The Data Processing Method for Small Samples and Multi-variates Series in GPS Deformation Monitoring

*Liu Guo-lin[1], Zheng Wen-Hua[1], Wang Xin-zhou[1] Zhang Lian-Peng[2]

([1] Geo-information Science & Engineering, Shandong University of Sci-Tech, Qingdao, Shandong, China)

([2] Dept. of Territory Information & Mapping Engineering, Xuzhou Normal University, Xuzhou, Jiangsu, China)

Abstract

Time series analysis is a frequently effective method of constructing model and prediction in data processing of deformation monitoring. The monitoring data sample must to be as more as possible and time intervals are equal roughly so as to construct time series model accurately and achieve reliable prediction. But in the project practice of GPS deformation monitoring, the monitoring data sample can't be obtained too much and time intervals are not equal because of being restricted by all kinds of factors, and it contains many variates in the deformation model moreover. It is very important to study the data processing method for small samples and multi-variates time series in GPS deformation monitoring. A new method of establishing small samples and multi-variates deformation model and prediction model are put forward so as to resolve contradiction of small samples and multi-variates encountered in constructing deformation model and improve formerly data processing method of deformation monitoring. Based on the system theory, a deformation body is regarded as a whole organism; a time-dependence linear system model and a time-dependence bilinear system model are established. The dynamic parameters estimation is derived by means of prediction fit and least information distribution criteria. The final example demonstrates the validity and practice of this method.

Keywords: Small Samples; Data processing; GPS deformation monitoring

1. Introduction

As everyone knows, No matter the earth's crust occur deformation or large building become deformed, the internal mechanism of deformation is not clearly, and affected by a lot of external factor. So it's difficult to describe the change as a whole and forecast the future tendency by general deformation. The problem would become simple when we use systems theory that a deformation body is regarded as a whole organism. Considering the internal movement as well as external factor, a time-dependence linear system model and a time-dependence bilinear system model would established in this paper.

We can gain finite observation data for establish small sample because deformation monitor restrained by manpower, material resources and financial resources. We couldn't choose many parameters due to finite observation data. The practical precision of the model reflect the reality is weak along with the reduced quantity of parameters; parameters estimation by general method couldn't achieve ideal result on the condition that finite observation data and quantity of parameters couldn't reduced. The dynamic parameters estimation is derived by means of prediction fit and least information distribution criteria base on small sample in this paper.

We may adopt the recursive estimation method in some literatures if just consider system time-dependence parameters estimation. In this paper we consider not only time-dependence of the system, but also the finite observation data. The time-dependence parameters in the model treated as continuous function and approximate by thrice B-spline function, thereby, time-dependence parameters estimation transform into non-time-dependence parameters estimation.

2. Time-dependence Deformation Monitor System Model

2.1 Time-dependence linear system model

Suppose deformation monitoring to a deformation body, we gained observation sequence y_i and u_i $(i=1,2,\cdots\cdots,m)$ in $t_1,t_2,\cdots\cdots,t_m$. Time-dependence linear system model is set up as follows:

$$\begin{aligned} & x(t)+a_1(t)x(t-1)+\cdots+a_m(t)x(t-m) \\ & = b_0(t)u(t)+b_1(t)u(t-1)+\cdots+b_n(t)u(t-1)+w(t) \end{aligned} \tag{1}$$

$$y(t)=c(t)x(t)+e(t) \tag{2}$$

x is state variable for describing movement law of system; y and u leave each other as deformation observation value and external factor; $w(t)$ and $e(t)$ are independent white noise with mean equal 0; $c(t),a_1(t),\cdots,a_m(t),b_0(t),\cdots,b_n(t)$ are time-dependence parameters.

From formula (2), we obtain

$$x(t)=y(t)/c(t)-e(t)/c(t) \tag{3}$$

According to formula (2) and (3), we obtain:

$$y(t)+A_1(t)y(t-1)+\cdots+A_m(t)y(t-m) \\ =B_0(t)u(t)+B_1(t)u(t-1)+\cdots+B_n(t)n(t-n)+w^{'}(t) \quad (4)$$

where, $A_i(t)=c(t)a_i(t)/c(t-i)$ ($i=1,2,\cdots\cdots,m$);

$B_j(t)=c(t)b_j(t)$ ($j=0,1,2,\cdots\cdots,n$);

$$w^{'}(t)=w(t)+\sum_{i=0}^{m}\frac{c(t)}{c(t-i)}a_i(t)e(t-i)$$

Based on deformation tendency and distance of one period, use free crunodes divide up time field $[t_1,t_m]$:

$$\Delta: t_1=\tau_0<\tau_1<\cdots<\tau_{r+1}=t_m$$

It can be shown by add new crunodes that:

$$\tau_{-3}=\tau_{-2}=\tau_{-1}=t_1=\tau_0<\tau_1<\cdots<\tau_{r+1} \\ =t_m=\tau_{r+2}=\tau_{r+3}=\tau_{r+4}$$

