Improving Covariance Based Adaptive Estimation for GPS/INS Integration

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Abstract

It is well known that the uncertainty of the covariance parameters of the process noise (Q) and the observation errors (R) has a significant impact on Kalman filtering performance. Q and R influence the weight that the filter applies between the existing process information and the latest measurements. Errors in any of them may result in the filter being suboptimal or even cause it to diverge. The conventional way of determining Q and R requires good a priori knowledge of the process noises and measurement errors, which normally comes from intensive empirical analysis. Many adaptive methods have been developed to overcome the conventional Kalman filter's limitations. Starting from covariance matching principles, an innovative adaptive process noise scaling algorithm has been proposed in this paper. Without artificial or empirical parameters to be set, the proposed adaptive mechanism drives the filter autonomously to the optimal mode. The proposed algorithm has been tested using road test data, showing significant improvements to filtering performance.

Keywords: GPS/INS, adaptive Kalman filter, stochastic modelling, covariance matching, process noise scaling

1. Introduction

Integrated positioning and navigation systems using GPS receivers and INS sensors have demonstrated great application potential in the areas of real-time navigation, mobile surveying and mapping, and location based services. Besides providing a full solution of position and attitude, the other benefits of integrating GPS and INS include the long term high positioning accuracy, the high update rate, the robustness to GPS signal jitter and interference, and the continuous calibration of INS errors. Despite various integration architectures, the central challenge of implementing such integrated systems is how well the GPS and INS measurement data can be fused to generate the optimal solution.

The Kalman filter (KF) is the most common technique for carrying out this task. The operation of the KF relies on the proper definition of a dynamic model and a stochastic model (Brown and Hwang, 1997). The dynamic model describes the propagation of system states over time. The stochastic model describes the stochastic properties of the system process noise and observation errors.

The uncertainty of the covariance parameters of the process noise (Q) and the observation errors (R) has a significant impact on the Kalman filtering performance (Grewal and Andrews, 1993; Grewal and Weil, 2001; Salychev, 2004). Q and R influence the weight that the filter applies between the existing process information and the latest measurements. Errors in any of them may result in the filter being suboptimal or even cause it to diverge.

The conventional way of determining Q and R requires good a priori knowledge of the process noise and measurement errors, which typically comes from intensive empirical analysis. In practice, the values are generally fixed and applied during whole application segment. The performance of the integrated systems suffers in two respects due to this inflexibility. First, process noise and measurement errors are dependent on the application environment and process dynamics. A precise evaluation of these in advance is unlikely to be possible. Second, the so-called KF "tuning" process is complicated, which is left to a few experts, and thus hampers its successful application across wider fields.

Many adaptive methods have been developed to overcome the conventional KF's limitations. Popular adaptive methods can be roughly classified into covariance scaling, multi-model adaptive estimation, and adaptive stochastic modelling (Hide et al., 2004a). The covariance scaling method improves the filter stability and convergent performance by introducing a multiplication factor to the state covariance matrix. The calculation of the scaling factor can either be fully empirical or based on some criteria derived from filter innovations (Hu et al., 2003; Yang, 2005; Yang and Gao, 2006; Yang and Xu, 2003). The multi-model adaptive estimation method requires a bank of simultaneously operating Kalman filters in which different stochastic models are employed. The output of multi-model adaptive estimation is the weighted sum of each individual filter's output. The weighting factor can be calculated using the residual probability function (Brown and Hwang, 1997; Hide et al., 2004b). Adaptive stochastic modelling includes innovation-based adaptive modelling (Mohamed and Schwarz, 1999) and residual-based adaptive modelling (Wang, 2000; Wang et al., 1999). It is well known that the innovation and residual sequences of the KF are a reliable indicator of the KF filtering performance. For an optimal filter, the innovation and residual sequences are white Gaussian noise (Brown and Hwang, 1997; Mehra, 1970). By online monitoring of the innovation and residual covariance, the adaptive stochastic modelling algorithm estimates directly the covariance matrices of process noise and measurement errors, and tunes them in real-time.

