# Using DGPS as An Acceleration Sensor for Airborne Gravimetry 

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#### Abstract

In airborne gravimetry, there are two data streams. One is the specific force measured by an air/sea gravimeter or accelerometers, the other is kinematic acceleration measured by DGPS. And the difference of them provides the gravity disturbance information. To satisfy the requirement of most applications, an accuracy of $1 \mathrm{mGal}\left(1 \mathrm{mGal}=10^{-5} \mathrm{~m} / \mathrm{s}^{2}\right)$ with a spatial resolution of 1 km is the aim of current airborne gravimetry.

There are two different methods to derive the kinematic acceleration. The generally used method is to differentiate the position twice, and the position can be calculated by commercial DGPS software. The main defect of this method is that integer ambiguities need to be fixed to get the precise position solution, but it's not a trivial thing for long base line. And to fix integer ambiguities, the noisier iono-free measurement is used. When differentiation is applied, noise is amplified and will influence the accuracy of acceleration. The other method is to get carrier phase acceleration by differentiate the carrier phase first, and then using the acceleration of GPS satellite to derive the vehicle acceleration. The main advantages include that fixing integer ambiguities is not needed anymore, position can be relaxed to about 10 meters, and smoother acceleration can be got since iono-free measurement is not needed. In some literatures, it's considered that the dynamic performance of the second method is inferior to that of the first. Through analysis, it is found that the performance degradation in dynamic environment results from the simplification of the GPS carrier phase observable model. And an iterative algorithm is presented to compensate the model error. Using a dynamic GPS data from an aeromagnetic survey, the importance of this compensation is showed at last.


Keywords: Airborne Gravimetry, DGPS, Acceleration.

## 1. Introduction

Airborne gravimetry is a method to measure the Earth's gravity over both inaccessible continental and oceanic regions (margins) with a spatial resolution wavelength $(\lambda)$ ranging from 1 to 10 km , thus filling the gap between land-based $(\lambda<1 \mathrm{~km})$ and spaceborne ( $\lambda>10 \mathrm{~km}$ over oceans and $\lambda>1000 \mathrm{~km}$ over land) techniques, see Verdun et al. (2003). The history of Airborne gravimetry can be traced back to the 50 's of the 20th century, see Thompson (1959), but until the realization of carrier phase DGPS in the late of 1980s that airborne gravimetry made a big progress towards fully operational.

The principle of airborne vector gravimetry is based on the Newton's equation of motion in the gravitational field of the earth. In the local-level frame( n ), the model of airborne vector gravimetry is expressed by the equation, see Bruton (2000)

$$
\begin{equation*}
\delta \mathbf{g}^{n}=\dot{\mathbf{v}}_{e}^{n}-\mathbf{C}_{b}^{n} \mathbf{f}^{b}+\left(2 \boldsymbol{\omega}_{i e}^{n}+\boldsymbol{\omega}_{e n}^{n}\right) \times \mathbf{v}_{e}^{n}-\boldsymbol{\gamma}^{n} \tag{1}
\end{equation*}
$$

Where $\delta \mathbf{g}^{n}$ is the gravity disturbance vector to be determined, $\dot{\mathbf{v}}_{e}^{n}, \mathbf{v}_{e}^{n}$ is the vehicle acceleration and velocity respectively, $\mathbf{C}_{b}^{n}$ is direction cosine matrix from the body frame(b) to the local-level frame $(\mathrm{n}), \mathrm{f}^{b}$ is the specific force in the body frame measured by three accelerometers, $\left(2 \boldsymbol{\omega}_{i e}^{n}+\boldsymbol{\omega}_{e n}^{n}\right) \times \mathbf{v}_{e}^{n}$ is the total of Coriolis and centripetal acceleration, $\gamma^{n}$ is the normal gravity vector.

Eq.(1) expresses the fact that the gravity disturbance vector can be extracted by subtracting specific force from kinematic acceleration after referring both to a common frame, often the
local-level frame. For an airborne scalar gravimetry, only the vertical component of $\delta \mathbf{g}^{n}$ needs to be determined.

