Engineering Realization of Full Attitude System Based On GPS Carrier Phase and MEMS IMU

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Abstract

This paper describes the design and realization of full attitude system based on MEMS IMU and GPS carrier phase. The work can be divided into two parts: First, initial heading is determined by using two GPS receivers. And this paper discusses the usage of space geometry conditions to reduce the range of ambiguity search. The method presented in this paper was tested on the static. On the static condition, an accuracy better than 0.06 degrees for heading for 3.48m long baseline has been achieved. Integration of GPS and low cost MEMS IMU are used to realize the real-time heading attitude system. Second, level attitude (pitch and roll) is determined using the method of frequency—velocity for the feedback control. At the same time, the method using the attitude based on MEMS IMU to help determination of the range of ambiguity search is proposed. The results done on the sea show that an alternative means to provide real-time, cost-effective, accurate and reliable attitude information for attitude surveys. Though motivated by a big ships application, the design can be applied to other vehicles.

Key words: MEMS IMU; GPS; full attitude determination; velocity for the feedback control

1. Introduction

Traditional attitude determination system is made up of medium or high level IMU, whose cost is very expensive, and whose accuracy is affected by gyro drift and accelerometer biases. It therefore becomes necessary to provide attitude survey system based on INS with regular external updates in order to bound the errors to an acceptable level. In the dynamic condition, initial alignment is very difficult to implement, its implement often relies on external aiding information such as GPS, and time of initial alignment is long.

Although GPS was primarily designed for precise positioning and time transfer, its potential for platform attitude determination was recognized at the early stages of the system development (Spinney, 1976; Ellis and Greswell,1979). Because attitude determination based on GPS shows cost-effective, high-accurate, low-volume, low-power-waste virtues, compared to attitude survey based on INS, its successful use shows the direction in developing low-cost attitude surveys system. Unfortunately, to achieve the required performance for many applications, Attitude survey based on GPS is often insufficient and must be augmented with other sensors. Inertial sensors are well suited for integration with satellite-based systems and have been successfully used in the past. However, previous investigations have typically used medium or high level inertial sensors to meet the most stringent accuracy requirements.

This paper investigates the simple, easily-implemented integration of MIMU with GPS for continuous high-accuracy attitude survey. Attitude system contains two GPS receivers and MIMU. First, initial heading is determined by using two GPS receivers. Using GPS initial heading, MIMU heading is determined.. Secondly, level attitude (pitch and roll) is determined using the method of frequency—velocity for the feedback control. At the same time, the method using the attitude based on MEMS IMU to help determination of the range of ambiguity search is proposed. The results done on the sea show that an alternative means to provide real-time, cost-effective, accurate and reliable attitude information for attitude surveys.

2. DETERMINATION OF HEADING

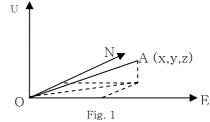
ATTITUDE

2.1 Heading Determination Based on GPS

From[1], GPS double differential observations are founded as follows:

$$\begin{cases} \Phi_b^{21} - N_b^{21} = \frac{1}{\lambda} R_b^{21} \bullet \vec{b} + \varepsilon \\ \Phi_b^{31} - N_b^{31} = \frac{1}{\lambda} \overline{R_b^{31}} \bullet \vec{b} + \varepsilon \\ \dots \\ \Phi_b^{n1} - N_b^{n1} = \frac{1}{\lambda} \overline{R_b^{n1}} \bullet \vec{b} + \varepsilon \end{cases}$$
(1)

Where R_b^{i1} are the unit directional vector from antenna 1 to the satellite i; \vec{b} is vector of baseline; N_b^{i1} is double differential ambiguity; Φ_b^{i1} is carrier phase fraction; λ is wavelengh of carrier.



Assuming that two GPS antennas is placed on the local level frame coordinates defined as in Fig 1. Antenna 1 is the origin of the coordinate system and antenna 2 is placed on A, whose coordinate is (x, y, z). the baseline OA from antenna 1 to antenna 2 defines the yaw. the formulas for computing yaw and pitch are immediately obtained as:

$$yaw = \tan^{-1}(z/x)$$

$$pitch = \tan^{-1}(y/\sqrt{x^2 + z^2})$$
(2)

Determination of yaw mainly relies on determination of the baseline vector OA. So, GPS Ambiguity Resolution must be found for determination of baseline vector OA.

