

# SBAS SIGNAL SYNCHRONIZATION

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## Abstract

In general DGPS system, the correction message is transferred to users by wireless modem. To cover wide area, many DGPS station should be needed. And DGPS users must have a wireless modem that is not necessary in stand-alone GPS. But SBAS users don't need a wireless modem to receive DGPS corrections because SBAS correction message is transmitted from the GEO satellite by L1 frequency band.

SBAS signal is generated in the GUS(Geo Uplink Subsystem) and uplink to the GEO satellite. This uplink transmission process causes two problems that are not existed in GPS. The one is a time delay in the uplink signal. The other is an ionospheric problem on uplink signal, code delay and carrier phase advance. These two problems cause ranging error to user. Another critical ranging error factor is clock synchronization. SBAS reference clock must be synchronized with GPS clock for an accurate ranging service.

The time delay can be removed by close loop control. We propose uplink ionospheric error correcting algorithm for C/A code and carrier. As a result, the ranging accuracy increased high. To synchronize SBAS reference clock with GPS clock, I reviewed synchronization algorithm. And I modified it because the algorithm didn't consider doppler that caused by satellites' dynamics. SBAS reference clock synchronized with GPS clock in high accuracy by modified algorithm. We think that this paper will contribute to basic research for constructing satellite based DGPS system.

**Keywords: DGPS, SBAS, Clock synchronization**

## Notation

d : Geodetic distance  
r : Measured range  
t : Time  
 $\phi$  : Carrier phases  
N : Integer cycle ambiguity  
 $\rho$  : Pseudo range  
I : Ionospheric delay  
T : Tropospheric delay  
 $\epsilon$  : other kind error  
x : State vector  
z : Observer vector  
H : State transition matrix  
Q : Process noise covariance matrix  
R : Measurement noise covariance matrix  
C : Control gain vector  
B : Receiver clock bias  
b : Satellite clock bias  
 $\alpha$  : Conversion constants from L1 ionospheric delay to C1 delay

## 1. Introduction

At the first time GPS is developed for the military purpose. The usage of civilian is getting larger because of the distinguished usability of GPS. GPS users are can get navigation data at instance in easy way. But the navigation data contains errors caused by the ranging method using satellites, so stand alone users are can only get about meter level accuracy. This

problem can be solved by using DGPS error correction message. If using DGPS error correction message users can get about centimeter level accuracy. In general case users are needs wireless data link to use DGPS error correction message. To solve this troublesome SBAS was developed. SBAS is sending DGPS error correction message by L1 signal. So users do not needs additional wireless data link instead receiving error correction data by GPS antenna. Besides SBAS sending data by L1 signal, users use it for additional pseudo range. SBAS GEO satellite functions just like GPS satellite. But the SBAS signal is different from GPS signal. GPS signal is generated in the GPS satellite but SBAS signal is generated in the ground station and uplinked to the GEO satellite. So SBAS signal is delayed on uplink. This delay must be compensated so that users can use SBAS signal without problem.

## 2. SBAS Summary

### 2.1 Components of SBAS

SBAS signal is generated in the ground station and then uplinked to the GEO satellite. So SBAS is needs following system components.

#### 2.1.1 GUS(GEO Uplink Subsystem)

Correction data is send from WMS to GUS. After then SBAS signal is generated in the GUS and uplinked to the GEO satellite. GUS contains signal generator and receiver.

#### 2.1.2 GEO Satellite

GEO satellite maintains high elevation angle so it provides low DOP to users. GEO satellite does not generate signal but transponders it. So GEO satellite is called 'bent-pipe' or 'transponder'. This is a main difference between GEO satellite and GPS satellite.

### 2.1.3 WRS(Wide-area Reference Station)

WRS collecting GPS signal which is used to make DGPS error collection data. The collected data is forwarded to WMS. WRS is strategically located to covers wide area.