Although there are at least three different approaches to measure specific force: air/sea gravimeter, gimbaled inertial navigation system and strapdown inertial navigation system, using carrier phase DGPS to derive kinematic acceleration is nearly the only choice which can satisfy the requirement of airborne gravimetry. From Eq.(1), it can be seen that for the quality of airborne gravimetry, both specific force and kinematic acceleration should have adequate accuracy. To satisfy the requirement of most applications, an accuracy of $1 \mathrm{mGal}\left(1 \mathrm{mGal}=10^{-5} \mathrm{~m} / \mathrm{s}^{2}\right)$ with a spatial resolution of 1 km is the aim of current airborne gravimetry, see Duquenne et al. (2002).
To derive the kinematic acceleration, the commonly used method is to differentiate the position twice, and the position can be calculated by commercial DGPS software, such as Waypoint GrafNav ${ }^{\circledR}$, it also be called as the position method. The main defect of this method is that integer ambiguities need to be fixed to get the precise position solution, but it is not a trivial thing for long base line, see Kennedy (2003). And to fix integer ambiguities, noisier L1/L2 iono-free measurement is used. When differentiation is applied, noise is amplified and will influence the accuracy of acceleration.
The other method is to get carrier phase acceleration by differentiate the carrier phase first, and then using the acceleration of GPS satellite to derive the vehicle acceleration, it also be called as the carrier phase method. The main advantages include that fixing integer ambiguities is not needed anymore, position accuracy can be relaxed to about 10 m , and smoother acceleration can be got since iono-free measurement is not needed, see Jekeli et al. (1997) and Kennedy (2003).

Jekeli tested the carrier phase method using both static and
dynamic data. The baseline length of the static data and dynamic data is about 2.5 km and 1200 km respectively. For static data, the acceleration can be determined to an accuracy of 1 mGal for 40 s averages. But for dynamic data, the result is far from satisfactory, The mean and standard derivation of the difference between the carrier phase method and the position method is 1.05 mGal and 5.18 mGal , see Jekeli et al. (1997). Currently, flying at a constant velocity is required in airborne gravimetry, and data during turning is discarded for lower quality, see Kennedy (2003). In fact, it is hard to guarantee this requirement at all time for the existance of gust, even a autopilot is equipped. So, improving the dynamic performance of acceleration determination is valuable for airborne gravimetry.

Through analysis, it is found that the performance degradation in dynamic environment results from the simplification of the GPS carrier phase observable model. And an iterative algorithm is presented to compensate the model error. Using a dynamic GPS data from an aeromagnetic survey, the importance of this compensation is showed in section 5 .

## 2. Commonly Used Carrier Phase Observable Model

In the following, satellites are indicated in the superscript and receivers in the subscript. The remote receiver is indicated as ' $k$ ', while the base receiver is indicated as ' $m$ '. Bolded quantities denote vectors or matrices. Superscript dots indicate time differentiation.

In Jekeli et al. (1997) and Kennedy (2003), the used carrier phase observable model is

$$
\begin{align*}
\rho_{k}^{j}\left(t_{k}\right)= & R_{k}^{j}\left(t_{k}, t^{j}\right)+c \cdot\left(\delta t_{k}\left(t_{k}\right)-\delta t^{j}\left(t^{j}\right)\right) \\
& -\lambda \cdot N_{k}^{j}\left(t_{0}\right)+\Delta R_{k}^{j}\left(t_{k}, t^{j}\right)+  \tag{2}\\
& \Delta_{k, \text { iono }}^{j}\left(t_{k}\right)+\Delta_{k, \text { trop }}^{j}\left(t_{k}\right)+\Delta_{k, \text { multi }}^{j}\left(t_{k}\right)+\varepsilon_{k}^{j}\left(t_{k}\right)
\end{align*}
$$