From literature[4] , we can work out thrice B-spline function series by use of crunodes cited just before as follows: $B_{-3}(t),B_{-2}(t),\cdots,B_r(t)$, then spline function denoted by

$$s(t)=\sum_{l=-3}^{r}c_l b_l(t) \quad (5)$$

Regard time-dependence parameters $A_i(t)$, $B_j(t)$ ($i=1,2,\cdots\cdots,m;\ j=0,1,\cdots,n$) in formula (4) as continuous function of t. and approximated by spline function (5), we obtain:

$$y(t)+\sum_{l=-3}^{r}a_{l1}B_l(t)y(t-1)+\cdots+\sum_{l=-3}^{r}a_{lm}B_l(t)y(t-m) \\ =\sum_{l=-3}^{r}\beta_{l0}B_l(t)u(t)+\cdots+\sum_{l=-3}^{r}\beta_{\ln}b_l(t)u(t-n)+w^{'}(t)+\varepsilon(t) \quad (6)$$

Where $\varepsilon(t)$ is approximate error, $\varepsilon(t)$ have stochastic behavior due to time-dependence parameters stochastic behavior. For estimate parameters conveniently, we use Moving average model to denote $w^{'}(t)+\varepsilon(t)$, namely

$$w^{'}(t)+\varepsilon(t)=\zeta(t)+\sum_{i=1}^{p}c_i\zeta(t-i) \quad (7)$$

Where $\{\varepsilon(t)\}$ is white noise series with mean equal 0. Using formula (6) and (7), denoted by matrix:

$$y(t)=\varphi^T(t)\theta+\zeta(t) \quad (8)$$

where

$$\varphi(t)=[-B_{-3}(t)y(t-1),\cdots,-B_r(t)y(t-1),\cdots,-B_{-3}(t) \\ y(t-m),\cdots,-B_r(t)y(t-m),B_{-3}(t)u(t),\cdots, \\ B_r(t)u(t),\cdots,B_{-3}(t)u(t-n),\cdots,B_r(t) \\ u(t-n),\zeta(t-1),\cdots,\zeta(t-p)]^T ,$$

$$\theta=[a_{-31},\cdots,a_{r1},\cdots,a_{-3m},\cdots a_{rn},\beta_{-30},\cdots, \\ \beta_{r0},\cdots\beta_{-3n},\cdots,\beta_{rn},c_1,\cdots c_p]^T$$

Time-dependence parameters estimation transforms into non-time-dependence parameters estimation pass through spline function approximate.

2.2 Time-dependence bilinear system model

Deformation of earth's crust is a complex system, deformation relate to not only the geological character, but also external factor, for example temperature, air pressure, level of underground water, temperature of surface, meanwhile, relate to earth's rotation, celestial gravitation as well as produce activity of human being. In such condition, linear system model can't reflect the objective condition all right, so non-linear system model at demand. However, non-linear system theory isn't perfect and algorisms are very complex, we adopt bilinear system model analyze and approximate non-linear system, so we set up bilinear system model as fellows:

$$x(t)+a_1x(t-1)+\cdots+a_px(t-p) \\ =a+b_0u(t-d)+b_1u(t-1-d)+\cdots \\ +b_qu(t-q-d)+\sum_{i=1}^{l}\sum_{j=1}^{m}\beta_{ij}x(t-i)u(u-j-d+1) \quad (9)$$

$$y(t)=a(t)x(t)+e(t) \quad (10)$$

Time-dependence of objective condition may reflected by the time-dependence observation vector and state vector. We don't describe parameters in (9) by time-dependence for estimate parameters and calculate conveniently. Formula (9) shows the relation of non-direct observation $x(t)$ and external factor $u(t)$. $y(t)$ is polluted observation by noise $e(t)$. d denote lag of deformation affect by external factor, $\{e(t)\}$ is white noise series with mean equal 0. If all $\beta_{ij}=0$ $(i=1,2,\cdots\cdots,m;\ j=0,1,\cdots,n)$, then, the model changes to linear system model. So β_{ij} is degree of the system deviate to linear.

We know from formula (10) that:

$$x(t) = y(t)/a(t) - e(t)/a(t) \tag{11}$$

Using (11) and(9), we obtain:

$$y(t) + \sum_{i=1}^{p} A_i y(t-i) = A + \sum_{i=0}^{q} B_i u(t-d-i) +$$
$$\sum_{i=1}^{l}\sum_{j=1}^{m} r_{ij} y(t-i)u(t-j-d+1) + e(t) + \tag{12}$$
$$\sum_{i=1}^{p} c_i e(t-1) + \sum_{i=1}^{l}\sum_{j=1}^{m} a_{ij} e(t-i)u(t-j-d+1)$$

Where, $A_i = a_i a(t)/a(t-i), i = 1,2\cdots,p;$

$B_i = b_i a(t), i = 0,1\cdots,q$; $A = aa(t);$

$c_i = a_i a(t)/a(t-i), i = 1,2,\cdots,p$;

$r_{ij} = \beta_{ij} a(t)/a(t-i), i = 1,2,\cdots,l;\ j = 1,2,\cdots,m$

$a_{ij} = -\beta_{ij} a(t)/a(t-i), i = 1,2,\cdots,l;\ j = 1,2,\cdots m$.

c_i and a_{ij} are supplementary variables.