(1) Volume of Ambiguity Search Space

The volume of the ambiguity search space can be obtained by Fig 2.

In Fig 2,at the same time, the satellite i and the satellite j can be observed by antennas 1 and 2. The distance between antennas 1 and 2 is L which is the length of the baseline, R_{12}^{i} and R_{12}^{j} are the unit directional vector from antenna 1 to the satellite i and satellite j, respectively; The difference between the

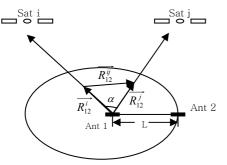


Fig. 2 volume of the ambiguity

 $\overrightarrow{R'_{12}}$ and $\overrightarrow{R'_{12}}$ is $\overrightarrow{R''_{12}}$; α is the angle between $\overrightarrow{R'_{12}}$ and $\overrightarrow{R''_{12}}$.

 R_{12}^{ij} , R_{12}^{i} , and R_{12}^{j} form a triangle. In the triangle , basing on law of cosines, we can obtain

$$\left| \overrightarrow{R}_{12}^{ij} \right| = 2\sin\left(\frac{\alpha}{2}\right)$$
(3)

Substituting Eqn (3) into double-difference equation:

$$\varphi_{12}^{ij} + N_{12}^{ij} = \frac{1}{\lambda} 2 \sin\left(\frac{\alpha}{2}\right) \cdot \left|\overrightarrow{OA}\right| \cos\theta + \varepsilon \tag{4}$$

Where φ_{12}^{ij} is the carrier phase decimal fraction, N_{12}^{ij} is the carrier phase integer. $|A_1A_2| = L$ is length of baseline and $|\cos \theta| \le 1$. So the volume of ambiguity search space N_{12}^{ij} is immediately obtained as

$$N_{12}^{ij} \in \left[-\frac{2L\sin(\frac{\alpha}{2})}{\lambda}, \frac{2L\sin(\frac{\alpha}{2})}{\lambda}\right]$$
(5)

Determination of Ambiguity Sets (2)

Many methods have developed for determining integer ambiguity. In this paper, an efficient method to construct the potential ambiguity sets on the surface of a sphere with the fixed baseline length is developed. This method is based on the space geometry condition and significantly improve the computational speed of the ambiguity searching process.

Using least squares ambiguity search method, only three

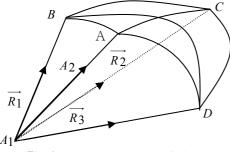


Fig. 3 space geometry restriction

double difference ambiguities from four primary satellites are searched. Supposed that four primary satellites have been chosen. The ambiguity search method basing on space geometry

conditions is introduced as Fig 3. In Fig 3, A_1A_2 is the vector of baseline; $\vec{R_1}$, $\vec{R_2}$, and $\vec{R_3}$ are difference between the unit vector from the master antenna to the reference satellite and the unit vector from the master antenna to the other satellites.

Now construct a spherical triangle for determining ambiguity sets. Assuming such a celestial sphere whose radius is infinite, A,B,C,and D are projections of A_2 , $\overrightarrow{R_1}$, $\overrightarrow{R_2}$, $\overrightarrow{R_3}$ on the celestial sphere, respectively, as indicated in Fig 3. Assuming N_{12}^{1} and N_{12}^{2} have been determined according to eqn.(5), how to solve N_{12}^3 according to N_{12}^1 and N_{12}^2 ,

$$\begin{cases} \varphi_{12}^{^{1}} + N_{12}^{^{1}} = \frac{1}{\lambda} \overrightarrow{R_{12}^{^{1}}} \bullet \overrightarrow{A_{1}} \overrightarrow{A_{2}} + \varepsilon \\ \varphi_{12}^{^{2}} + N_{12}^{^{2}} = \frac{1}{\lambda} \overrightarrow{R_{12}^{^{2}}} \bullet \overrightarrow{A_{1}} \overrightarrow{A_{2}} + \varepsilon \end{cases}$$
(6)