Where

| $t_{k}, t^{j}$ | are the epochs of the reception and transmission of the carrier phase; |
| :---: | :---: |
| $\rho_{k}^{j}$ | is the observed phase pseudorange between satellite $j$ |
| $\lambda$ | is the wavelength; |
| c | is the speed of light in vacuum; |
| $R_{k}^{j}$ | is the distance from the satellite $j$ at transmit time to the receiver $k$ at receive time; |
| $N_{k}^{j}$ | is the integer ambiguity number; |
| $\delta t_{k}$ | is the clock error of receiver $k$; |
| $\delta t^{j}$ | is the clock error of satellite $j$; |
| $\Delta R_{k}^{j}$ | is the ephemeris error; |
| $\Delta_{k, \text { iono }}^{j}$ | is the ionospheric delay; |
| $\Delta_{k, \text { trop }}^{j}$ | is the tropspheric delay; |
| $\Delta_{k, \text { multi }}^{j}$ | is the multipath effect; |
| $\varepsilon_{k}^{j}$ | is the remaining noise; |

The first and second time derivatives of the phase pseudorange, called the phase pseudorange rate and pseudorange acceleration, are given by, see Jekeli et al. (1997)

$$
\begin{align*}
& \dot{\rho}_{k}^{j}\left(t_{k}\right)=\dot{R}_{k}^{j}\left(t_{k}, t^{j}\right)+c\left(\delta \dot{t}_{k}\left(t_{k}\right)-\delta \dot{t}^{j}\left(t^{j}\right)\right)+\dot{\varepsilon}_{k}^{j}\left(t_{k}\right)  \tag{3}\\
& \ddot{\rho}_{k}^{j}\left(t_{k}\right)=\ddot{R}_{k}^{j}\left(t_{k}, t^{j}\right)+c\left(\delta \ddot{t}_{k}\left(t_{k}\right)-\delta \ddot{t}^{j}\left(t^{j}\right)\right)+\ddot{\varepsilon}_{k}^{j}\left(t_{k}\right) \tag{4}
\end{align*}
$$

Where
$\dot{R}_{k}^{j} \quad$ is the line-of-sight range rate between receiver $k$ and satellite $j$;
$\ddot{R}_{k}^{j} \quad$ is the line-of-sight range acceleration between receiver $k$ and satellite $j$;

In Eq.(3)(4), the first and second derivatives of the ephemeris error, ionospheric delay, tropospheric delay and multipath effect are lumped into the remaining noise $\dot{\varepsilon}_{k}^{j}$ and $\ddot{\varepsilon}_{k}^{j}$.

To eliminate the receiver clock error, the "single betweensatellite" differenced phase pseudorange rate and pseudorange acceleration can be formed

$$
\begin{align*}
& \dot{\rho}_{k}^{i, j}=\dot{R}_{k}^{i, j}+c\left(\delta t^{i}\left(t^{i}\right)-\delta \dot{t}^{j}\left(t^{j}\right)\right)+\dot{\varepsilon}_{k}^{i, j}  \tag{5}\\
& \ddot{\rho}_{k}^{i, j}=\ddot{R}_{k}^{i, j}+c\left(\delta \ddot{t}^{i}\left(t^{i}\right)-\delta \ddot{t}^{j}\left(t^{j}\right)\right)+\ddot{\varepsilon}_{k}^{i, j} \tag{6}
\end{align*}
$$

In Eq.(5)(6), the time arguments have been omitted for simplicity. To further eliminate the satellite clock error, double differenced phase pseudorange rate and pseudorange acceleration can be formed

$$
\begin{align*}
& \dot{\rho}_{m, k}^{i, j}=\dot{R}_{m, k}^{i, j}+\dot{\varepsilon}_{m, k}^{i, j}  \tag{7}\\
& \ddot{\rho}_{m, k}^{i, j}=\ddot{R}_{m, k}^{i, j}+\ddot{\varepsilon}_{m, k}^{i, j} \tag{8}
\end{align*}
$$

In the derivation of the double differenced phase pseudorange rate and pseudorange acceleration, one subtle term that the satellite phases not arriving at the two receivers simultaneously for the path lengths are different is omitted. Jekeli analysized the effect of the term on the double differenced phase pseudorange, and thought that it can be ignored for a baseline length of 100 km , see Jekeli et al. (1997). And satellite positions, velocities and accelerations at every second are used to derive the acceleration of the vehicle, see Kennedy (2003).

