A Design of Adaptive Controller for Transportation System with Dynamic Friction

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Abstract

In this paper, we propose an adaptive control algorithm to improve the position accuracy and reduce the nonlinear friction effects for linear motion servo system. Especially, the considered system includes not only the variation of the mass of the mover but also the friction change by the normal force. To adapt to these problems, we designed the controller with the mass estimator and the compensator by observing the variation of normal force. Finally, the numerical simulation results are presented in order to show the effectiveness of the proposed method to improve the position accuracy compared to other control methods.

Keywords: LuGre friction model, dynamic friction, adaptive controller, mass estimator, normal force, Stribeck effect, position tracking control, Lyapunov function candidates.

1. Introduction

In the general mechanical servo system, the friction deteriorates the performance of the controller due to its nonlinear characteristics. In particular, the friction phenomenon causes steady-state tracking errors, limits cycles in the position and velocity control system, although gains of the controller are well-tuned in the linear system model. Even if the sensor uses higher accuracy level, it is difficult to improve the tracking performance of the position to the same level with a general control method such as a PID type. Therefore, many friction models were proposed and compensation methods have been actively researched (Armstrong, et al., 1994).

The nonlinear friction model is divided into static and the dynamic models. The static model is characterized by different combinations of stiction, Coulomb friction, viscous friction, and Stribeck effect. However, the performance of the compensator based on the static model of friction is not always satisfactory in a high precision position or low velocity tracking control because this model includes neither hysteretic phenomena by stick-slip nor the variation of break-away force or dead zone by stiction. To obtain more accurate friction compensation, the dynamic friction model is desired. Dynamic friction model has been proposed to analyze friction characteristic or design compensator from the Dahl's model (Bliman, 1992). Recently, a new dynamic model named LuGre model is proposed by Wit, et al., in 1995. It is a very popular compensation system because of its simple first-order structure and rich characterization of friction properties. In the LuGre model, there are some parameters that assume the coefficients for the stiffness, damping, viscous friction, Stribeck velocity, etc. And these are varying by variations of the normal force, lubrication state, temperature, and the characteristics of material and mechanical structure, etc.

In this paper, we consider that the variation of mover's mass varies depending on loading and unloading. The normal force variation occurs by it and other parameters. Therefore, the proposed control system is composed of the main position controller and the friction compensator part. Also a mass estimator was designed by the adaptive control law. The mass estimator is described in section 3.3 of this paper. Its output is used as a scale factor for tuning gains in the main position controller and an initial value of normal force observer in adaptive friction compensator. We then performed a simulation to evaluate its adaptation and performance of position accuracy for control systems of other control systems. Others are considered only when the PID controller is applied and adding compensators that are a fixed type and adaptive type without the mass estimator and gain tuner.

2. Mathematical Modeling

The linear motion servo system with nonlinear friction and variation of mover's mass can be shown as figure 1. The figure shows the mover's mass M_1 changing to $M_1 + M_2$. The system dynamic equation can be described by Equation (1).

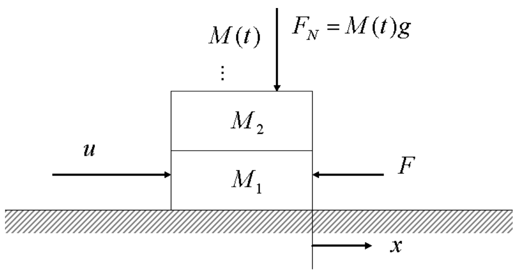

Fig. 1. Configuration of the considered system and each component

$$M\frac{d^2x}{dt^2} = u - F \qquad (1)$$

where,

x	mover position
M	mass
u	control input(thrust force)
F	friction force.

$$F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v \qquad (2)$$

where,

z	average deflection of elastic bristles.
v	mover's velocity
σ_0	coefficient of stiffness
σ_1	damping coefficient
σ_2	coefficient of viscous.