From Eqn.(6), we can determine angle AB between $R_{1,2}^{(1)}$ and $\overrightarrow{A_1A_2}$,and angle \overrightarrow{AC} between $\overrightarrow{R_{12}^2}$ and $\overrightarrow{A_1A_2}$; because $\overrightarrow{R_{12}^1}$, $\overrightarrow{R_{12}^2}$, and $\overrightarrow{R_{12}^3}$ are known , angle \overrightarrow{BC} between $\overrightarrow{R_{12}^1}$ and $\overrightarrow{R_{12}^2}$,angle \overrightarrow{DC} between $\overrightarrow{R_{12}^2}$ and $\overrightarrow{R_{12}^3}$, angle \overrightarrow{BD} between $\overrightarrow{R_{12}^{i}}$ and $\overrightarrow{R_{12}^{i}}$ are known. In the spherical triangle $\triangle ABC$, we can get,

cos

$$AC = \cos AB \cdot \cos BC +$$
(7)

$$\sin AB \cdot \sin BC \cdot \cos \angle ABC$$

From Eqn.(7), $\angle ABC$ can be determined immediately. In spherical triangle $\triangle BCD$, can obtain

$$\cos \overrightarrow{DC} = \cos \overrightarrow{BD} \cdot \cos \overrightarrow{BC} + \sin \overrightarrow{BD} \cdot \sin \overrightarrow{BC} \cdot \cos \angle DBC$$
(8)

From Eqn.(8) $\angle DBC$ can be determined immediately .Because $\cos \angle ABD = \cos(\angle ABC \pm \angle DBC) =$ (0)

$$\cos \angle ABC \cdot \cos \angle DBC \pm \sin \angle ABC \cdot \sin \angle DBC$$

So we obtain $\angle ABD$. In spherical triangle ABD,

$$\cos AD = \cos AB \cdot \cos BD + \sin AB \cdot \sin BD \cdot \cos \angle ABD$$
(10)

From Eqn.(10), cos AD can be determined immediately. Once $\cos \widehat{AD}$ is determined, angle \widehat{AD} between $\overrightarrow{R_{12}^3}$ and $\overrightarrow{R_{12}^3}$ can be obtained, so

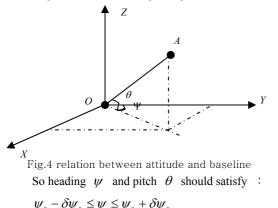
$$\varphi_{12}^{3} + N_{12}^{3} = \frac{1}{\lambda} \left| \overrightarrow{R_{12}^{3}} \right| \cdot \left| \overrightarrow{A_{1}A_{2}} \right| \cdot \cos \widehat{AD}$$
(11)

From Eqn.(11), N_{12}^3 can be determined.

Based on Eqn.(11), there are only two trial values for N_{12}^{3} (rounded to the nearest integers) to be tested for each integer trial set(N_{12}^1, N_{12}^2). Assuming that the ambiguity search ranges for N_{12}^1 and N_{12}^2 are ± 20 cycles, then the total potential ambiguity sets to be tested are $41 \times 41 \times 2 = 3362$, as opposed to $41^3 = 68921$ in a brute force check where N_{12}^3 is not solved using the known fixed baseline length. This greatly speeds up the computation of the ambiguity search process.

2.2 MIMU Aiding GPS Heading Determination

Supposed the length of baseline is L, heading provided by MIMU is ψ_0 , whose bound is $\psi_0 \pm \delta \psi_0$, pitch provided by MIMU θ_0 , whose bound is $\theta_0 \pm \delta \theta_0$



$$\theta_{0} - \delta\theta_{0} \le \theta \le \theta_{0} + \delta\theta_{0}$$

$$(12)$$

In fig.4, according to the length of baseline |b| = |OA| = L.