The online stochastic modelling method has been investigated for GPS/INS integration, and some initial results have been published (e.g. Ding et al., 2006). Besides the successful implementation and test, one limitation is that the estimation algorithm is very sensitive to colour noise and a change in the observed satellites. Theoretically, this sensitivity can be due to two reasons. First, the covariance estimation of the innovation and residual sequences is very noisy due to the use of a short data segment, the coloured noises, and the non-stationary noise property during a short time span. On the other hand, smoothing covariance estimation by increasing the estimation window size would degrade the dynamic response of the adaptive mechanism. Secondly, with a limited number of rough covariance observations it is difficult to derive precise process noise and observation error estimates. Considering the large matrix dimension of process noise when the INS Psi model is used, full estimation becomes virtually impossible.

Hence, a more robust algorithm with fewer adaptive parameters is desirable. Starting from the covariance matching principles, an innovative process noise scaling algorithm has been developed. Without artificial or empirical parameters to be set, the proposed adaptive mechanism has the ability to drive the filter autonomously to the optimal mode. This proposed algorithm has been analysed using road test data. Significant improvements to the filtering performance have been noticed.

In Section 2, the Kalman filter and the online stochastic modelling algorithm are introduced. Then a new covariance based process noise scaling method is derived. The test results are presented in Section 3.

2. Adaptive Kalman filtering

2.1 Conventional Kalman filter

Considering a multivariable linear discrete system for the integrated GPS/INS system:

$$\mathbf{x}_k = \mathbf{\Phi}_{k-1}\mathbf{x}_{k-1} + \mathbf{w}_{k-1} \quad (1)$$

$$\mathbf{z}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k \quad (2)$$

where $\mathbf{x}_k$ is (n×1) state vector, $\mathbf{\Phi}_k$ is (n×n) transition matrix, $\mathbf{z}_k$ is (r×1) observation vector, $\mathbf{H}_k$ is (r×n) observation matrix.

$\mathbf{w}_k$ and $\mathbf{v}_k$ are uncorrelated white Gaussian noise sequence with means and covariances:

$$\begin{aligned} &E\left\{\mathbf{w}_k\right\} = E\left(\mathbf{v}_k\right) = 0 \\ &E\left\{\mathbf{w}_k\mathbf{v}_i^T\right\} = 0 \\ &E\left\{\mathbf{w}_k\mathbf{w}_i^T\right\} = \begin{cases}\mathbf{Q}_k & i = k \\ 0 & i \neq k\end{cases} \\ &E\left\{\mathbf{v}_k\mathbf{v}_i^T\right\} = \begin{cases}\mathbf{R}_k & i = k \\ 0 & i \neq k\end{cases} \end{aligned} \quad (3)$$

where $E\{\cdot\}$ denotes the expectation function. $\mathbf{Q}_k$ and $\mathbf{R}_k$ are covariance matrix of process noise and observation errors, respectively. The KF state prediction and state covariance prediction are:

$$\begin{aligned} \hat{\mathbf{x}}_k^- &= \mathbf{\Phi}_{k-1}\hat{\mathbf{x}}_{k-1} \\ \hat{\mathbf{P}}_k^- &= \mathbf{\Phi}_{k-1}\hat{\mathbf{P}}_{k-1}\mathbf{\Phi}_{k-1}^T + \mathbf{Q}_{k-1} \end{aligned} \quad (4)$$

where $\hat{\mathbf{x}}_k$ denotes the KF estimated state vector; $\hat{\mathbf{x}}_k^-$ the predicted state vector for the next epoch; $\hat{\mathbf{P}}_k$ the estimated state covariance matrix; and $\hat{\mathbf{P}}_k^-$ the predicted state covariance matrix. The Kalman measurement update algorithms are:

$$\begin{aligned} \mathbf{K}_k &= \hat{\mathbf{P}}_k^-\mathbf{H}_k^T\left(\mathbf{H}_k\hat{\mathbf{P}}_k^-\mathbf{H}_k^T + \mathbf{R}_k\right)^{-1} \\ \hat{\mathbf{x}}_k &= \hat{\mathbf{x}}_k^- + \mathbf{K}_k\left(\mathbf{z}_k - \mathbf{H}_k\hat{\mathbf{x}}_k^-\right) \\ \hat{\mathbf{P}}_k &= \left(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k\right)\hat{\mathbf{P}}_k^- \end{aligned} \quad (5)$$

where $\mathbf{K}_k$ is the Kalman gain, which defines the updating weight between new measurements and predictions from the system dynamic model.

The innovation sequence is defined as:

$$\mathbf{d}_k = \mathbf{z}_k - \mathbf{H}_k\hat{\mathbf{x}}_k^- \quad (6)$$

and the residual sequence as:

$$\boldsymbol{\varepsilon}_k = \mathbf{z}_k - \mathbf{H}_k\hat{\mathbf{x}}_k \quad (7)$$

2.2 Online stochastic modelling

Since the Kalman filtering is equivalent to a recursive least squares estimation process, the KF measurement update can be rewritten in the form of (e.g., Wang et al., 1999):

$$\begin{bmatrix}\mathbf{z}_k \\ \hat{\mathbf{x}}_k^-\end{bmatrix} = \begin{bmatrix}\mathbf{H}_k \\ \mathbf{I}\end{bmatrix}\mathbf{x}_k + \begin{bmatrix}\mathbf{v}_k \\ \mathbf{e}_k^-\end{bmatrix} \quad (8)$$

where vector $\mathbf{e}_k^-$ denotes the state prediction errors, and $\mathbf{I}$ the identity matrix. Equation (8) shows that at each epoch, the filter update is an optimal blending of existing information from the predicted states and the information of the new measurements. The associated covariance matrix is:

$$\mathbf{C} = \begin{bmatrix}\mathbf{R}_k & 0 \\ 0 & \hat{\mathbf{P}}_k^-\end{bmatrix} \quad (9)$$

Given that

$$\mathbf{l} = \begin{bmatrix}\mathbf{z}_k \\ \hat{\mathbf{x}}_k^-\end{bmatrix}, \text{ and } \mathbf{A} = \begin{bmatrix}\mathbf{H}_k \\ \mathbf{I}\end{bmatrix} \quad (10)$$

where $\mathbf{l}$ is the observation vector, and $\mathbf{A}$ the design matrix. The optimal estimation of the states is:

$$\hat{\mathbf{x}}_k = \left(\mathbf{A}^T\mathbf{C}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{C}^{-1}\mathbf{l} \quad (11)$$

The least squares estimation residual is:

$$\boldsymbol{\varepsilon}_k = \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_k \\ = \left(\mathbf{I} - \mathbf{H}_k \mathbf{K}_k\right) \mathbf{d}_k \tag{12}$$

By applying the error propagation law to the above equation, one obtains:

$$E\left\{\boldsymbol{\varepsilon}_k \boldsymbol{\varepsilon}_k^T\right\} = E\left\{\mathbf{v}_k \mathbf{v}_k^T\right\} - \mathbf{H}_k \hat{\mathbf{P}}_k \mathbf{H}_k^T \tag{13}$$

$$\hat{\mathbf{R}}_k = E\left\{\boldsymbol{\varepsilon}_k \boldsymbol{\varepsilon}_k^T\right\} + \mathbf{H}_k \hat{\mathbf{P}}_k \mathbf{H}_k^T \tag{14}$$