The equation of bristle deflection z and the positive definite function that relies on factors such as surface material characteristics can be described by Equations (3) and (4) respectively.

$$\frac{dz}{dt} = v - \frac{\sigma_0 |v|}{g(v)} z \tag{3}$$

$$g(v) = F_C + (F_S - F_C) e^{-(v/v_s)^2} \tag{4}$$

where,

F_C	Coulomb friction level
F_S	stiction force level
v_s	Stribeck velocity.

In this paper, we need to modify Equation (3) for considering the mass and the normal force variation system as follows:

$$\frac{dz}{dt} = v - \theta \frac{\sigma_0 |v|}{g(v)} z \tag{5}$$

where, θ is an uncertain friction scale factor that is normalized by the initial mass $M(0)$. Therefore, the relationship between the actual variation of mass and θ can be noted as follows:

$$\theta(t) = \frac{M(0)}{M(t)} \tag{6}$$

The steady state relation is $\dot{z} = 0$, and the steady state friction F_{SS} can be rewritten as Equation (7). Figure 2 shows the surface of the steady state friction model for mass, velocity, and friction force.

$$\begin{aligned} F \approx F_{SS} &= g(v)\,\mathrm{sgn}(v) + \sigma_2 v \\ &= \{F_C + (F_S - F_C) e^{-(v/v_s)^2}\}\,\mathrm{sgn}(v) + \sigma_2 v \end{aligned} \tag{7}$$

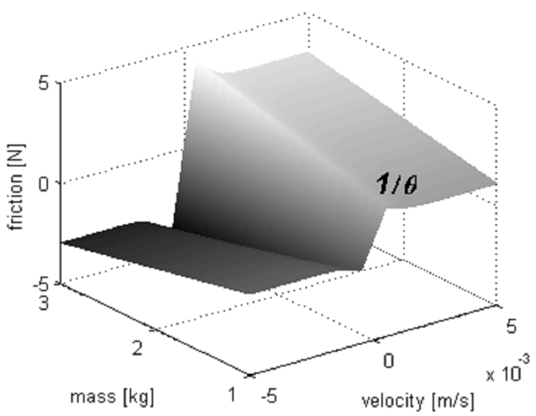

Fig. 2. Steady state friction surface for scale factor θ

3. Design of Control System

3.1 Fixed-type Friction Compensator

There are many compensation methods for the nonlinear friction force, and it needs to identify parameters for the friction model using this experiment. It can then be a design compensator based on a friction model by the observer (Wit, *et al*., 1995 and 2002; Celik, 1991; Jeons, 2000; Kim and Ha, 2002). The observer for the state of the bristle displacement z and the friction can be designed as Equation (8) and observation errors for these are described as Equation (9), respectively (Wit, *et al*., 1995).

$$\begin{aligned} \frac{d\hat{z}}{dt} &= v - \frac{\sigma_0|v|}{g(v)}\hat{z} - ke, \quad k > 0 \\ \hat{F} &= \sigma_0 \hat{z} + \sigma_1 \frac{d\hat{z}}{dt} + \sigma_2 v \end{aligned} \tag{8}$$

$$\begin{aligned} \frac{d\tilde{z}}{dt} &= -\frac{\sigma_0|v|}{g(v)}\tilde{z} + ke, k > 0 \\ \tilde{F} &= \sigma_0 \tilde{z} + \sigma_1 \frac{d\tilde{z}}{dt} \end{aligned} \tag{9}$$

where, $\tilde{F} = F - \hat{F}$ and $\tilde{z} = z - \hat{z}$. In order to converse the observer error, the friction error, and the position error $e = x - x_d$ to zero asymptotically, control law can be designed as follows:

$$u = -MH(s)e + \hat{F} + M\frac{d^2 x_d}{dt^2} \tag{10}$$

For stability analysis, it needs to choose an error equation and it can also be described as Equation (11) by using the friction estimate error given by Equation (9). Then, the transfer function for the entire error of the considered system is described as Equation (12).