### 2.1.4 WMS(Wide area Master station)

In WMS received GPS data from WRS and processed it to make DGPS error correction message.

## 2.2 Signal Characteristic

SBAS signal is generated in C band and converted into L1 band in the GEO satellite.

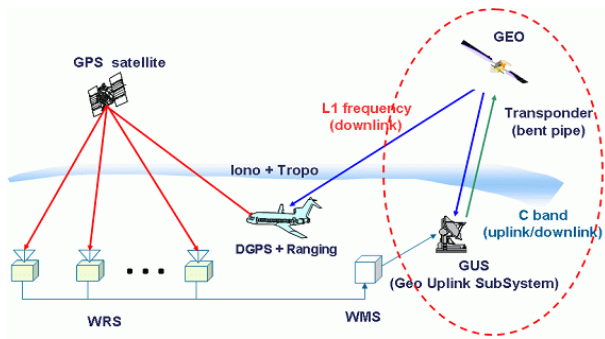
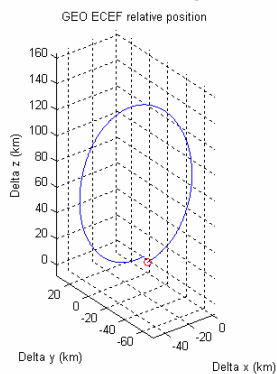


Figure 1. SBAS overview

## 3. Range error components

### 3.1 Satellite ephemeris error

SBAS signal is generated in the GUS and then, uplinked to the GEO satellite. In the GEO satellite, the signal is converted into L1 signal and then, downlinked to user. In user's point of view, SBAS signal contains range error; GUS to GEO satellite distance. So SBAS signal had to be compensated error that is GUS to GEO satellite distance. GEO satellite does not maintain fixed position so the distance is not constant. The distance should be controlled by feedback control algorithm.



### 3.2 Ionospheric error

SBAS signal is generated in the GUS then uplinked to the GEO

satellite. In the satellite the signal is converted into L1 band then downlinked. So, SBAS signal passes ionosphere twice. The ionospheric delay, generated in uplink time, is additional error bias to users. When signal is passing ionosphere C/A code is elayed and carrier advanced. The C/A code and carrier delay is about same magnitude with different sign. In this paper Klobuهار model is used

$$i = I(I) \cdot Q(\theta)$$

$$I = A_1 + A_2 \cos\left[\frac{2\pi(\tau - A_3)}{A_4}\right]$$

$\tau$ : Local time

$A_1 = 5 \times 10^{-9}$  (s) (Night time constant)

$$A_2 = \alpha_1 + \alpha_2 \phi_M + \alpha_3 \phi_M^2 + \alpha_4 \phi_M^3$$

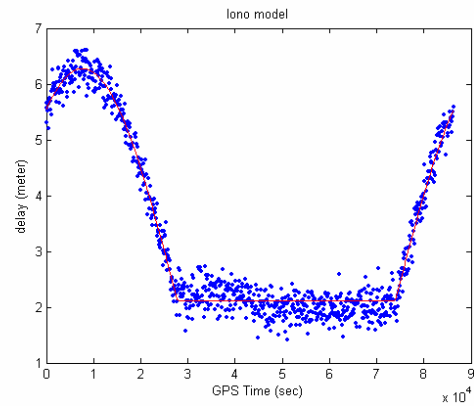
$A_3 = 14:00$  (Local peak time)

$$A_4 = \beta_1 + \beta_2 \phi_M + \beta_3 \phi_M^2 + \beta_4 \phi_M^3$$
 (Period)

$\phi_M$ : Geomagnetic latitude of ionospheric pierce point

$\alpha_i, \beta_i$ : parameters

$$I: [\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4]^T$$



### 3.3 Tropospheric error

Tropospheric error, generated when uplink, is also additional error source to users compare with GPS signal. Tropospheric error can be modeled by Black's model. By contrast to ionospheric error tropospheric error is same through C/A code and carrier.