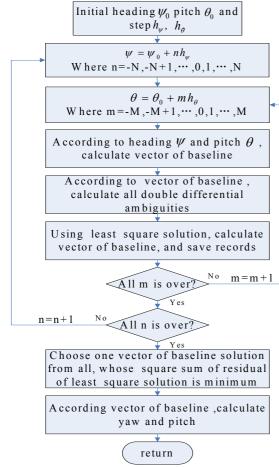


Fig.5 attitude determination flow chart with MIMU aiding

heading ψ_0 and pitch θ_0 , vector of baseline can be determined.

Let vector of baseline as $\vec{b} = (x, y, z)^T$, and components of the vector are :

$$\begin{cases} x = L \cos \theta \sin \psi \\ y = L \cos \theta \cos \psi \\ z = L \sin \theta \end{cases}$$
(13)

According to Equ.(1), we can determinate double differential ambiguities. Detailed flow chart is given as Fig.5. In Fig.5, N and M are determined as follows:

$$N = round(\delta \psi_0 / h_{\psi})$$
(14)

$$M = round(\delta\theta_{_{0}} / h_{_{\theta}})$$

Where round() denotes the nearest integers.

3. PITCH AND ROLL DETERMINATION

3.1 System-level Error Equations

Attitude error equations and velocity error equations can be expressed as following,:

$$\overline{\psi} = -\overline{\omega}_{in}^{n} \times \overline{\psi} - C_{b}^{n} \delta \overline{\omega}_{ib}^{b} + \delta \overline{\omega}_{in}^{n}$$
(15)

$$\delta \overline{v} = \overline{f}^n \times \overline{\psi} + C_b^n \delta \overline{f}^b$$

Navigation coordinate used in this paper is NUE. Equation (15) can be divided into the level channel and heading channel.

Because of determination of heading, we only consider level channel. Level attitude error equation and velocity error equations can be simplified as follows:

$$\begin{bmatrix} \dot{\psi}_{x} \\ \dot{\psi}_{z} \end{bmatrix} = \begin{bmatrix} 0 & -\omega_{ie} \sin L \\ \omega_{ie} \sin L & 0 \end{bmatrix} \begin{bmatrix} \psi_{x} \\ \psi_{z} \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} C_{b}^{n} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta \omega_{ibx}^{b} \\ \delta \omega_{ibz}^{b} \end{bmatrix} - \begin{bmatrix} 0 \\ \omega_{ie} \cos L \end{bmatrix} \psi_{y}$$

$$\begin{bmatrix} \delta \dot{v}_{x}^{n} \\ \delta \dot{v}_{z}^{n} \end{bmatrix} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \begin{bmatrix} \psi_{x} \\ \psi_{z} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} C_{b}^{n} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta f_{x}^{b} \\ \delta f_{z}^{b} \end{bmatrix}$$
(16)

Where ψ_x is roll; ψ_z is pitch; L is latitude.

$$B_{11} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} C_b^n \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} ,$$

$$= \begin{bmatrix} \cos \varphi \cos \theta & \sin \varphi \cos \gamma + \cos \varphi \sin \theta \sin \gamma \\ -\sin \varphi \cos \theta & \cos \varphi \cos \gamma - \sin \varphi \sin \theta \sin \gamma \end{bmatrix}$$

$$\overline{X} = [\psi_x \quad \psi_z \quad \delta v_x^n \quad \delta v_z^n]^T ,$$

$$\overline{W} = \begin{bmatrix} \delta \omega_{ibx}^b & \delta \omega_{ibz}^b & \delta f_x^b & \delta f_z^b & \psi_y \end{bmatrix}^T .$$

We can get level error equations:
$$\overline{X} = A\overline{X} + B_1 \overline{W}$$
(18)

Where

$$A = \begin{bmatrix} 0 & -\omega_{ie} \sin L & 0 & 0 \\ \omega_{ie} \sin L & 0 & 0 & 0 \\ 0 & g & 0 & 0 \\ -g & 0 & 0 & 0 \end{bmatrix}$$
$$B_{1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ B_{11} & 0 & 0 & -\omega_{ie} \cos L \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3.2 Determination of Level Attitude

Measurement error model can be expressed as: Г

$$\begin{bmatrix} z_{x} \\ z_{z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \psi_{x} \\ \psi_{z} \\ \delta v_{x}^{n} \\ \delta v_{z}^{n} \end{bmatrix} - \begin{bmatrix} \xi_{x} \\ \xi_{z} \end{bmatrix} = C\overline{X} - \overline{\xi}$$
(19)

П

Where $\overline{\xi}$ is velocity measurement noise.

