GPS Satellite Orbit Prediction Based on Unscented Kalman Filter

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ABSTRACT

In GPS Positioning, the error of satellite orbit will affect user's position accuracy directly, it is important to determine the satellite orbit precise. The real-time orbit is needed in kinematic GPS positioning, the precise GPS orbit from IGS would be delayed long time, so orbit prediction is key to real-time kinematic positioning. We analyze the GPS predicted ephemeris, on the base of comparison of EKF and UKF, a new orbit prediction method is put forward based on UKF in this paper, the result shows that UKF improves the orbit predicted precision and stability. It offers a new method for others satellites orbit determination as Galileo, and so on.

Keyword: Unscented Kalman Filter, Extended Kalman Filter, GPS Satellite Orbit Prediction, Orbital Integrator, Dynamic Model, State Transition Matrix

1. Introduction

In GPS precise positioning , orbit determining and navigation, GPS satellites are looked as observation targets with known locations, therefore, GPS satellite orbit error would affects the precision of user' receiver in precise point positioning (PPP), directly. In real time kinematic PPP, the final IGS orbit products will be distributed after thirteen days, even, the rapid products will be distributed after seventeen hours [1], as table 1. It is difficult to implement real-time kinematic positioning with these GPS ephemeris products. So, we must to utilize ultra-rapid orbit, broadcast ephemeris or predicted orbit.

Table 1 GPS satellite orbit and clock correction

		accuracy	latency	update	Sample interval
Broadcast	orbits	160cm	RT		daily
	Sat.clocks	7ns			
Ultra-Rapid (predicted half)	orbits	10cm	RT	4 times daily	15min
	Sat.clocks	5ns			
Ultra-Rapid (observed half)	orbits	<5cm	3 hours	4 times daily	15min
	Sat.clocks	0.2ns			
Rapid	orbits	<5cm	17hours	daily	15min
	Sat.clocks	0.1ns			5min
Final	orbits	<5cm	13 days	weekly	15min
	Sat.clocks	0.1ns			5min

There are two main GPS orbit prediction methods; they are geometric and dynamic method. Geometric method means extrapolation orbit with function model or mathematic process based on observed orbit, it is simple and convenient in computation but low extrapolation precision, instability and limited extrapolation length; dynamic one includes analytical method and numerical method, that is getting parameter states and state transition matrix with satellite motion equation and measurement one, and then integration orbit, its merit is high orbit prediction precision and extrapolation stability, but it is suffered from satellite motion model and perturbing force, and difficult to compute and predict. To make sure stability and liability precision of orbit prediction, dynamic method to predict orbit is used commonly. Aim to dynamic prediction as study object, according to the comparison of advantages and flaws of extended kalman filter (EKF) and unscented kalman filter (UKF), a new GPS satellite motion orbit prediction method is put forward in this paper.

EKF is a most common used recursive filter algorithm in nonlinear system, it is applied to nonlinear parameter estimation widely, including nonlinear motion system, nonlinear systematic state estimation and parameter estimation, ground vehicle navigation system and so on, its main idea is to develop the nonlinear motion equation and measurement equation to the first-order with Taylor Series, and then process it with traditional kalman filter. But the linearized second-order even high-order will reduce EKF estimation precision because of interceptive error, even lead to filter divergence, instability. Meanwhile, it is not adapt to motion equation with too many perturbed items, and it needs more time-consuming calculations. It is showed that it is little effect to filter initial stage precision from linear error and model error of motion equation nonlinear system, and main effect to long time recursive precision after analyzing the error of nonlinear system linearization, the effect from measurement equation nonlinear system linearization error is opposite to that of motion equation nonlinear system. It is showed that linearization error of nonlinear system is the key effect to whether kalman filter is valid or not [2, 3]. The general idea of UKF is similar to that of EKF; the key difference between them is that they determine predicted mean value and covariance matrix with different ways. UKF estimates nonlinear transformed probability density of random variable but not developed nonlinear function with Taylor Series, the processing steps of UKF is avoid to linearizing nonlinear motion equation and measurement equation as EKF, but using Unscented Transformation-UT, that is, processing nonlinear motion equation and measurement equation with nonlinear method directly, and it is not necessary to calculate Jacobians, so the algorithm has superior implementation properties to the EKF.