When the residual covariance $E\left\{\boldsymbol{\varepsilon}_k \boldsymbol{\varepsilon}_k^T\right\}$ is available, the covariance of the observation error $\hat{\mathbf{R}}_k$ can be estimated directly from equation (14). Calculation of the residual covariance $E\left\{\boldsymbol{\varepsilon}_k \boldsymbol{\varepsilon}_k^T\right\}$ normally uses a limited number of samples of the innovation sequence:

$$E\left\{\boldsymbol{\varepsilon}_k {\boldsymbol{\varepsilon}_k}^T\right\} = \frac{1}{m}\sum_{i=0}^{m-1} \boldsymbol{\varepsilon}_{k-i} \boldsymbol{\varepsilon}_{k-i}^T \tag{15}$$

where m is the 'estimation window size'. For equation (15) to be valid, the residual sequence has to be ergodic and stationary over the m steps. The result of equation (15) is unbiased estimates of the autocorrelation (equivalent to covariance with zero mean) of the residual sequence (Papoulis, 1991). The estimation converges to the true value as the window size becomes larger. However, large window size and the stationary condition are contradictory requirements for kinematic GPS/INS applications. Choosing an appropriate window size is a trade-off between estimation stability and estimation accuracy (Ding et al., 2006; Wang et al. 1999).

As discussed earlier, practical implementation of an online stochastic modelling algorithm faces many challenges. One critical factor influencing stochastic modelling accuracy is ensuring precise and smooth estimation of the innovation and residual covariance from data sets with limited length. Furthermore, the stochastic parameters are closely coupled with each other when using current estimation algorithms (Ding et al., 2006), which makes the correct estimation more difficult.

2.2 Scaling of process noise

To improve the robustness of the adaptive filtering algorithm to innovation (and residual) covariance estimation bias, a new process noise scaling method is proposed here. From equation (6), the predicted covariance of the innovation sequence can be expressed as:

$$\mathbf{S}_k = \mathbf{H}_k \hat{\mathbf{P}}_k^- \mathbf{H}_k^T + \mathbf{R}_k \tag{16}$$

In addition, the real covariance of the innovation sequence can be approximated using:

$$Cov\left\{\mathbf{d}_k\right\} = \frac{1}{m}\sum_{j=0}^{m-1} \mathbf{d}_{k-j} \mathbf{d}_{k-j}^T \tag{17}$$

For an optimal filter, the predicted innovation covariance should be equal to the one directly calculated from the innovation sequence. Any deviation between them can be ascribed to the wrong definition of $\hat{\mathbf{P}}_k^-$ and/or $\mathbf{R}_k$. Attempting to resolve two variable matrices with one constraint is not practical, as discussed above. Considering that the performance of the Kalman filter relies on the relative magnitudes of $\hat{\mathbf{P}}_k^-$ and $\mathbf{R}_k$, and $\mathbf{R}_k$ has several other ways to be assessed in GPS/INS integration, $\mathbf{R}_k$ is assumed to be perfectly known for adaptation purposes. So,

$$\frac{1}{m}\sum_{j=0}^{m-1} d_{k-j} d_{k-j}^T = \mathbf{H}_k \tilde{\mathbf{P}}_k^- \mathbf{H}_k^T + \mathbf{R}_k \tag{18}$$

where $\tilde{\mathbf{P}}_k^-$ denotes the estimation of the process noise prediction. Attempting to directly resolve the $\tilde{\mathbf{P}}_k^-$ from equation (18) is not practical, although a partial adaptation might be possible.

To simplify, define the scaling factor as:

$$\alpha = \frac{trace\left\{\mathbf{H}_k \tilde{\mathbf{P}}_k^- \mathbf{H}_k^T\right\}}{trace\left\{\mathbf{H}_k \hat{\mathbf{P}}_k^- \mathbf{H}_k^T\right\}} = \frac{trace\left\{\frac{1}{m}\sum_{j=0}^{m-1} d_{k-j} d_{k-j}^T - \mathbf{R}_k\right\}}{trace\left\{\mathbf{H}_k \hat{\mathbf{P}}_k^- \mathbf{H}_k^T\right\}} \tag{19}$$

where the scaling factor α implies a rough ratio between the calculated innovation covariance and the predicted one. Since