Practically, to ignore the term is reasonable for relative positioning when baseline is shorter than 100 km . But is it really reasonable for the determination of acceleration? In the next section, this problem will be analysized.

## 3. Error of Simplified Carrier Phase Observable Model

### 3.1 Carrier Phase Observable Model

Eq.(2) can be expressed as the function of $t_{k}$, the reception epoch of the carrier phase, see Zhang (2004)

$$
\begin{align*}
\rho_{k}^{j}(t)= & R_{k}^{j}(t)\left[1-\frac{1}{c} \dot{R}_{k}^{j}(t)\right]+c\left[1-\frac{1}{c} \dot{R}_{k}^{j}(t)\right] \delta t_{k}(t) \\
& +\frac{1}{2} \ddot{R}_{k}^{j}(t) \cdot\left[\frac{1}{c} R_{k}^{j}(t)\right]^{2}-c \cdot \delta t^{j}(t)  \tag{9}\\
& -\lambda \cdot N_{k}^{j}\left(t_{0}\right)+\Delta R_{k}^{j}(t)+\Delta_{k, \text { iono }}^{j}(t) \\
& +\Delta_{k, \text { trop }}^{j}(t)+\Delta_{k, \text { multi }}^{j}(t)+\varepsilon_{k}^{j}(t)
\end{align*}
$$

Where, $t_{k}$ is simplified as $t$.
Eq.(9) can be rearranged as

$$
\begin{align*}
& \rho_{k}^{\prime j}(t)=\rho_{k}^{j}(t)+\delta \rho_{k}^{j}(t) \\
& \quad=R_{k}^{j}(t)+c \cdot\left(\delta t_{k}(t)-\delta t^{j}(t)\right)-\lambda \cdot N_{k}^{j}\left(t_{0}\right)+\Delta R_{k}^{j}(t)  \tag{10}\\
& \quad+\Delta_{k, \text { iono }}^{j}(t)+\Delta_{k, \text { trop }}^{j}(t)+\Delta_{k, \text { multi }}^{j}(t)+\varepsilon_{k}^{j}(t)
\end{align*}
$$

Where

$$
\begin{align*}
\delta \rho_{k}^{j}(t) & =\frac{1}{c} R_{k}^{j}(t) \dot{R}_{k}^{j}(t)+\dot{R}_{k}^{j}(t) \delta t_{k}(t) \\
& -\frac{1}{2} \ddot{R}_{k}^{j}(t) \cdot\left[\frac{1}{c} R_{k}^{j}(t)\right]^{2} \tag{11}
\end{align*}
$$

Comparing Eq.(2) and Eq.(10), they are the same from the form. $\delta \rho_{k}^{j}(t)$ can be treated as phase pseudorange error coming from the simplification of observable model. The first and second time derivatives of $\delta \rho_{k}^{j}(t)$ are phase pseudorange rate error $\delta \dot{\rho}_{k}^{j}(t)$ and pseudorange acceleration error $\delta \ddot{\rho}_{k}^{j}(t)$.

We will study the three terms on the right side of Eq.(11) respectively.

### 3.2 Testing Data

We use one static data and one dynamic data to test the magnitude of $\delta \rho_{k}^{j}(t), \delta \dot{\rho}_{k}^{j}(t)$ and $\delta \ddot{\rho}_{k}^{j}(t)$. The baseline length of the static data is about 22.9 km . The dynamic data is coming from an aeromagnetic survey. Both are provided by AGRS (Aerogeophysical Survey and Remote Sensing Center, China). Figure 1 shows the horizontal profile of the dynamic data, the red star * denotes the start point. Figure 2 and 3 show the curves of position and velocity respectively. Figure 4 shows the separation between master and remote receiver.


Figure 1. Horizontal profile of dynamic data


Figure 2. The curves of position


Figure 3. The curves of velocity


Figure 4. Separation between master and remote

### 3.3 Error of The First Term

Figure 5 shows the double differenced phase pseudorange error, pseudorange rate error and pseudorange acceleration error for the static data. The forth curve in Figure 5 is the phase pseudorange acceleration error after 120s low pass filtering. From this figure, we can see that the error of the first term is small enough to be ignored.