The most common orbit prediction and correction with dynamic approach is to use numerical method to calculate precise ephemeris and state transition matrix because of nuisance force model, but it is obtained by solving two groups of ordinary differential equations, complicated and nuisance calculation[5], we can calculate state transition matrix with analytical method , this method is not only very efficient for the case that the orbit determining or prediction arc is not too long as GPS real-time kinematic PPP, but also it can avoid to the integration of two groups of differential equations at the same time.

2. Dynamic model

There are two choices to orbit parameters, one is satellite orbit elements $\psi\ (a,e,i,\Omega,\omega,M)$, and the other is satellite coordinate vector $\vec{r}$ and velocity vector $\dot{\vec{r}}$. In LEO satellite precise orbit determination, the common approach is looking on satellite coordinate vector $\vec{r}$ and velocity vector $\dot{\vec{r}}$ as orbit parameter because of complex force model, but sometimes, we take on satellite orbit elements $\psi\ (a,e,i,\Omega,\omega,M)$ or coordinate vector and velocity vector $(\vec{r},\dot{\vec{r}})$ as orbit parameter to medium or high earth orbiter satellite because high altitude orbiter and ignoring some tiny perturbed items as GPS.

(1) Taking on satellite orbit elements as orbit parameters

Satellite perturbed equation is as follow:

$$\ddot{\vec{r}} = -\frac{GM}{r^3}\vec{r} + \vec{a}_p \tag{1}$$

$\vec{a}_p = \vec{a}_{nonspherical} + \vec{a}_{sun-radiation} + \vec{a}_{3-body} + \vec{a}_{others}$, the detail form of every variable in perturbed acceleration see in Ref. [6].

Satellite motion model (GPS satellite orbit elements differential equation) can express as follows [7]:

$$\dot{a} = \sqrt{\frac{p}{GM}} \cdot \frac{2a}{1-e^2} \cdot \left\{ e \cdot \sin v \cdot R + \frac{p}{r} \cdot S \right\} \tag{2}$$

$$\dot{e} = \sqrt{\frac{p}{GM}} \cdot \{\sin v \cdot R + (\cos v + \cos E) \cdot S\} \tag{3}$$

$$i^{(1)} = \frac{r \cdot \cos(\omega + v)}{n \cdot a^2 \cdot \sqrt{1-e^2}} \cdot W \tag{4}$$

$$\dot{\Omega} = \frac{r \cdot \sin(\omega + v)}{n \cdot a^2 \cdot \sqrt{1-e^2} \cdot \sin i} \cdot W \tag{5}$$

$$\dot{\omega} = \frac{1}{e} \cdot \sqrt{\frac{p}{GM}} \cdot \left\{ -\cos v \cdot R + \left(1 + \frac{r}{p}\right) \cdot \sin v \cdot S \right\} - \cos i \cdot \dot{\Omega} \tag{6}$$

$$\dot{M} = \frac{1}{na} \cdot \frac{1-e^2}{e} \cdot \left\{ \left(\cos v - 2e\frac{r}{p}\right) \cdot R - \left(1 + \frac{r}{p}\right) \cdot \sin v \cdot S \right\} + \frac{3}{2} \cdot \frac{n}{a} \cdot (t - t_0) \cdot \dot{a} \tag{7}$$

In formula (2) ~(7), $p = a \cdot (1-e^2)$ is ellipse parameter, v is anomaly, r is the range from satellite to earth-center, GM is the earth's gravitational constant, n is mean motion, R, S, W are accelerations of radial, along-track and cross-track, respectively.

In short-arc orbit prediction, the position and velocity at t epoch can derive from mean orbit elements at the same epoch, directly, after that, period item of earth non-spherical gravitational field perturbing force can be showed by satellite coordinate and velocity in rectangle coordinate system to distinguish long periodic item and short periodic item, and deduce corresponding coordinate updates and velocity ones, for so, it can avoid the problem of singularity in computing periodic of orbit elements variation[8]. It can relax the choice condition to orbit elements and applied to all type of orbits prediction.