$$\hat{\mathbf{P}}_k^- = \mathbf{\Phi}_{k-1} \hat{\mathbf{P}}_{k-1} \mathbf{\Phi}_{k-1}^T + \mathbf{Q}_{k-1} \tag{20}$$

after substituting equation (20) into equation (19), α can be expressed as:

$$\alpha = \frac{trace\left\{\mathbf{H}_k \left(\mathbf{\Phi}_{k-1} \hat{\mathbf{P}}_{k-1} \mathbf{\Phi}_{k-1}^T + \tilde{\mathbf{Q}}_{k-1}\right) \mathbf{H}_k^T\right\}}{trace\left\{\mathbf{H}_k \left(\mathbf{\Phi}_{k-1} \hat{\mathbf{P}}_{k-1} \mathbf{\Phi}_{k-1}^T + \mathbf{Q}_{k-1}\right) \mathbf{H}_k^T\right\}} \tag{21}$$

Based on equations (19) and (21), an intuitive adaptation rule is defined as:

$$\hat{\mathbf{Q}}_k = \mathbf{Q}_{k-1}\sqrt{\alpha} \tag{22}$$

The square root in equation (22) is used to contribute a smoothing effect, which is not essential. Directly tuning $\hat{\mathbf{P}}_k^-$ based on equation (19) is not considered due to the concerns of its effect on filtering smoothness and parameter consistency.

Compared with existing process noise scaling methods, the distinct feature of this proposed algorithm is that α can be a scaling factor either larger than one or smaller than one, which provides a full range of options to tune $\hat{\mathbf{Q}}_k$. Only when the predicted innovation covariance and the calculated innovation covariance are consistently equal, α is stabilised at value one.

The innovation covariance still needs to be estimated using equation (17). When compared with the adaptive stochastic modelling method, the process noise scaling method is more robust to covariance estimation bias due to fewer parameters involved in the tuning, and the tuning is a smooth feedback. However, since only the overall magnitude of $\hat{\mathbf{Q}}_k$ is tuned rather than individual elements, optimal allocation of noise to each individual source can not be achieved. This is one fundamental difference between the adaptive stochastic modelling methods and the covariance scaling methods.

3. Testing

3.1 Test Configuration

The tests involved two sets of Leica System 530 GPS receivers and one BEI C-MIGITS II (DQI-NP) INS unit. One of the GPS receivers was set up as static reference station, and the other one on top of the test vehicle together with the INS unit. The data were stored on the GPS receiver's PCMCIA card and a Notebook PC for post processing. The BEI's C-MIGITS II has its own GPS receiver (the MicroTracker) to synchronise the INS data to GPS time.

Table 1 shows the DQI-NP's technical data for reference. The specified parameters were used in setting up the Q estimation in the standard Kalman filtering process. Figure 1 shows the ground track of the test vehicle.

Table 1. DQI-NP's technical data

	Gyro	Accelerometer
Bias	5 deg/hr	500 µg
Scale factor	500 ppm	800 ppm
Random walk/ white noise	0.035 deg/sqrt(hr)	180 µg/sqrt(hr)

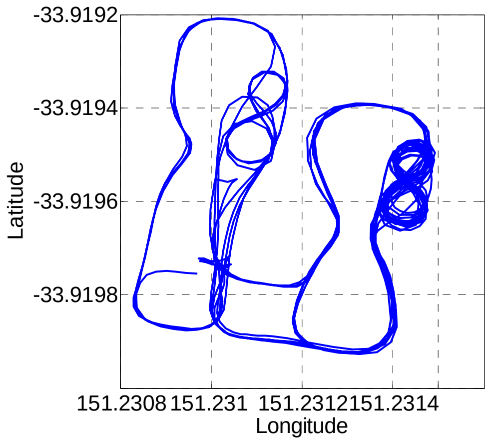

Figure 1. Ground track of the test vehicle

3.2 Data and analysis

Figure 2 shows the RMSs of the adaptive Kalman filter's states derived from the covariance matrix $\hat{\mathbf{P}}_k$. Only the RMS values of the first three diagonal elements have been shown. The trends for the remaining states are similar. It is clear that the overall filter operation is stable and converged. The bump at about 100 epochs indicates the effectiveness of the adaptive process noise scaling algorithm. The window size used for the innovation covariance calculation is 64.