Figure 5. Errors of the first term(static data)
Figure 6 shows the double differenced errors for dynamic data. From the first curve of Figure 6 and Figure 4, we can see that the phase pseudorange error is related to the separation between master and remote. From Figure 6 and Figure 2~3, we can see that the phase pseudorange rate and pseudorange acceleration errors are very small when the plane is flying at a constant, but the errors will be enlarged more than 100 times during maneuvers. The reason is that the motion of the remote receiver is unrelated to the master receiver, so the errors can not be weakened through difference processing.


Figure 6. Errors of the first term(dynamic data)

### 3.4 Error of The Second Term

The clock error $\delta t_{k}$ of a NovAtel OEM 4 receiver is less than 200 ns , see Kennedy (2003). Here, we use the maximum, ie, $\delta t_{k}=200 \mathrm{~ns}$.

Figure 7 shows the errors between remote receiver and satellite $\mathrm{PRN}=1$, dynamic data is used. From this figure, we can see that the errors of the second term can be ignored.


Figure 7. Errors of the second term(dynamic data)

### 3.5 Error of The Third Term

Figure 8 shows the errors between remote receiver and satellite $\mathrm{PRN}=1$ for static data. It can be seen that the errors of the third term can be ignored.

Figure 9 shows the double differenced errors of the third term for dynamic data. Although the errors are much smaller than those of the first term, they can not be ignored for high accuracy acceleration determination.

Through the above analysis, three conclusions can be summarized as follows:
(1) the phase pseudorange acceleration error of the first term can reach several hundred mGal during maneuvers, it must be compensated to derive the vehicle acceleration.
(2) the pseudorange acceleration error of the third term can reach several mGal during maneuvers, it also needs to compen-
sated to improve the accuracy of vehicle acceleration.
(3) the pseudorange acceleration error of the second term is small enough to be ignored.

In the next section, we will discuss how to determine the vehicle acceleration using the carrier phase observables.


Figure 8. Errors of the third term(static data)


Figure 9. Error of the third term(dynamic data)

## 4. Math Model of Acceleration Determination

The elaborate derivation of the algorithm of the carrier phase method can be found in Jekeli et al. (1997) and Kennedy (2003). Here, only main equations are presented.

The basic function to determine vehicle acceleration is

$$
\begin{equation*}
\ddot{R}_{k}^{j}=\mathbf{e}_{k}^{j} \cdot\left(\ddot{\mathbf{x}}^{j}-\ddot{\mathbf{x}}_{k}\right)+\frac{1}{R_{k}^{j}}\left[\left|\dot{\mathbf{x}}_{k}^{j}\right|^{2}-\left(\dot{R}_{k}^{j}\right)^{2}\right] \tag{12}
\end{equation*}
$$

Where
$\ddot{R}_{k}^{j} \quad$ is the line-of-sight range acceleration between receiver $k$ and satellite $j$;
$\dot{R}_{k}^{j} \quad$ is the line-of-sight range rate between receiver $k$ and satellite $j$;
$R_{k}^{j} \quad$ is the line-of-sight range between receiver $k$ and satellite $j$;
$\mathbf{e}_{k}^{j} \quad$ is the unit direction vector between receiver k and
satellite j ;
$\ddot{\mathbf{x}}^{j} \quad$ is the acceleration of satellite $j$
$\dot{\mathbf{x}}_{k}^{j} \quad$ is the relative velocity between receiver k and satellite j;
$\ddot{\mathbf{x}}_{k} \quad$ is the acceleration of receiver $k$ to be determined.

Except for the receiver acceleration, the remaining unknown in Eq.(12) is the satellite-receiver relative velocity. It can be solved using the equation.

$$
\begin{equation*}
\dot{R}_{k}^{j}=\mathbf{e}_{k}^{j} \cdot \dot{\mathbf{x}}_{k}^{j} \tag{13}
\end{equation*}
$$

The "single between-satellite" differenced pseudorange rate(Eq.(5)) or double differenced pseudorange rate(Eq.(7)) can be used to solve for relative velocity $\dot{\mathbf{x}}_{k}^{j}$.