(2) Taking on satellite coordinate and velocity vector as orbit parameters

Taking on coordinate vector $\vec{r}$ and velocity vector $\dot{\vec{r}}$ as orbit parameter, state vector Z is:

$$Z = \begin{pmatrix} \vec{r} \\ \dot{\vec{r}} \end{pmatrix} \tag{8}$$

Corresponding state differential equations are as follows:

$$\begin{cases} \dot{X} = f(X, t; \varepsilon) \\ t = t_0, X(t_0) = X_0 \end{cases} \tag{9}$$

Where, ε is the perturbed parameter. The solution of formula (9) can be expressed as:

$$X = X(t, t_0, X_0; \varepsilon) \tag{10}$$

Corresponding conditional equation is got from measurement equation, the measurement equation is:

$$Y = H(X, t) + v \tag{11}$$

Where, v is the random error in observation. Let conditional equation as follow:

$$y = Fx_0 \tag{12}$$

Computing x_0 from formula (12), the estimated state vector X_0 (updated orbit) is:

$$X_0 = X_0^* + x_0 \tag{13}$$

Update the state matrix by updated orbit, and predict the orbit of next epoch, analogizing along.

(3) Transformation between orbit elements and ECEF coordinates and velocity

Supported that the coordinate and velocity of GPS satellite in ECEF is $(x,\ y,\ z)$ and $(\dot{x},\ \dot{y},\ \dot{z})$, respectively, and coordinate is $(\xi_s,\ \eta_s,\ \zeta_s)$ in orbit coordinate system, ignore the polar effect, the transformation of GPS satellite coordinate between ECEF and orbit coordinate system is[9,10]:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R_3(GAST)R_3(-\Omega)R_1(-i)R_3(-\omega_s)\begin{pmatrix} \xi_s \\ \eta_s \\ \zeta_s \end{pmatrix} \quad (14)$$

Where, $GAST$ is Greenwich mean sidereal time, ECEF is Earth Centered Earth Fixed.

$$R_3(GAST) = \begin{pmatrix} \cos(GAST) & \sin(GAST) & 0 \\ -\sin(GAST) & \cos(GAST) & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$R_3(-\Omega) = \begin{pmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$R_1(-i) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{pmatrix},$$

$$R_3(-\omega_s) = \begin{pmatrix} \cos\omega_s & -\sin\omega_s & 0 \\ \sin\omega_s & \cos\omega_s & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (15)$$

Supported,

$$k = R_3(-\Omega)R_1(-i)R_3(-\omega_s) \quad (16)$$

According to the formula (3-21) in Ref. [9], then formula (16) is obtained ulteriorly:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R_3(GAST)k\begin{pmatrix} a_s(\cos E_s - e_s) \\ a_s\sqrt{1-e_s^2}\sin E_s \\ 0 \end{pmatrix} \quad (17)$$

The corresponding relationship between satellite velocity and orbit elements are:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = -R_3(GAST)k\begin{pmatrix} a_s(\cos E_s - e_s) \\ a_s\sqrt{1-e_s^2}\sin E_s \\ 0 \end{pmatrix} + R_3(GAST)k\begin{pmatrix} -\frac{a_s^2 n}{r}\sin E_s \\ \frac{a_s^2 n}{r}\sqrt{1-e_s^2}\cos E_s \\ 0 \end{pmatrix} \quad (18)$$

3. General Law of UKF method

(1) Basic idea

In the processing of UKF, firstly, choosing sigma points of adapted to sample mean and sample covariance matrix of systematic vector based on systematic a priori information, if the filtering value and filtering error covariance matrix are $\hat{x}_{k-1}$ and P_{k-1} at $k-1$ epoch respectively, we can obtain n points by row or column of $\left(\sqrt{(n+\lambda)P_{k-1}}\right)_i$, also, we can obtain the other n points by row or column of $\left(-\sqrt{(n+\lambda)P_{k-1}}\right)_i$, their sample mean and sample covariance are $\hat{x}_{k-1}$ and P_{k-1}, adding the central point, the total points are $(2n+1)$, their filtering value and filtering error covariance matrix are $\hat{x}_{k-1}$ and P_{k-1}, secondly, substituting these $(2n+1)$ points into system nonlinear equation to transformation directly, then, predicting sample mean $\hat{x}_{k|k-1}$ and sample covariance $P_{k|k-1}$ by weighting transformed $(2n+1)$ points, finally, calculating prediction measurements by prediction state matrix, and updating prediction state parameter by observed measurement[11], analogizing along.