$$\Delta \rho_{Trop} = \int (n-1) dh$$

$n$ : Refractive index

$h$ : Height of the troposphere

$(n-1)$ : Refractive

Refractivity

$$N = (n-1)10^6$$

$$N = 77.6 \frac{P}{T} + 3.73 \cdot 10^5 \frac{e}{T^2}$$

$P$  = pressure

$T$ : temperature

$e$ : partial water vapour pressure

### 3.4 Clock error

To use SBAS signal for ranging service, SBAS reference time must be synchronized with GPS time. If SBAS and GPS time are not synchronized then pseudo range does not correct. So clock synchronization is the most important part.

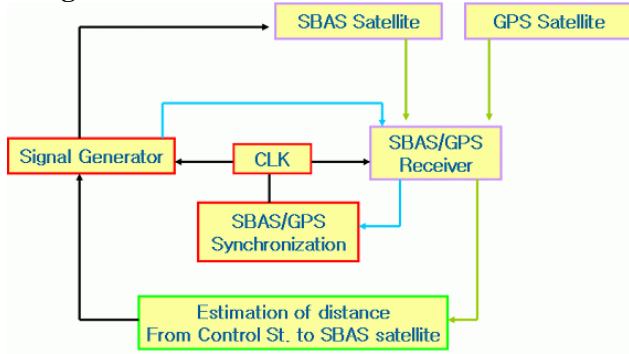
### 3.5 Inter frequency bias

SBAS using two different signals those are L1/L5. When uplinked, the signals are converted into L1/L5 signal in the GEO satellite. Delay appeared in the signal path when it converted. L1 and L5 signal passes different signal path so delay is different L1/L5.

### 3.6 Receiver error

When receiver processing the GPS and SBAS signal, noise can be included the signal. This noise dilutes the pseudo range accuracy. Receiver noise is not constant. It is depends hardware structure and environment.

## 4. Signal Control



### 4.1 C/A code Control

The range delay and ionospheric delay which are generated in uplink must be compensated using feedback control loop. In GPS case the measured range is like below. But in SBAS the measured range contains additional delays which are range delay and ionospheric delay.

$$\rho_{GPS} = d + I_{down} + T_{down} + \varepsilon_{\rho}$$

$$\rho_{SBAS} = d_{SBAS} + I_{up} + T_{up} + d + I_{down} + T_{down} + \varepsilon_{\rho}$$

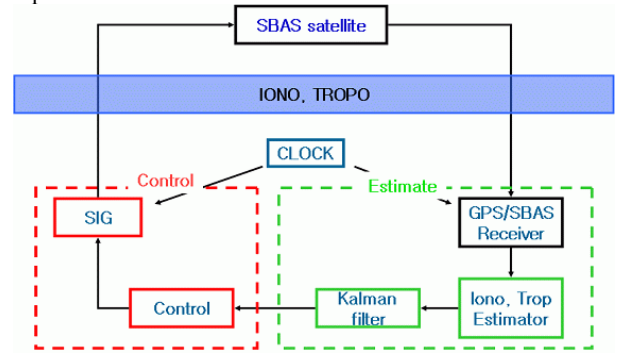
#### 4.1.1 Ionospheric delay control

Ionosphere delays code phase and advance carrier phase. In this paper we assume the ionospheric delay can be measure precisely by dual frequency (L1/L2) receiver and use that value for compensate SBAS signal delay.

#### 4.1.2 Code phase delay control

The feedback control for compensate GUS to the GEO satellite distance delay using SBAS signal. First time guess GUS to the GEO satellite distance and generate signal advanced the guessed value. After then receiving the signal by receiver, compare it with guessed value. If guessed value and receiving values are different then the difference are input to the feedback control

loop.



To compensate GUS to GEO satellite distance, two algorithms are used. First part is estimate the pseudo range and second is error compensation feedback control part. The estimate parameter is GUS to GEO satellite distance, rate and acceleration. The state vector is like below.