$$\begin{aligned} e &= \frac{1}{Ms^2 + MH(s)}(-\tilde{F}) = \frac{\sigma_1 s + \sigma_0}{Ms^2 + MH(s)}(-\tilde{z}) \\ &= -G(s)\tilde{z} \end{aligned} \tag{11}$$

$$G(s) = \frac{\sigma_1 s + \sigma_0}{Ms^2 + H(s)} \tag{12}$$

The error state equation for the transfer function $G(s)$ can be described as Equation (13). The position controller $H(s)$ is possible to a PID controller form generally, while the integral function can be described by $K_i/(\tau s + 1)$ in order to be satisfied with SPR.

$$\begin{aligned} \frac{d\zeta}{dt} &= A\zeta + B(-\tilde{z}) \\ e &= C^T \zeta \end{aligned} \tag{13}$$

Also if the transfer function $G(s)$ is SPR (strictly positive real), a unique matrix $P = P^T > 0$, $Q = Q^T > 0$ satisfying $PA + A^T P = -Q$ and $PB = C^T$ exists. *Lyapunov function candidate* for closed-loop position control system can be chosen as Equation (14) by the *Kalman-Yakubovitch Theorem* (Khalil, 1992). And, if $\zeta \neq 0$, dV/dt can be written as Equation (15). Therefore, $e = C^T\zeta$ converges asymptotically to zero and the system is stable.

$$V = \zeta^T P \zeta + \frac{\tilde{z}^2}{k} \tag{14}$$

$$\begin{aligned}\frac{dV}{dt} &= -\zeta^T Q\zeta - 2\zeta^T PB\tilde{z} + \frac{2}{k}\tilde{z}\frac{d\tilde{z}}{dt} \\ &= -\zeta^T Q\zeta - 2e\tilde{z} + \frac{2}{k}\tilde{z}(-\frac{\sigma_0|v|}{g(v)}\tilde{z} + ke) \\ &= -\zeta^T Q\zeta - \frac{2}{k}\frac{\sigma_0|v|}{g(v)}\tilde{z}^2 \\ &\le -\zeta^T Q\zeta < 0\end{aligned} \tag{15}$$

3.2 Normal Force Adaptive Compensator

In the general transportation system, there are works of loading and unloading on the mover. These variations of mover's mass occur when there are changes in the normal force. Then, it has consequences for the friction force.

The friction state z and friction estimates can be described as Equation (16) by using Equation (5). When the estimates of scale factor of the normal force are $\hat{\theta}_1$ and it can be designed to Equation (17) by the adaptive law for the second-order system.

$$\frac{d\hat{z}}{dt} = v - \hat{\theta}_1\frac{\sigma_0|v|}{g(v)}\hat{z} - ke, \quad k > 0$$
$$\hat{F} = \sigma_0\hat{z} + \sigma_1\frac{d\hat{z}}{dt} + \sigma_2 v \tag{16}$$

$$\frac{d\hat{\theta}_1}{dt} = -\gamma_1\frac{\sigma_0|v|}{g(v)}\hat{z}(z_m - \hat{z}) \tag{17}$$

where, z_m is the filtered state model of z and it has the relationship $z_m = u_f - a_f$. The filtered signals u_f and a_f are described as following.

$$u_f = \frac{1}{\sigma_1 s + \sigma_0}u \tag{18}$$

$$a_f = \frac{ms}{\sigma_1 s + \sigma_0}v \tag{19}$$

For stability analysis, if $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$ then the equations of $\tilde{z}$ and $\tilde{\theta}_1$ can be obtained as follows:

$$\frac{d\tilde{z}}{dt} = -\theta_1\frac{\sigma_0|v|}{g(v)}\tilde{z} - \tilde{\theta}_1\frac{\sigma_0|v|}{g(v)}\hat{z} + ke$$
$$\frac{d\tilde{\theta}_1}{dt} = \gamma_1\frac{\sigma_0|v|}{g(v)}\hat{z}\tilde{z} \tag{20}$$