(2) UKF algorithm and state estimation [2, 11]

Supported that systematic motion model and measurement model are as follows:

$$x_{k+1} = F(x_k, u_k, w) \quad (19)$$

$$y_k = H(x_k, v_k, w),\ \ k \in [0,\ \ \infty] \quad (20)$$

Initialize with:

$$\hat{x}_0 = E[x_0],\ P_0 = E\left[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T\right] \quad (21)$$

$$\hat{x}_0^a = E[x^a] = \left[\hat{x}_0^T \quad 0 \quad 0\right]^T \quad (22)$$

$$P_0^a = E\left[(x_0^a - \hat{x}_0^a)(x_0^a - \hat{x}_0^a)^T\right] = \begin{bmatrix} P_0 & 0 & 0 \\ 0 & P_u & 0 \\ 0 & 0 & P_v \end{bmatrix} \quad (23)$$

Nonlinear function propagation:

$$\gamma_i = g(\chi_i),\ i = 0,1,\cdots,2n \quad (24)$$

$$\bar{y} \approx \sum_{i=0}^{2n} W_i^{(m)}\gamma_i \quad (25)$$

$$P_y \approx \sum_{i=0}^{2n} W_i^{(c)}\{\gamma_i - \bar{y}\}\{\gamma_i - \bar{y}\}^T \quad (26)$$

Calculate the sigma points:

$$\chi_{k-1}^{a}=\left[\hat{x}_{k-1}^{a} \quad \hat{x}_{k-1}^{a}+\sqrt{(n+\lambda)P_{k-1}^{a}} \quad \hat{x}_{k-1}^{a}-\sqrt{(n+\lambda)P_{k-1}^{a}}\right] \quad (27)$$

Where, $x^{a}=\left(x^{T} \quad u^{T} \quad v^{T}\right)^{T}, \chi^{a}=\left((\chi^{x})^{T} \quad (\chi^{u})^{T} \quad (\chi^{v})^{T}\right)^{T}$.

Time update:

$$\chi_{k|k-1}^{x}=F\left[\chi_{k-1}^{x}, \chi_{k-1}^{u}\right] \quad (28)$$

$$\hat{x}_{k}^{-}=\sum_{i=0}^{2n} W_{i}^{(m)} \chi_{i,k|k-1}^{x} \quad (29)$$

$$P_{k}^{-}=\sum_{i=0}^{2n} W_{i}^{(c)}\left\{\chi_{i,k|k-1}^{x}-\hat{x}_{k}^{-}\right\}\left\{\chi_{i,k|k-1}^{x}-\hat{x}_{k}^{-}\right\}^{T} \quad (30)$$

$$\gamma_{k|k-1}=H\left[\chi_{k|k-1}^{x}, \chi_{k-1}^{v}\right] \quad (31)$$

$$\hat{y}_{k}^{-}=\sum_{i=0}^{2n} W_{i}^{(m)} \gamma_{i,k|k-1} \quad (32)$$

Measurement update:

$$P_{\hat{y}_k \hat{y}_k}=\sum_{i=0}^{2n} W_{i}^{(c)}\left\{\gamma_{i,k|k-1}-\hat{y}_{k}^{-}\right\}\left\{\gamma_{i,k|k-1}-\hat{y}_{k}^{-}\right\}^{T} \quad (33)$$

$$P_{x_k y_k}=\sum_{i=0}^{2n} W_{i}^{(c)}\left\{\chi_{i,k|k-1}-\hat{x}_{k}^{-}\right\}\left\{\gamma_{i,k|k-1}-\hat{y}_{k}^{-}\right\}^{T} \quad (34)$$