We use linear feedback: $U^{n} = -K[z_{x}, z_{z}]^{T}$, and

let $K = \begin{bmatrix} 0 & K_2 & K_3 & 0 \\ K_1 & 0 & 0 & K_4 \end{bmatrix}^T$, so system-level closed loop

error equations can be expressed as:

$$\begin{bmatrix} \psi_{x}(S) \\ \psi_{z}(S) \\ \delta v_{x}^{n}(S) \\ \delta v_{z}^{n}(S) \end{bmatrix} = (SI - A - KC)^{-1} \times \begin{bmatrix} \left[\psi_{x}(0) \\ \psi_{z}(0) \\ \delta v_{x}^{n}(0) \\ \delta v_{z}^{n}(0) \end{bmatrix} + L \begin{cases} \delta \omega_{ibx}^{b} \\ \delta \omega_{ibz}^{b} \\ \delta f_{x}^{b} \\ \delta f_{z}^{b} \\ \delta \psi_{y} \end{bmatrix} + KL\{\xi\} \end{bmatrix}$$
(20)

Where $L\{.\}$ is the Lapilace transformed operator.

Ignoring $\omega_{ie} \sin L$, we can get:

$$(SI - A - KC)^{-1} = \begin{bmatrix} \frac{S + K_4}{S^2 + K_4 S - gK_1} & 0 & 0 & \frac{-K_1}{S^2 + K_4 S - gK_1} \\ 0 & \frac{(S + K_3)}{S^2 + K_3 S + gK_2} & \frac{-K_2}{S^2 + K_3 S + gK_2} & 0 \\ 0 & \frac{g}{S^2 + K_3 S + gK_2} & \frac{S}{S^2 + K_3 S + gK_2} & 0 \\ \frac{-g}{S^2 + K_4 S - gK_1} & 0 & 0 & \frac{S}{S^2 + K_4 S - gK_1} \end{bmatrix}$$

$$(21)$$

Closed loop system eigenvalues are:

$$\frac{-K_3 \pm \sqrt{K_3^2 - 4gK_2}}{2}, \frac{-K_4 \pm \sqrt{K_4^2 + 4gK_1}}{2}$$
(22)

Closed loop system eigenvalues must situate on the left portion of S-plane, so using the method of frequency-velocity for the feedback control, feedback coefficient matrix K can be designed.

According to level channel system-level error equations(18), closed loop sysem equation can be expressed as:

$$\dot{\overline{X}} = A\overline{X} + B_1\overline{W} - K \begin{bmatrix} z_x \\ z_z \end{bmatrix} = A\overline{X} + B_1(\overline{W} + \overline{U}^n) \quad (23)$$

Where $K = \begin{bmatrix} 0 & K_2 & K_3 & 0 \\ K_1 & 0 & 0 & K_4 \end{bmatrix}$, $U^n = -K[z_x, z_z]^T$.

According to $U^n = -K[z_x, z_z]^T$, we can get error control as:

$$\begin{bmatrix} U_{\omega x}^{n} \\ U_{\omega z}^{n} \end{bmatrix} = \begin{bmatrix} K_{1} z_{z} \\ K_{2} z_{x} \end{bmatrix}, \begin{bmatrix} U_{f x}^{n} \\ U_{f z}^{n} \end{bmatrix} = \begin{bmatrix} K_{3} z_{x} \\ K_{4} z_{z} \end{bmatrix}$$
(24)

So control variables channel for the level are: $U_{\omega}^{n} = \left[U_{\omega x}^{n}, 0, U_{\omega z}^{n} \right]^{T}$ and $U_{f}^{n} = \left[U_{f x}^{n}, 0, U_{f z}^{n} \right]^{T}$.