Now we use double differenced pseudorange acceleration(Eq.(8)) to solve for receiver acceleration $\ddot{\mathbf{x}}_{k}$. Combining Eq.(12) and Eq.(8), the system equation for satellite $i$, satellite $j$, remote receiver $k$ and master receiver $m$ is

$$
\begin{align*}
&\left(\mathbf{e}_{k}^{i}-\mathbf{e}_{k}^{j}\right) \cdot \ddot{\mathbf{x}}_{k}-\left(\mathbf{e}_{k}^{i} \cdot \ddot{\mathbf{x}}^{i}-\mathbf{e}_{k}^{j} \cdot \ddot{\mathbf{x}}^{j}\right) \\
&-\left(\frac{1}{R_{k}^{i}}\left[\left|\dot{\mathbf{x}}_{k}^{i}\right|^{2}-\left(\dot{R}_{k}^{i}\right)^{2}\right]-\frac{1}{R_{k}^{j}}\left[\left|\dot{\mathbf{x}}_{k}^{j}\right|^{2}-\left(\dot{R}_{k}^{j}\right)^{2}\right]\right)+\left(\mathbf{e}_{m}^{i} \cdot \ddot{\mathbf{x}}^{i}-\mathbf{e}_{m}^{j} \cdot \ddot{\mathbf{x}}^{j}\right)  \tag{14}\\
&+\left(\frac{1}{R_{m}^{i}}\left[\left|\dot{\mathbf{x}}^{i}\right|^{2}-\left(\dot{R}_{m}^{i}\right)^{2}\right]-\frac{1}{R_{m}^{j}}\left[\left|\dot{\mathbf{x}}^{j}\right|^{2}-\left(\dot{R}_{m}^{j}\right)^{2}\right]\right)+\ddot{\rho}_{k, m}^{i, j}(t)=\ddot{\varepsilon}_{k, m}^{i, j}
\end{align*}
$$

To solve for receiver acceleration $\ddot{\mathbf{x}}_{k}$, a minimum of four satellites is required. For a total of $n$ satellites, Eq.(14) can be written as a matrix form

$$
\begin{equation*}
\mathbf{V}=\mathbf{A} \ddot{\mathbf{x}}_{k}-\mathbf{L} \tag{15}
\end{equation*}
$$