$$K_{k}=P_{x_k y_k} P_{\hat{y}_k \hat{y}_k}^{-1} \quad (35)$$

$$\hat{x}_{k}=\hat{x}_{k}^{-}+K_{k}\left(y_{k}-\hat{y}_{k}^{-}\right) \quad (36)$$

$$P_{k}=P_{k}^{-}-K P_{\hat{y}_k \hat{y}_k} K_{k}^{T} \quad (37)$$

The n dimensional random variable x with mean $\bar{x}$ and covariance P_{xx} is approximated by $2n+1$ weighted points given by:

$$\chi_{0}=\bar{x}, \quad W_{0}^{(m)}=\frac{\lambda}{n+\lambda}, \quad i=0$$

$$\chi_{i}=\bar{x}+\left(\sqrt{(n+\lambda)P_{xx}}\right)_{i},$$

$$W_{0}^{(c)}=\frac{1}{n+\lambda}+\left(1-\alpha^{2}+\beta\right), \quad i=1,2,\cdots,n$$

$$\chi_{i}=\hat{x}-\left(\sqrt{(n+\lambda)P_{xx}}\right)_{i}, \quad W_{i}^{(m)}=W_{i}^{(c)}=\frac{1}{2(n+\lambda)},$$

$$i=n+1, n+2, \cdots, 2n \quad (38)$$

Where, $x^{a}=\left[x^{T} \quad u^{T} \quad v^{T}\right]$, $\chi^{a}=\left[(\chi^{x})^{T} \quad (\chi^{u})^{T} \quad (\chi^{v})^{T}\right]$, $\lambda=\alpha^{2}(n+\kappa)-n$ is scale parameter, α is for 10^{-3}, κ is the second scale parameter, common is zero, β is showed a priori information of x (if it obey to Gauss distribution, β =2), if dynamic noise vector obeys standard normal distribution, $\lambda=3-n$; $\left(\sqrt{(n+\lambda)P_{k-1}}\right)_{i}$ is the i low or i column of root square of matrix $(n+\lambda)P_{k-1}$, $W_{i}=\lambda/(n+\lambda)$, $(i=0)$, $W_{i}^{(m)}=W_{i}^{(c)}=1/[2(n+\lambda)]$, $(i\neq 0)$ is the weight of corresponding point i, subscript m and c are mean index and covariance one, respectively, respectively. We can obtain different parameter and estimated error variance matrix with the different m and c, it is worth to discuss the value of m and c. What is more, from the above mentioned processing, we can see that the Jacobians matrix is not necessary to be calculated in UKF processing.

4. GPS orbit prediction based on UKF

(1) Predicting GPS orbit with initial coordinate

Supported that orbit coordinate $(\hat{x}_0, \ \hat{y}_0, \ \hat{z}_0)$ and $(\hat{x}_1, \ \hat{y}_1, \ \hat{z}_1)$ at t_0, t_1 is known respectively, therefore, we can obtain the estimated velocity at t_1 as follows:

$$\hat{\dot{x}}_{1}=\frac{\hat{x}_{1}-\hat{x}_{0}}{t_{1}-t_{0}}, \hat{\dot{y}}_{1}=\frac{\hat{y}_{1}-\hat{y}_{0}}{t_{1}-t_{0}}, \hat{\dot{z}}_{1}=\frac{\hat{z}_{1}-\hat{z}_{0}}{t_{1}-t_{0}} \quad (39)$$

Now, we can obtain satellite initial coordinate and velocity as follows:

$$\hat{r}_{1}=\begin{pmatrix}\hat{x}_{1} \\ \hat{y}_{1} \\ \hat{z}_{1}\end{pmatrix}, \quad \hat{\dot{r}}_{1}=\begin{pmatrix}\hat{\dot{x}}_{1} \\ \hat{\dot{y}}_{1} \\ \hat{\dot{z}}_{1}\end{pmatrix} \quad (40)$$