Also, it can choose the *Lyapunov function candidate* V as follows:

$$V = \zeta^T P\zeta + \frac{1}{\gamma_1}\tilde{\theta}_1^2 + \frac{\tilde{z}^2}{k} \tag{21}$$

and the time derivative of Equation (21), dV/dt is described by

$$\begin{aligned}\frac{dV}{dt} &= -\zeta^T Q\zeta - \theta_1\frac{\sigma_0|v|}{g(v)}\tilde{z}^2 \\ &\quad + \tilde{\theta}_1(-\tilde{z}\frac{\sigma_0|v|}{g(v)}\hat{z} + \frac{1}{\gamma_1}\frac{d\tilde{\theta}_1}{dt}) \\ &= -\zeta^T Q\zeta - \theta_1\frac{\sigma_0|v|}{g(v)}\tilde{z}^2 \\ &\le -\zeta^T Q\zeta < 0\end{aligned} \tag{22}$$

3.3 Adaptive Controller with Mass Estimator

Let's assume the parameters of dynamic friction and the initial mover's mass are known. However, these are changed during the time of control because of loading and unloading. Therefore, we designed the mass estimator that can scale the position controller $H(s)$ and constrict the estimates of the normal force factor more quickly.

Figure 3 shows the proposed control system and the system equation with control which is described below:

$$\begin{aligned}M\frac{d^2x}{dt^2} &= u - F \\ &= -\hat{M}H(s)e + \hat{M}\frac{d^2x_d}{dt^2} + \hat{F} - F\end{aligned} \tag{23}$$

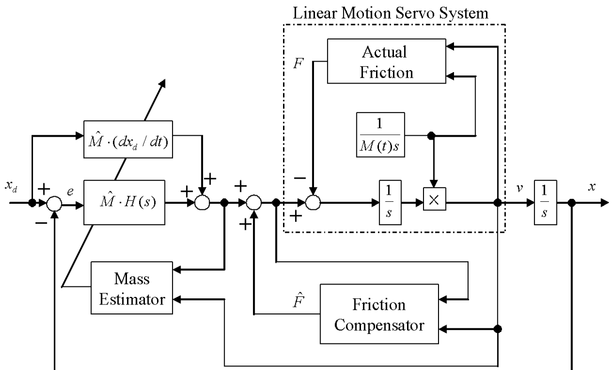

Fig. 3. Control system structure with the mass estimated adaptive controller and compensator

To design the mass estimator, it needs to assume there are no friction and other external forces in the system by the ideal compensator. The relation can then be obtained as follows (Ioannou, *et al.*, 1996):

$$u = M\frac{d^2x}{dt} = Ma = \frac{a}{\theta_2} \tag{24}$$

where, θ_2 is the estimates of mass, and we can know $\theta_2 = 1/M = a(t)/u(t)$. Then, its estimate can be written as Equation (25) and shown by Figure 4.

$$\hat{\theta}_2 = \frac{1}{\hat{M}(t)} = \frac{\hat{a}(t)}{u(t)} \tag{25}$$

To analyze the convergence of the mass estimator, let's define the estimate of the system output as $\varepsilon = a - \hat{a}$, and the error of estimate as $\tilde{\theta}_2 = \hat{\theta}_2 - \theta_2$. Then, ε can be rewritten as follows:

$$\varepsilon = a - \hat{a} = \theta_2 u - \hat{\theta}_2 u = -\tilde{\theta}_2 u \tag{26}$$

In order to minimize ε, the cost function is defined by

$$J(\hat{\theta}_2) = \frac{\varepsilon^2}{2} = \frac{(a - \hat{\theta}_2 u)^2}{2} \tag{27}$$

When $\nabla J(\hat{\theta}_2)$ is $(a - \hat{\theta}_2 u)u = 0$, $J(\hat{\theta}_2)$ has a minimum. Therefore, the equation of $\tilde{\theta}_2$ can be rewritten as Equation (28) by using the gradient method.