Figure 3 shows the history of the estimated scaling factor evolving with time. As expected, after the initial turbulence, the scaling factor quickly settles to a value of about one.

For the optimal Kalman filter, both innovation and residual magnitudes should be minimised. Figure 4(a) shows that the magnitude of the innovations is within 0.1m. After measurement update, the magnitude of residuals is within 0.02m, as illustrated in Figure 4(b). Since the necessary and sufficient condition for the optimality of a Kalman filter is that the innovation sequence should be white, the autocorrelation of the innovation sequence is plotted in Figure 5, which shows clear white noise features. A further check of the whiteness can be carried out using the method introduced by Mehra (1970).

Figures 6 and 7 show two groups of accelerometer bias and gyro bias estimates for comparison purposes. The process noise parameters used by the standard extended Kalman filter are calculated according to the manufacturer's technical specification. It can be seen that all three configurations have generated similar estimates. The standard extended Kalman filter provides the smoothest estimation. The estimates using the process scaling method are much nosier, which implies that it responds quicker to signal changes. The estimates on the Z axis have the worst consistency. This may be due to its weak observability, since during the tests with the ground vehicle the Z axis has the least dynamics. Another reason could be that gravity uncertainties were not properly scaled. They may behave differently from the INS noises.

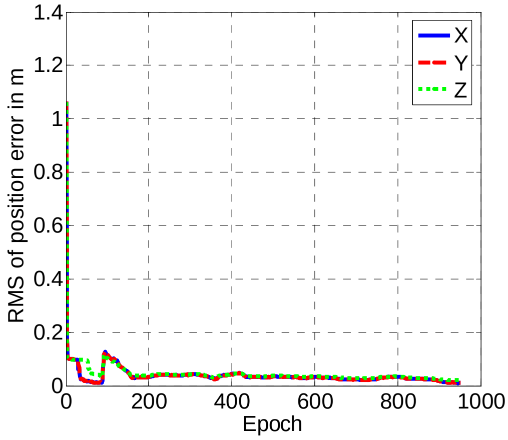

Figure 2. RMSs of the adaptively estimated Kalman filter states

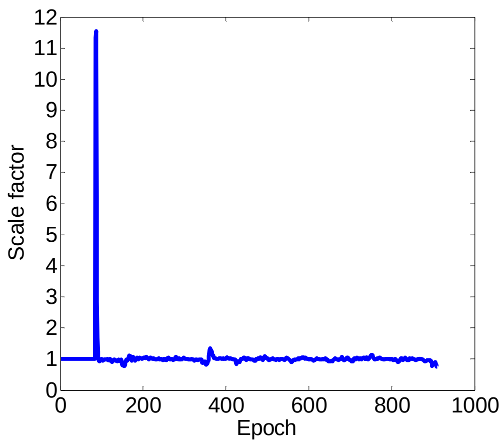

Figure 3. The estimated scaling factor sequence

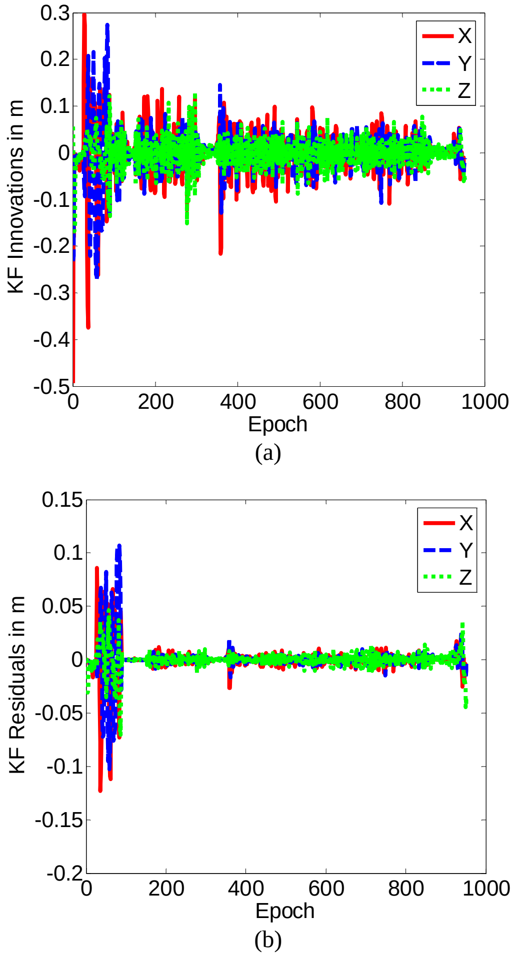

Figure 4. (a) Innovation sequence (b) Residual sequence

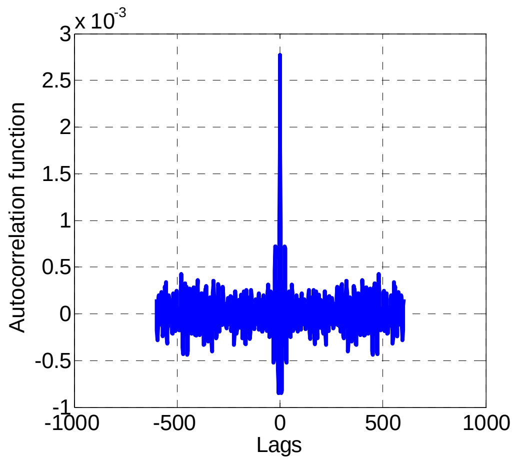

Figure 5. Autocorrelation of innovation sequence (unbiased)

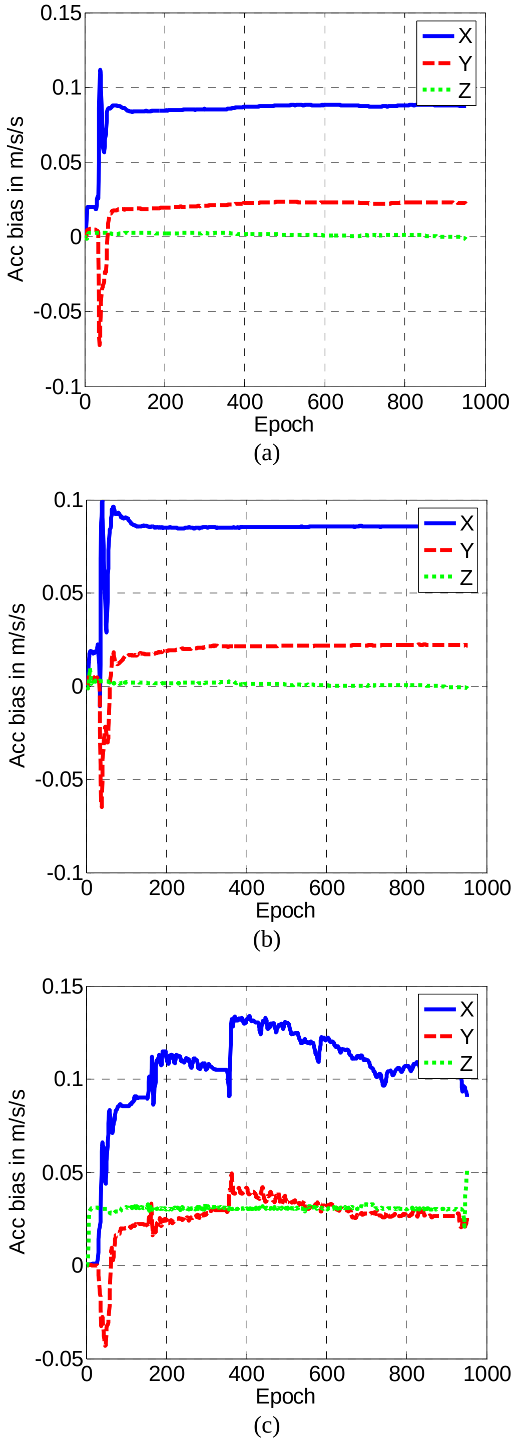

Figure 6. Estimates of accelerometer bias using different methods (a) Standard Extended KF (b) Extended KF with online stochastic modelling (c) Extended KF with the proposed process scaling algorithm