The corresponding estimated error variance matrixes are:

$$Var\left(\hat{r}_{1}\right)=P_{r}, \quad Var\left(\hat{\dot{r}}_{1}\right)=\frac{P_{r}}{\left(t_{1}-t_{0}\right)^{2}} \quad (41)$$

Where, P_r is the estimated error variance matrix of $\hat{r}_1$ at t_1, we can calculate the estimated orbit elements $\hat{\psi}_1$ of ψ_1 at t_1 by $(\hat{r}_1 \quad \hat{\dot{r}}_1)$, the corresponding estimated error variance matrix is [12]:

$$Var\left(\hat{\psi}_{1}\right)=\begin{pmatrix}\frac{\partial \hat{r}_{1}}{\partial \hat{\psi}_{1}} \\ \frac{\partial \hat{\dot{r}}_{1}}{\partial \hat{\psi}_{1}}\end{pmatrix}^{-1}\begin{pmatrix}Var\left(\hat{r}_{1}\right) & 0 \\ 0 & Var\left(\hat{\dot{r}}_{r}\right)\end{pmatrix}\begin{pmatrix}\frac{\partial \hat{r}_{1}}{\partial \hat{\psi}_{1}} \\ \frac{\partial \hat{\dot{r}}_{1}}{\partial \hat{\psi}_{1}}\end{pmatrix}^{-T} \quad (42)$$

Denoting orbit elements with $\psi(a,e,i,\Omega,\omega,M)$, we can calculate orbit elements as state vector by satellite initial position and velocity $(\hat{r}_1 \quad \hat{\dot{r}}_1)$ at t_1 epoch with UKF algorithm, the satellite nonlinear dynamic model and measurement equation in known, if the state filtering $\hat{\psi}(k|k)$

and filtering error matrix $P(k|k)$ at t_k epoch are known, the steps of predicting and correcting orbit at t_{k+1} epoch are as follows:

(1) Determining dynamic model and measurement equation of satellite motion system;

We should determine dynamic model and measurement equation of satellite motion system, obtain the initial value of orbit prediction.

(2) Calculating and determining $2n+1$ sigma points based on formula (27);

(3) Calculating state parameter prediction x_{k+1}^{-};

Obtaining weight $W_i^{(m)}$ of corresponding point and sigma point $\chi_{i,k}^{x}$ from formula (38), computing x_{k+1}^{-} of t_{k+1} epoch by state filtering value x_k at t_k epoch according to formula (29). Where, the key to this algorithm is the determining of weight of sigma point and that of state parameter and estimated error variance matrix.

(4) Calculating predicted error variance matrix $P(k+1|k)$, see formula (30);

$$P(k+1|k)=\Phi(k+1,k)P(k|k)\Phi(k+1,k)^T+\varepsilon(k+1) \qquad (44)$$

Where, $\Phi(k+1,k)$ is the state transition matrix from epoch t_k to t_{k+1}, see in Ref.[13] as follows:

$$\begin{cases}\mathring{\Phi}(t_{k+1},t_k)=B(t)_k\,\Phi(t_k,t_0)\\ \Phi(t_0,t_0)=known\end{cases} \qquad (45)$$

We can compute $\Phi(k+1,k)$ according to formula (45), where, $B(t)_k$ is linear matrix developed from state equation (9) with Taylor series in EKF algorithm; $\varepsilon(k+1)$ is variance matrix of model error, to compensate by processing noise.

(5) Calculating gain matrix K_k;

We can compute $P_{\hat{y}_k\hat{y}_k}$ and $P_{x_k y_k}$ from formula (33) and (34), respectively, then, calculate gain matrix K_k from $P_{\hat{y}_k\hat{y}_k}$ and $P_{x_k y_k}$.