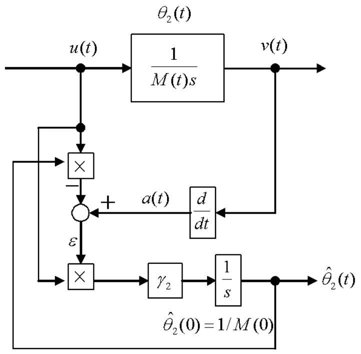

Fig. 4. Block diagram of the mass estimator for the plant without actual friction and compensator

$$\frac{d\hat{\theta}_2}{dt} = -\gamma_2 \nabla = \gamma_2 (a - \hat{\theta}_2 u) u = \gamma_2 \varepsilon u \tag{28}$$

where, $\hat{\theta}_2(0) = 1/M(0)$ and $\gamma_2 > 0$. To analysis the stability of the whole system, the Lyapunov function candidate V can be chosen as follows:

$$V = \zeta^T P \zeta + \frac{1}{\gamma_1}\tilde{\theta}_1^2 + \frac{\tilde{z}^2}{k} + \frac{1}{\gamma_2}\tilde{\theta}_2^{\,2} \tag{29}$$

Then, dV/dt can be obtained as Equation (30) under the conditions that $\dot{\tilde{\theta}}_2 = \dot{\hat{\theta}}_2 - \dot{\theta} = \gamma_2 \varepsilon u - \dot{\theta}_2 = -\gamma_2 u^2 \tilde{\theta}_2$ and $\dot{\theta}_2 = 0$.

$$\begin{aligned} \frac{dV}{dt} &= -\zeta^T Q \zeta - \theta_1 \frac{\sigma_0 |v|}{g(v)} \tilde{z}^2 \\ &\quad + \tilde{\theta}_1 \left(-\tilde{z}\frac{\sigma_0 |v|}{g(v)} \hat{z} + \frac{1}{\gamma_1}\frac{d\tilde{\theta}_1}{dt}\right) + \frac{2\tilde{\theta}_2 \dot{\tilde{\theta}}_2}{\gamma_2} \\ &= -\zeta^T Q \zeta - \theta_1 \frac{\sigma_0 |v|}{g(v)} \tilde{z}^2 - 2u^2 \tilde{\theta}_2^2 \\ &= -\zeta^T Q \zeta - \theta_1 \frac{\sigma_0 |v|}{g(v)} \tilde{z}^2 - 2\varepsilon^2 \le -\zeta^T Q \zeta < 0 \end{aligned} \tag{30}$$

4. Simulation Results

In this paper, we performed the numerical simulations to verify the proposed control system about the position tracking control using the Matlab/Simulink module. These results include the asymptotic tracking performances for the distance errors, the friction estimates, and the mass and normal force estimate, etc. The performances were compared with four other cases. These are described as follows:

· Case I: PID type controller
· Case II: PID+compensator with fixed parameters
· Case III: Normal force adaptive compensator
· Case IV: III+scaling $H(s)$
· Case V: Adaptive controller with mass estimator

where, in gains tuning of $H(s)$, Matlab NCD(nonlinear control design blockset) toolbox was used. And $H(s)$ structure is Equation (31). The parameters used here are described by table 1. It was referenced to be compared by (Wit, *et al*., 1995).

$$H(s) = E(s)(K_p + K_d s + \frac{K_i}{\tau s + 1}) \tag{31}$$

Table 1 The used parameters

Parameters	Values	Parameters	Values
σ_0	100,000 [N/m]	load mass	1 [kg]
σ_1	$\sqrt{1000{,}000}$ [Ns/m]	γ_1	500,000
σ_2	0.4 [Ns/m]	γ_2	1.038
F_C	1 [N]	K_i	4
F_S	1.5 [N]	K_p	3
v_s	0.001 [m/s]	K_d	6
initial M	1 [kg]	τ	0.1

The simulation scenario is following, (i) is described the variation of mover's mass and (ii) is the reference trajectory.

(i) $M(t) = \begin{cases} 1\,[kg], & 0 \le t < 10 \\ 2\,[kg], & 10 \le t < 20 \\ 3\,[kg], & 20 \le t \le 30 \end{cases}$

(ii) $x_d(t) = 1 + \sin(0.1\pi \cdot t + 1.5 \cdot \pi)$

Figures 5 and 6 show the results of the displacement and its error, respectively. Figure 7 shows the control input. Though there are loading works, the proposed control scheme shows that the system output is unique for the varying friction.

