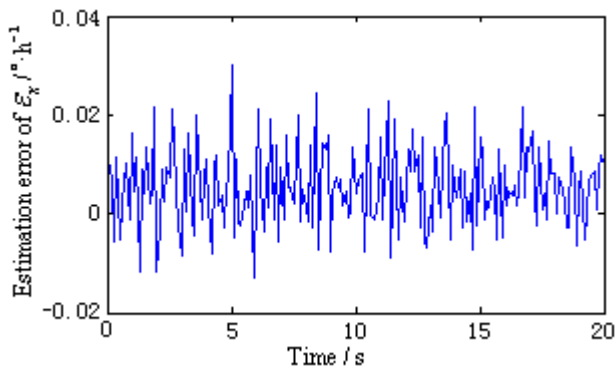


Figure 4 Estimation error of  $\varepsilon_x$

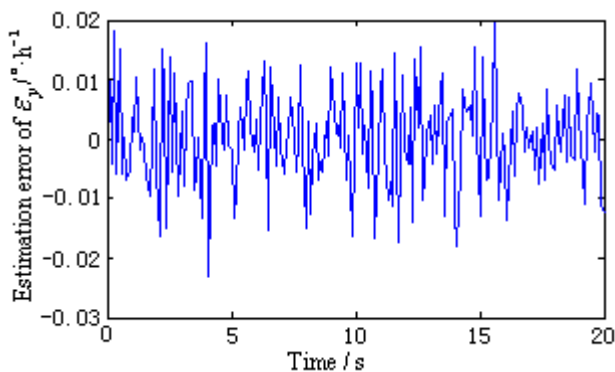


Figure 5 Estimation error of  $\varepsilon_y$

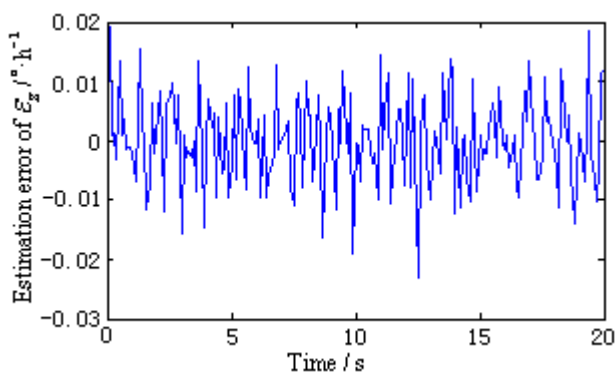


Figure 6 Estimation error of  $\varepsilon_z$

Simulation results are given in figures 1-6, which figures 1-3 are the errors of attitude angle and figures 4-6 show the estimation errors of gyro drift rate.

Figures 1-6 show that the filter works well. After less than 5 s, it has already converged rapidly. According to the computer simulation results, the errors of the two leveling attitude angles are about 5", and the error of the azimuth angle is about 1.5'. Computer simulation results also verify that the optimal time of the three misalignment angles is less than 5 s. Figures 4-6 show that three gyros drift rate are estimated accurately. While the traditional initial alignment has the alignment accuracy of 10" for leveling attitude angles and 2-5' for azimuth angle, with the alignment time of 20-50 s.

## 5. Conclusion

Because of the poor observability of the system, it is hard to give attention to the accuracy and speed of alignment by the traditional initial alignment technology. In order to solve this problem, through analyzing the characteristic of the SINS stationary alignment seriously, a brand-new SINS stationary alignment approach is proposed by means of establishing new system model and measurement model. The new approach could complete the SINS stationary alignment process through the outputs of IMU without any external information.

According to the observability analysis, the alignment model established in this paper is completely observable, which brings the excellent performance of the filter as well as the high accuracy and speed of the alignment. Compared with the traditional initial alignment technology, the new approach has such advantages as follows: it could estimate all states including east gyro drift rate and azimuth misalignment angle accurately in short time; higher alignment accuracy; shorter alignment time; more self-contained and less calculation. Therefore, the new approach is a preferable choice for SINS stationary self-alignment.

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