(6) Measurement correction

The orbit prediction at t_{k+1} epoch can be numerical integrated with satellite motion equation by KSG or 21-order Runge-Kutta integrator according to the above calculated state transition matrix and gain matrix at t_k. Then, we can update prediction, estimated error variance matrix and state transition matrix with observed measurement, see in formula (35) ~ (37), as a matting for next epoch. The perturbing item in satellite motion equation, besides earth-central force, includes earth's non-spherical gravitational field perturbing (about $5\times10^{-5}\,m/s^2$), force from third-body effects (about $5\times10^{-6}\,m/s^2$), solar radiation pressure (about $10^{-7}\,m/s^2$) to medium or high earth orbiter satellite as GPS and so on, and ignoring some tiny perturbed items as earth tidal correction [9].

(7) Predicting state parameter at t_{k+2} by t_{k+1} epoch, redo step (2) to (6), analogizing along.

(8) Deriving non- integral step solution with interpolation;

It can meet the need of orbit determination precision with poly-orders (10orders) Lagrange interpolation method.

(2) Predicting GPS satellite orbit with prediction ephemeris

The navigation message distributed from GPS satellite includes broadcast ephemeris and prediction ephemeris, taking on initial prediction ephemeris of navigation message as initial value, we can obtain the required step satellite orbit with formula (2) ~ (7) by KSG or 21-order Runge-Kutta integrator (with short arc）, deriving non-integral step solution with interpolation, interpolate GPS satellite orbit with poly-orders (10orders) Lagrange interpolation method to every epoch. Then update measurement, estimated error variance matrix and state transition matrix with observed measurement, and integrate along, analogizing along. The detailed definition, expression and calculation formula see in Ref. [15].

5. Result analysis

To test the reliability and precision of above mentioned algorithm, we choose the IGS orbit products and ephemeris prediction parameter on 1st, May, 2006 (the Number of week is 1373) [17], then transform the broadcast ephemeris of this day into WGS-84 coordinate system, and contrast the transformed coordinate to the predicted orbit obtained with the above mentioned algorithm, the initial orbit elements are in table 2 as follows:

Table2. Initial orbit elements

M_0	3.05523608442	Ω_0	0.882840250320
e	$0.969721737783\times10^{-2}$	i_0	0.957271722804
$\sqrt{A}$	$5.15370683479\times10^{3}$	ω	2.59834658731
T_{oa}	93568	$\dot{\Omega}$	$-0.77528229362\times10^{-8}$

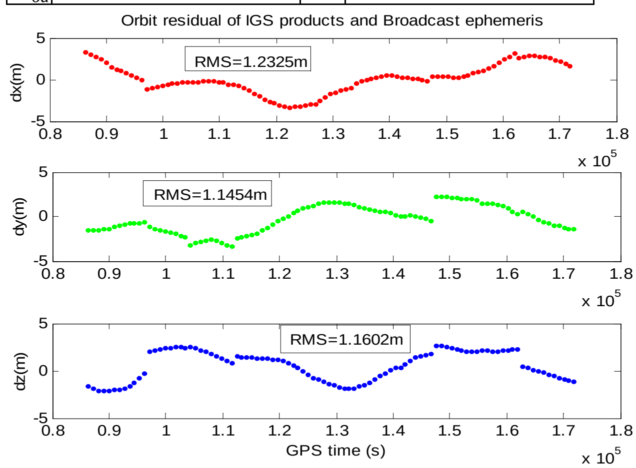

Fig.1. the residual of calculated orbit and IGS orbit products

the sampling interval of prediction ephemeris of navigation in figure 1 is 15min. on 1st, May, 2006, we transform the ephemeris predict parameter into WGS-84 coordinate system, comparing the coordinate in WGS-84 transformed from ephemeris predict parameter of navigation to IGS products, it is tested that the residual between the coordinate transformed from ephemeris predict parameter and IGS products is 2.044 meters (1.2325m, 1.1454m and 1.1602m).