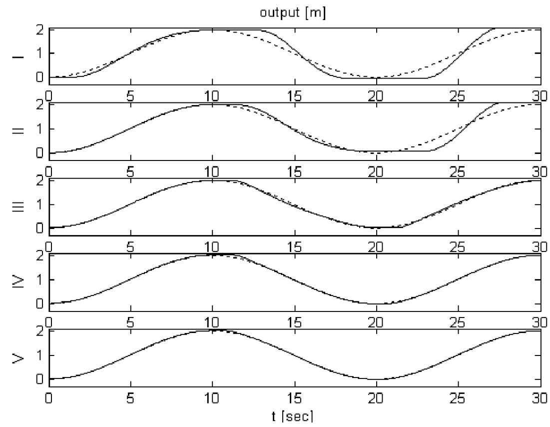

Fig. 5. Distance variations (solid line) for the reference trajectory (dotted line)

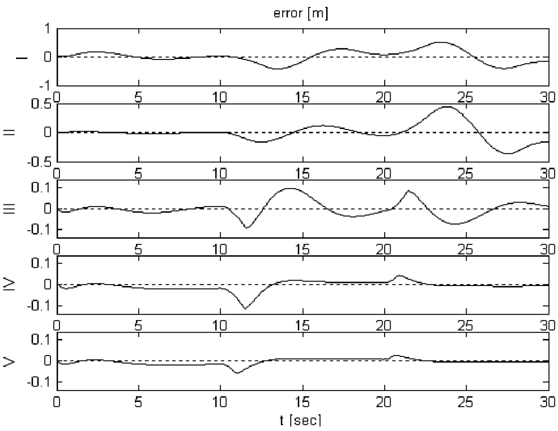

Fig. 6. Variations of the position error

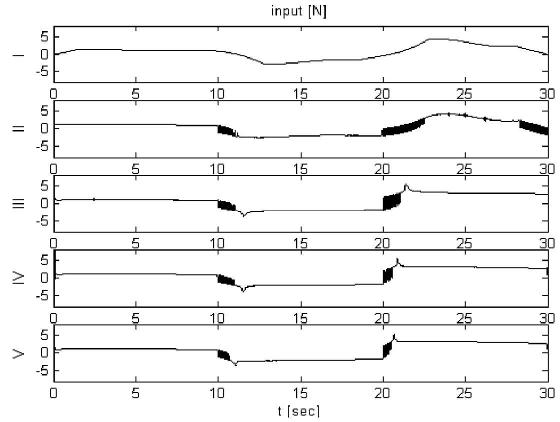

Fig. 7. Control input

Figure 8 shows the friction model output and estimates and figure 9 presents the errors between the friction and the compensator output. As the proposed scheme comes nearer, we can see that the range of trigger was reduced furthermore. From estimates of the normal force and mover's mass shown in Figures 10 and 11, we can see that the performance of convergence is increased. In particular, the proposed method reduced the time of convergence and error by using two factors $\hat{\theta}_1$ and $\hat{\theta}_2$. Estimates of the mass can be deduced by these two factors. Furthermore, figure 12 shows that the integrated root-mean square error of the proposed scheme was less than others until the final time.

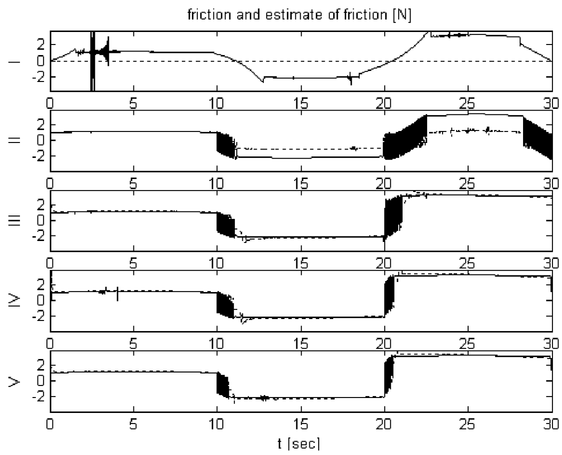

Fig. 8. Friction variations (solid line) and estimates of the friction (dotted line)