Where

$$
\begin{aligned}
& \mathbf{V}=\left[\begin{array}{llll}
\ddot{\varepsilon}_{k, m}^{1,2} & \ddot{\varepsilon}_{k, m}^{1,3} & \cdots & \ddot{\varepsilon}_{k, m}^{1, n}
\end{array}\right]^{T} \\
& \ddot{\mathbf{x}}_{k}=\left[\begin{array}{lll}
\ddot{\mathrm{x}}_{k x} & \ddot{\mathrm{x}}_{k y} & \ddot{\mathrm{x}}_{k z}
\end{array}\right]^{T} \\
& \mathbf{A}=\left[\begin{array}{ccc}
\mathbf{e}_{k x}^{1}-\mathbf{e}_{k x}^{2} & \mathbf{e}_{k y}^{1}-\mathbf{e}_{k y}^{2} & \mathbf{e}_{k z}^{1}-\mathbf{e}_{k z}^{2} \\
\mathbf{e}_{k x}^{1}-\mathbf{e}_{k x}^{3} & \mathbf{e}_{k y}^{1}-\mathbf{e}_{k y}^{3} & \mathbf{e}_{k z}^{1}-\mathbf{e}_{k z}^{3} \\
\vdots & \vdots & \vdots \\
\mathbf{e}_{k x}^{1}-\mathbf{e}_{k x}^{n} & \mathbf{e}_{k y}^{1}-\mathbf{e}_{k y}^{n} & \mathbf{e}_{k z}^{1}-\mathbf{e}_{k z}^{n}
\end{array}\right] \\
& {\left[\left(\mathbf{e}_{k}^{1} \cdot \ddot{\mathbf{x}}^{1}-\mathbf{e}_{k}^{2} \cdot \ddot{\mathbf{x}}^{2}\right)+\left(\frac{1}{R_{k}^{1}}\left[\left|\dot{\mathbf{x}}_{k}^{1}\right|^{2}-\left(\dot{R}_{k}^{1}\right)^{2}\right]-\frac{1}{R_{k}^{2}}\left[\left|\dot{\mathbf{x}}_{k}^{2}\right|^{2}-\left(\dot{R}_{k}^{2}\right)^{2}\right]\right)\right.} \\
& -\left(\mathbf{e}_{m}^{1} \cdot \ddot{\mathbf{x}}^{1}-\mathbf{e}_{m}^{2} \cdot \ddot{\mathbf{x}}^{2}\right)-\left(\frac{1}{R_{m}^{1}}\left[\left|\dot{\mathbf{x}}^{1}\right|^{2}-\left(\dot{R}_{m}^{1}\right)^{2}\right]-\frac{1}{R_{m}^{2}}\left[\left|\dot{\mathbf{x}}^{2}\right|^{2}-\left(\dot{R}_{m}^{2}\right)^{2}\right]\right)-\ddot{\rho}_{k, m}^{1,2}(t) \\
& \left(\mathbf{e}_{k}^{1} \cdot \ddot{\mathbf{x}}^{1}-\mathbf{e}_{k}^{3} \cdot \ddot{\mathbf{x}}^{3}\right)+\left(\frac{1}{R_{k}^{1}}\left[\left|\dot{\mathbf{x}}_{k}^{1}\right|^{2}-\left(\dot{R}_{k}^{1}\right)^{2}\right]-\frac{1}{R_{k}^{3}}\left[\left|\dot{\mathbf{x}}_{k}^{3}\right|^{2}-\left(\dot{R}_{k}^{3}\right)^{2}\right]\right) \\
& \mathbf{L}=\quad-\left(\mathbf{e}_{m}^{1} \cdot \ddot{\mathbf{x}}^{1}-\mathbf{e}_{m}^{3} \cdot \ddot{\mathbf{x}}^{3}\right)-\left(\frac{1}{R_{m}^{1}}\left[\left|\dot{\mathbf{x}}^{1}\right|^{2}-\left(\dot{R}_{m}^{1}\right)^{2}\right]-\frac{1}{R_{m}^{2}}\left[\left|\dot{\mathbf{x}}^{2}\right|^{2}-\left(\dot{R}_{m}^{2}\right)^{2}\right]\right)-\ddot{\rho}_{k, m}^{1,3}(t) \\
& \left(\mathbf{e}_{k}^{1} \cdot \ddot{\mathbf{x}}^{1}-\mathbf{e}_{k}^{n} \cdot \ddot{\mathbf{x}}^{n}\right)+\left(\frac{1}{R_{k}^{1}}\left[\left|\dot{\mathbf{x}}_{k}^{1}\right|^{2}-\left(\dot{R}_{k}^{1}\right)^{2}\right]-\frac{1}{R_{k}^{n}}\left[\left|\dot{\mathbf{x}}_{k}^{n}\right|^{2}-\left(\dot{R}_{k}^{n}\right)^{2}\right]\right) \\
& \left.-\left(\mathbf{e}_{m}^{1} \cdot \ddot{\mathbf{x}}^{1}-\mathbf{e}_{m}^{n} \cdot \ddot{\mathbf{x}}^{n}\right)-\left(\frac{1}{R_{m}^{1}}\left[\left|\dot{\mathbf{x}}^{1}\right|^{2}-\left(\dot{R}_{m}^{1}\right)^{2}\right]-\frac{1}{R_{m}^{n}}\left[\left|\dot{\mathbf{x}}^{n}\right|^{2}-\left(\dot{R}_{m}^{n}\right)^{2}\right]\right)-\ddot{\rho}_{k, m}^{1, n}(t)\right]
\end{aligned}
$$

The least square estimation for receiver acceleration is

$$
\begin{equation*}
\ddot{\mathbf{x}}_{k}=\left(\mathbf{A}^{\mathrm{T}} \cdot \mathbf{A}\right)^{-1} \cdot \mathbf{A} \cdot \mathbf{L} \tag{16}
\end{equation*}
$$