To estimate the state value Kalman filter is used. Time and measurement update equation is like below.

$$\Phi = \begin{bmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \quad x = \begin{bmatrix} r_s \\ r_s' \\ r_s'' \end{bmatrix}$$

$$\hat{x}_k^- = \Phi \hat{x}_k$$

$$P_k^- = \Phi P_{k-1} \Phi^T + b Q b^T$$

$$z = Hx + v = [2 \quad 0 \quad 0]x + v$$

$$K_k = P_k^- + K_k (z_k - H \hat{x}_k^-)$$

$$\hat{x}_k = \hat{x}_k^- + K_k (z_k - H \hat{x}_k^-)$$

$$P_k = (I - K_k H) P_k^-$$

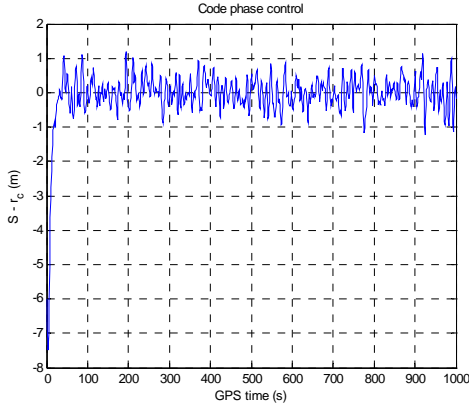
In feedback control loop uses the estimated state.

$$Control(k+1) = \begin{bmatrix} 1 & 1 & \frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} Control(k) + \begin{bmatrix} \frac{1}{6} \\ \frac{1}{2} \\ 1 \end{bmatrix} C \varepsilon$$

$$Control = \begin{bmatrix} r_c' \\ r_c'' \\ r_c \end{bmatrix} \quad \varepsilon = \begin{bmatrix} d_s' - r_c' \\ d_s'' - r_c'' \\ d_s - r_c \end{bmatrix}$$

$C$  = Control gain

The simulation result is like below.



The RMS value is 0.42425(m) is in 200~1000(s).

## 4.2 Carrier phase control

In uplink time, carrier phase also delayed like code phase. And ionosphere advances carrier phase. If do not compensate it users are can not tracking carrier signal correct. The carrier phase ranging measurement equation is like below.

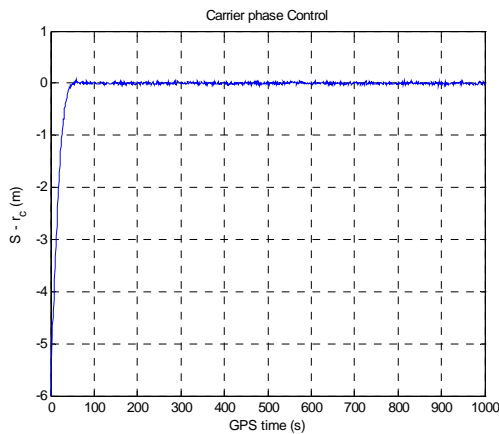
$$\begin{aligned}\phi_{GPS} &= d - I_{down} + T_{down} + N\lambda + \varepsilon_{\phi} \\ \phi_{SBAS} &= d_{SBAS} - I_{up} + T_{up} \\ &\quad + d - I_{down} + T_{down} + N\lambda + \varepsilon_{\phi}\end{aligned}$$

### 4.2.1 Ionospheric control

Ionosphere delays code phase and advance carrier phase. Like code phase control loop we assume that the ionospheric delay can be measure using dual frequency receiver and use that value for carrier phase control loop.

### 4.2.2 Carrier phase delay control

Carrier phase delay control loop use the same algorithm like code phase control loop. The simulation result is like below.



The RMS value is 0.01672(m) in 200~1000(s).