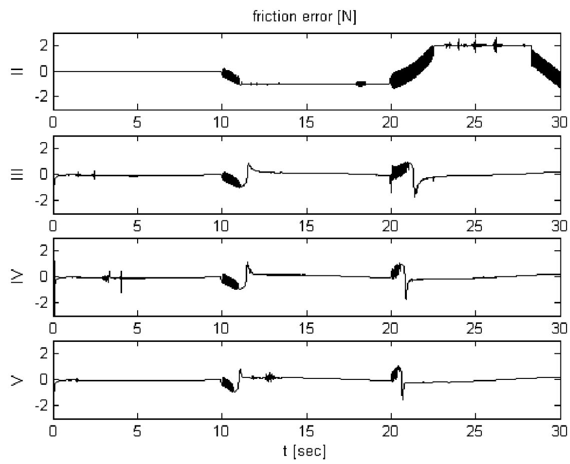

Fig. 9. The friction estimate errors

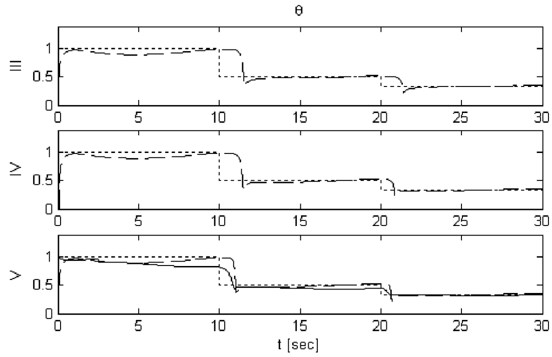

Fig. 10. Variations of the normal force (dotted line) and its estimates ($\hat{\theta}_1$: dashed line, $\hat{\theta}_2$: solid line)

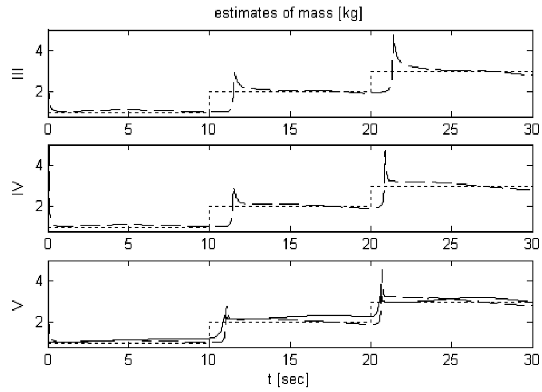

Fig. 11. Variations of mass and its estimates

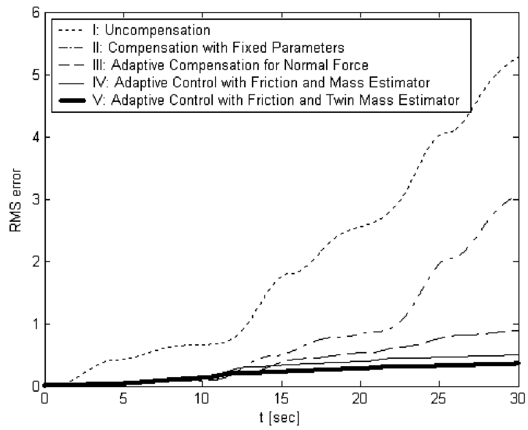

Fig. 12. Variations of integration of RMS error

5. Conclusion

Generally, In the case of a low speed and a precision position control, the friction has a bad influence on the system. But then it is impossible to control by measuring the friction. Therefore, many modeling and compensation methods have been proposed until now.

We proposed the adaptive control system with the mass estimator in this paper for the nonlinear dynamic friction. Therefore, this method got the adaptation on the compensation and the position controller gain scheduling for various mover's load mass. However, in the mass estimator and the normal force observer, the velocity of convergence must be improved more rapidly near the start time. Furthermore, it needs to estimate other parameters in the friction model for more precision control.

Acknowledgment

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