## 5. Analysis in Acceleration Domain

In this section, we will use both the static data and dynamic data to evaluate the effect of the compensation of model error. For Eq.(9) is a implicit equation, the unknown line-of-sight range rate and acceleration present on the right side of Eq.(9), so an iterative algorithm is used to compensate the model error $\delta \rho_{k}^{j}(t)$. At the initial step(no iterative), $\delta \rho_{k}^{j}(t)$ is set to zero, that is equivalent to Eq.(2); Then at the first iteration step, the calculated receiver velocity and acceleration can be used to compensate for the model error; And it is similar for the next steps.
For static data, the acceleration of the remote receiver is zero and this provides a truth-value. The statistics of the acceleration error are listed in Table 1.

Table 1. Statistics of the error(static data )

| iteration <br> step | Mean Value(mGal) |  |  | Standard Deviation(mGal) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | North | East | Down | North | East | Down |
| 0 | 0.054 | 0.038 | -0.121 | 0.690 | 0.734 | 1.715 |
| 1 | 0.054 | 0.038 | -0.121 | 0.689 | 0.739 | 1.715 |
| 2 | 0.054 | 0.037 | -0.121 | 0.690 | 0.738 | 1.715 |

It can be seen that the acceleration error due to the model simplification is very small, that is to say the adoption of Eq.(2) is valid for static data. The curves of the first iteration step are shown in Figure 10.


Figure 10. Acceleration error of static data

For dynamic data, there is no standard value for comparison. Acceleration computed from the position method is used as comparison values. Identical differentiators and low pass filters are applied to both the carrier phase method and the position method to get comparable accelerations.

The statistics of the difference between the carrier phase method and the position method are listed in Table 2. The curves when no iteration is used are shown in Figure 11. The curves of the first iteration step are show in Figure 12. It should be noted that the whole data including turning sections is used for comparison.

Table 2. Statistics of the difference(dynamic data )

| iteration <br> step | Mean Value(mGal) |  |  | Standard ${ }^{3}$ Deviation(mGal) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | North | East | Down | North | East | Down |
| 0 | -0.093 | -0.148 | 0.114 | 2.968 | 2.428 | 5.586 |
| 1 | 0.273 | 0.489 | 2.631 | 140.0 | 184.8 | 20.48 |
| 2 | 0.270 | 0.492 | 2.643 | 140.0 | 184.9 | 20.48 |
| 3 | 0.271 | 0.499 | 2.641 | 140.0 | 184.9 | 20.49 |

At first glance, the difference between the two methods seems to be amplified by the iterative algorithm. There may be two possible but opposite reasons. One is the foregoing analysis is wrong, the other is that neither the position method nor the carrier phase method using simplified model can work properly for dynamic data. Care should be taken on Table 2, the difference converges to constant value after the first iteration step. Supposing that the first reason is tenable, the difference should not converge. So we can concluded that the second reason is the fact. It seems that the horizontal components are more sensitive to the simplification of observable model, it can be explained that the maneuvers are often done in the horizontal plane.

To further validate this conclusion, multi master station dynamic data should be used to test the internal consistency between different remote-master combinations.


Figure 11. Difference between two methods(no iteration)


Figure 12. Difference between two methods $\left(1^{\text {st }}\right.$ iteration $)$

## 6. Conclusion

The determination of the vehicle kinematic acceleration is one of the key issues in airborne gravimetry. Currently, carrier phase DGPS is widely used to get the vehicle kinematic acceleration. The commonly used method is to twice differentiate the position calculated by commercial DGPS software. The other is called the carrier phase method, which has many advantages over the former, such as fixing integer ambiguities is not needed anymore, position accuracy can be relaxed to about 10 m , etc.

In this paper, the carrier phase observable model for acceleration determination is analysized. It is found the commonly used simplified model can not work properly in dynamic environment. An iterative algorithm is used to compensate the model error.

Multi master station dynamic data should be used to the test the validity of the new algorithm in the future work.

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