## 5. Clock Synchronization

### 5.1 Clock modeling

GPS satellite use atomic clock because to get highly precise time. SBAS reference clock also uses atomic clock but there exist clock bias still GPS clock and SBAS clock. The clock bias is time varying function. This bias must be compensated to get accurate range. And SBAS clock and GPS clock must be synchronize precisely.

### 5.2 Clock synchronization

Clock can be modeling using Allan variance parameter. The clock modeling equation is 2nd order Markov sequence.

$$\begin{bmatrix} phase \\ freq \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} phase \\ freq \end{bmatrix}_k + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}_k$$

$$E[WW^T] = \begin{bmatrix} \frac{h_0}{2} \Delta t + 2h_{-1} \Delta t^2 + \frac{2}{3} \pi^2 h_{-2} \Delta t^3 & 2h_{-1} \Delta t + \pi^2 h_{-2} \Delta t^2 \\ 2h_{-1} \Delta t + \pi^2 h_{-2} \Delta t^2 & \frac{h_0}{2 \Delta t} + 2h_{-1} + \frac{8}{3} \pi^2 h_{-2} \Delta t \end{bmatrix}$$

$$drift \cong \sqrt{2 \cdot \ln 2 \cdot h_{-1}}$$

For Rubidium

$$h_0 = 2E - 20$$

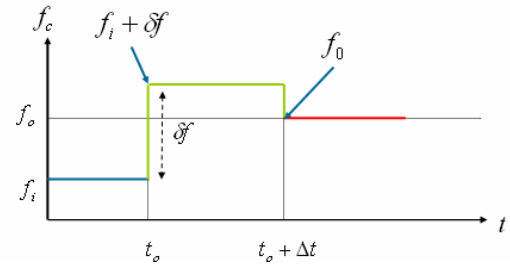
$$h_{-1} = 7E - 24$$

$$h_{-2} = 4E - 29$$

In clock synchronization part, SBAS clock synchronized with GPS clock using 4 steps.

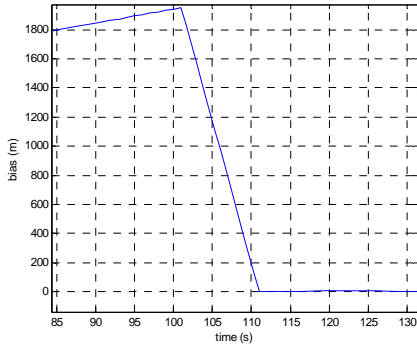
### 5.2.1 Step function

Using step function part is controls SBAS clock rapidly to synchronize with GPS. In step function part clock synchronization is did roughly. The initial clock bias is set to 1000m



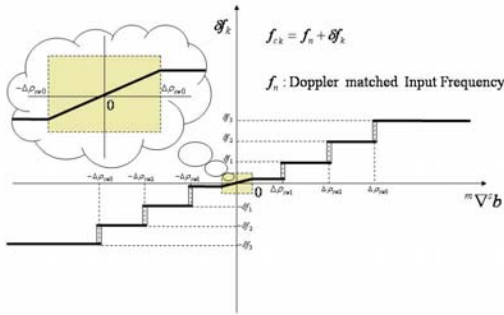
$$\begin{aligned}\delta f &= \frac{G \nabla^S \dot{b}(t_o)}{\lambda} + \frac{G \nabla^S b(t_o)}{\Delta t \cdot \lambda} \\ \delta f &= \frac{G \nabla^S \dot{b}(t_o)}{\lambda} + \frac{G \nabla^S b(t_o)}{\Delta t \cdot \lambda}\end{aligned}$$

The simulation result is represented bellow.



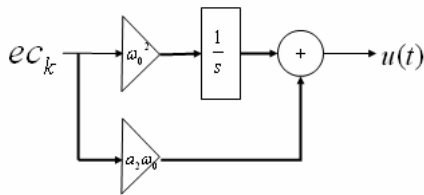
### 5.2.2 Stair function

Stair function is some kind of feedback control. The input is depends bias value.