Substituting
$$U_{\omega}^{n} = \left[U_{\omega x}^{n}, 0, U_{\omega z}^{n}\right]^{T}$$
 and $U_{f}^{n} = \left[U_{fx}^{n}, 0, U_{fz}^{n}\right]^{T}$ into Equ.(15), get:

$$\dot{\overline{\psi}} = -\overline{\omega}_{in}^{n} \times \overline{\psi} - C_{b}^{n} \delta \overline{\omega}_{ib}^{b} + \delta \overline{\omega}_{in}^{n} + U_{\omega}^{n}$$

$$\dot{\delta \overline{\psi}} = \overline{f}^{n} \times \overline{\psi} + C_{b}^{n} \delta \overline{f}^{b} + U_{f}^{n}$$
(25)

4. TEST RESULT

The performance of MIMU used in attitude surveying system is as follows Table 1.

Table 1 Performance of MIMU						
Gyro drift (long period)	20°/h	Accelerometer drift (long period)	3×10 ⁻³ g			
Gyro drift (short period)	1°/h (2 min)	Accelerometer drift (short period)	1×10 ⁻³ g (2 min)			
Gyro scale factors	300ppm	Accelerometer scale factors	500ppm			
Gyro random noise	0.06°/h	Accelerometer random noise	50×10 ⁻⁶ g /h			

Two GPS receivers adopted are CMC Allstar OEM with 1 HZ sampling frequency, and programmer is realized by C. Two data sets were analyzed. The first one was the static test and the second one was the dynamic test on a big ship on the sea.

4.1 Static Test Results

Firstly, accuracy of attitude based GPS is evaluated for open area applications. In Fig 6, the real-time yaw and pitch results are obtained, respectively. An accuracy better than 0.06 degree for heading for 3.48m long baseline has been achieved, and the accuracy for pitch is little worse than the accuracy for yaw.

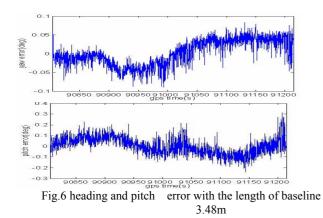
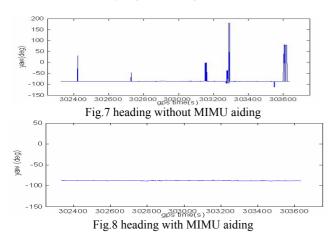


Table 2 is two group tests, and show heading information without MIMU aiding and with aiding, respectively. No.1 is tested on the open sky area, and No.2 is tested on worse environment. From Table 2 rating of success of attitude determination can be improved with MIMU aiding.

Table 2	Compared results between GPS attitude without MIMU aiding and

GPS attitude with aiding							
No.	Length of baseline (m)	With MIMU Aiding?	Number of epochs calculated	Number of epochs calculated successfully	Rate of success		
1	1 1.059m	No	1207	1165	96.52%		
		Yes	1207	1207	100%		
2	1.10	No	1549	723	46.67%		
1.12m	Yes	1549	1549	100%			

No.1 test is drawn by Fig.7 and Fig. 8 as follows:

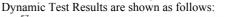


4.2 Dynamic Test Results

Dynamic test was conducted in the sea, on July ,2005. Place three GPS receivers on the big ship, and the ship was at anchor ashore. Adding one GPS receiver is for improving rate of GPS attitude determination success rate. On the ship, the level attitude reference system was installed as level attitude reference. GPS/MIMU attitude determination system set-up are shown as follows:



Fig 9 GPS/MIMU attitude determination system set-up on the ship



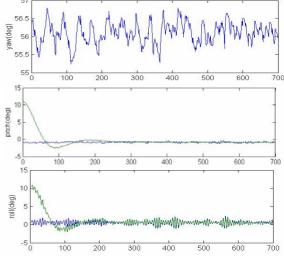


Fig10 Attitude results when the ship being at anchor compared to standard roll reference

From Fig.10, level attitude can converge about 180s compared to the level attitude reference system.

5. Conclusion

Throughout simple integration of low-cost MIMU and GPS, a medium-precision and high-reliability full attitude surveying system is designed. An accuracy better than 0.06 degree for heading for 3.48m long baseline has been achieved, level attitude depends on accuracy of accelerometer. The method used in this paper can be implemented simply, and its computation load is less than other method.

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