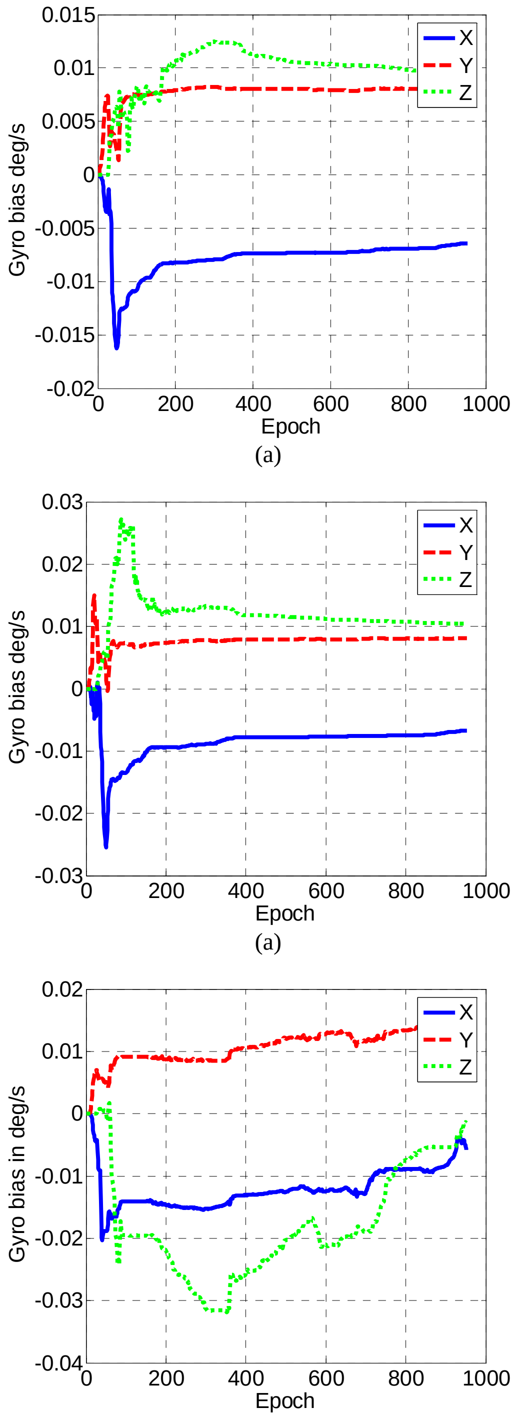

Figure 7. Estimates of gyro bias using different methods (a) Standard Extended KF (b) Extended KF with online stochastic modelling (c) Extended KF with the proposed process scaling algorithm

4. Conclusion

Over the past few decades, adaptive KF algorithms have been intensively investigated to reduce the uncertainty of the covariance matrices of the process noise (Q) and the observation errors (R). The covariance matching method is one of the most promising solutions. Based on the covariance matching principle, the individual Q and R elements can be adaptively estimated online using the stochastic modelling method. However, it demonstrates vulnerability to the innovation and residual covariance estimation biases, and is not scalable to a large number of parameters. In this paper, an innovative covariance based adaptive process noise scaling method has been proposed and tested. This method is reliable, robust, and suitable for practical implementations. The initial tests have demonstrated significant improvements of the filtering performance. However, optimal allocation of noise to each individual source is not possible using scaling factor methods, which is a topic for further investigation.

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