### 5.2.3 FLL

FLL part is synchronize SBAS clock frequency with GPS clock frequency. 2nd order loop filter is used in FLL part.



$$ec_k = {}^G \nabla^S \hat{\phi}_k - {}^G \nabla^S \hat{d}_k$$

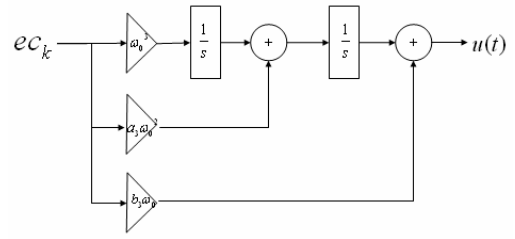
$$\hat{\theta}_{k+1} = \hat{\theta}_k + T \cdot \omega_0^2 \cdot ec_k$$

$$\hat{\theta}_{k+1} = \hat{\theta}_k + T \cdot (\hat{\theta}_{k+1} + a_2 \cdot \omega_0 \cdot ec_k)$$

$$\therefore u_k = \hat{\theta}_{k+1} + a_2 \cdot \omega_0 \cdot ec_k$$

### 5.2.4 PLL

In PLL part, SBAS and GPS carrier phase are synchronized. 3rd order loop filter is used in PLL part.



$$ec_k = {}^G \nabla^S \hat{\phi}_k - {}^G \nabla^S \hat{d}_k$$

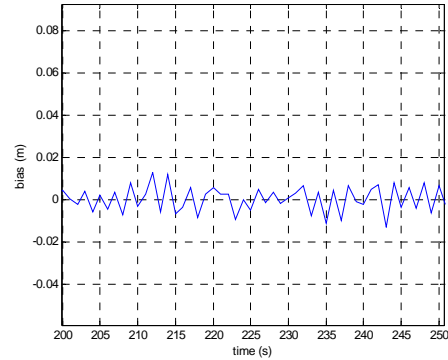
$$\hat{\theta}_{k+1} = \hat{\theta}_k + T \cdot \omega_0^3 \cdot ec_k$$

$$\hat{\theta}_{k+1} = \hat{\theta}_k + T \cdot (\hat{\theta}_{k+1} + a_3 \cdot \omega_0^2 \cdot ec_k)$$

$$\hat{\theta}_{k+1} = \hat{\theta}_k + T \cdot (\hat{\theta}_{k+1} + a_3 \cdot \omega_0^2 \cdot ec_k)$$

$$\therefore u_k = \hat{\theta}_{k+1} + b_3 \cdot \omega_0 \cdot ec_k$$

The simulation result is like below.



At PLL control loop, the clock bias error in RMS is 0.00458(m).

## 6. Conclusion

SBAS signal pass different signal path compare with GPS signal. SBAS signal is generated in the GUS and then uplinked to the GEO satellite and downlink to the users. So SBAS signal contains additional range errors that does not contained in GPS signal. First is GUS to GEO satellite distance and second is ionospheric delay which generated in uplink time. Those errors can be compensated by advancing the C/A code phase and carrier phase. To compensate the GUS to GEO satellite distance we use feedback control loop to control the code phase and carrier phase. Through this control loop SBAS signal range can be compensated by excluding the distance error. So users can get correct SBAS pseudo range.

And the ionospheric delay must be compensated which are generated in uplink time. This delay also acts like range error to users. Ionosphere delays code phase and advances carrier phase. Consider about this character we control the code and carrier to compensate the error.

To serve ranging service to users SBAS reference clock must be synchronized with GPS clock. We use synchronization algorithm which was used in pseudolite synchronize. Pseudolite is fixed in ground so does not have dynamics but GEO satellite have dynamics so we modify the algorithm. After this work the SBAS reference clock synchronize with GPS clock.

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