Now, we predict GPS orbit with above mentioned dynamic model, measurement model, KSG integrator and initial orbit prediction ephemeris based on EKF and UKF algorithm, respectively, to test the liability and accuracy or UKF, just taking earth's gravitational field non-spherical perturbing, third-body perturbing and solar radiation pressure into account, and taking on IGS final orbit products as reference, the result analysis is showed in table 3 as follows:

Table3. The comparisons result (RMS: cm)

time	algorithm	x	y	z	Position
1-hour	EKF	10.2	9.9	10.5	17.67
	UKF	8.3	7.7	8.8	14.34
6-hour	EKF	20.6	21.0	21.8	36.61
	UKF	16.0	16.5	18.0	29.19
24-hour	EKF	40.0	42.5	41.9	71.85
	UKF	30.4	31.6	32.0	54.28

Under taking the same perturbing items into account, the calculation result showed that the error is relative large because we simplified the perturbing force ,but the result from UKF is a bit better than that of EKF, it is showed that the UKF algorithm improved the precision of orbit determination than that of EKF, the more important point is that UKF has the untouchable merits than EKF, what is more, in the calculation, we found that the calculation time is more shorten with UKF than EKF with the arc increasing, and we can not statistic them because of paper length. If the integration step is too long, and avoiding to substituting processing noise into model error, all UKF and EKF are divergence, but the UKF divergence time is later than that of EKF, it is showed that the EKF is affect by linearity model error of nonlinear system except for integration step, inherent model error. Therefore, it is valid to improve the stability and precision of GPS orbit prediction based on UKF. It offers a new method for some others satellites orbit determination as Galileo.

References

1. http://igscb.jpl.nasa.gov/components/prods.html。

2. Eric A.Wan and Rudolph van der Merwe, The Unscented Kalman Filter for Nonlinear Estimation, IEEE: 2000 P153-158.

3.Rachel Kleinbauer, Kalman Filtering Implementation with Matlab[R], Stuttgart University, Helsinki, 2004.11.

4. Mathieu St-Pierre, Denis Gingras, Comparsion between the Unscented Kalman Filter and the Extended Kalman Filter for the position estimation module of an integrated navigation information system, 2004 IEEE Intelligent Vehicles Symposium, University of Parma, Parma, Italy, June 14-17,2004.

5.ZHANG Qiang, LIU Lin, An Analytical Method of Computing the State Transition Matrix in Orbit Determination [J], Acta Astronomica Sinica, 1999.5 Vol.40, No.2.P113-121.(In Chinese)

6. David A., Vallodo, Scort S.Carter, Accurate Orbit Determination from Short-arc Dense Observational Data, JAS915.

7. JIAO Wenhai, Study on Construct Satellite Navigation System Coordinate Reference [R], Shanghai Astronomical Observatory, Chinese Academy of Sciences, 2003. (In Chinese)

8. LIU Lin, WANG Yanrong, An Analytical Method for Satellite Orbit Forecasting, Acta Astronomica Sinica, 2005.7, Vol.46, No.3.P307-313. (In Chinese)

9.ZHOU Zhong mo, YI Jiejun, ZHOU Qi, GPS Satellite Surveying Theory and Application [M], Beijing: Surveying and Mapping Publishing House, 1999. (In Chinese)

10. WEI Ziqing, GE Maorong, Mathematic Model of GPS Relative Positioning [M], Beijing: Surveying and Mapping Publishing House, 1998. (In Chinese)

11.Simon J.Julier, Jeffrey K.Uhlmann, A New Extension of the Kalman Filter to Nonlinear Systems[C], Proceeding of Aerosense: the 11th Int. Symp.on Aerospace/Defence Sensing Simulation and Controls, 1997.

12. JIA Peizhang, XIONG Yongqing, An Orbit Determining Algorithm with Onboard GPS using Kalman Filter [J], Acta Astronomica Sinica, 2005.10, Vol46, No.4. (In Chinese)

13. HUANG Cheng, FENG Chugang, SLR Data Processing and its Software Implememtation (teaching materials), Shanghai Astronomical Observatory, Chinese Academy of Sciences, 2003. (In Chinese)

14. Byron D.Tapley, Bob E.Schutz, George H.Born, Statistical Orbit Determination, ELSEVIER ACADEMIC PRESS, 2004.

15. LI Hongtao, et al, GPS Applied Program Design [M], Beijing: Scientific Publishing House, 1999. (In Chinese)

16. SOPAC & CSRC, ftp://lox.ucsd.edu, San Diego, University of California.

17. ftp://garner.ucsd.edu

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