When

$$\varphi(t) = [1, -y(t-1), \cdots, -y(t-p), u(t-d), \cdots,$$
$$u(t-d-q), y(t-1)u(t-d), \cdots,$$
$$y(t-l)u(t-m-d+1), e(t), e(t-1), \cdots e(t-p),$$
$$e(t-1)u(t-d), \cdots, e(t-l)u(t-m-d+1)]^T$$

$$\theta = [A, A_1, \cdots, A_P, B_0, \cdots, B_q, r_{11}, \cdots, r_{lm}, c_1, \cdots, c_p, a_{11}, \cdots, a_{lm}]^T$$

Then formula (12) may denoted by

$$y(t) = \varphi^T(t)\theta + e(t) \tag{13}$$

Thereby time-dependence bilinear system model change to linear forecast error model.

3. Dynamic Parameters Estimation

Suppose observation system f , may described by parameters model with sample space is Y , parameters aggregation is Θ and density distribution function is

$$P = \{p(y|\theta), \theta \in \Theta\} \tag{14}$$

Suppose X is n-times repeated trials to f , Bayesian forecast base on density function $p(y|\theta)$ of X denoted by

$$q(y|x) = \int_\theta p(y|\theta)p(\theta|x)d\theta \tag{15}$$

Where $p(\theta|x)$ is posterior density function base on prior probability $p(\theta)$ and X . $Aitchison$[5] using prediction fit criterion:

$$\int_\Theta p(\theta)d\theta \int_X p(X|\theta)dx \int_Y p(y|\theta)\log\{q(y|x)/r(y|x)\}dy \tag{16}$$

And prove it always plus, so Bayesian forecast method is better than others. $Murry$[6] points out that forecast density function fits the true density function better than other method. From literature [7], when the true density function is normal distribution, the forecast density function is student distribution. So student distribution fits the probability density function quite well when small sample from normal distribution collectivity.

Suppose N independent observation of Y from normal distribution collectivity $y_1, y_2, \cdots, y_N$, y_i have mean value of $\varphi^T(t)\theta$ and same variance σ^2 , follow above, we know:

$$p(y_i|\theta,\sigma^2) = std(r, \varphi^T(i)\theta, \sigma^2)$$
$$= \frac{\Gamma(\frac{r+1}{2})}{(\pi t)^{\frac{1}{2}}\sigma\Gamma(\frac{1}{2})}\{1 + (r\sigma^2)^{-1}(y_i - \varphi^T(i)\theta)^2\}^{-\frac{r+1}{2}} \tag{17}$$

Where $r = N-1,\ N$ is quantity of sample, r is free degree. As independent of the samples, likelihood function is:

$$L = p(y|\theta,\sigma^2) = \prod_{i=1}^{N} p(y_i|\theta,\sigma^2)$$
$$= \frac{\Gamma^N(\frac{r+1}{2})}{(\pi r)^{\frac{1}{2}}\sigma^N\Gamma^N(\frac{r}{2})}\prod_{i=1}^{N}\{1 + (r\sigma^2)^{-1}(y_i - \varphi^T(i)\theta)^2\}^{-\frac{r+1}{2}} \tag{18}$$

Logarithmetics (18), obtain that

$$\ln L = N\ln\Gamma(\frac{r+1}{2}) - \frac{N}{2}\ln(\pi r) - N\ln\sigma -$$
$$N\ln\Gamma(\frac{r}{2}) - \frac{r+1}{2}\sum_{i=1}^{N}\ln[1 + (r\sigma^2)^{-1}(y_i - \varphi^T(i)\theta)^2] \tag{19}$$

Base on Bayesian

$$\ln p(\theta,\sigma^2|Y) = \ln p(Y|\theta,\sigma^2) + \ln p(\theta,\sigma^2) - \ln p(Y)$$
$$= N\ln\Gamma(\frac{r+1}{2}) - \frac{r}{2}\ln(\pi r) - N\ln\sigma - N\ln\Gamma(\frac{1}{2}) -$$
$$\frac{r+1}{2}\sum_{i=1}^{N}\ln[1 + (r\sigma^2)^{-1}(y_i - \varphi^T(i)\theta)^2] + \ln p(\theta,\sigma^2) - \ln p(Y) \tag{20}$$

We must make (21) reach extreme in order to enable (20) reach extreme.

$$J(\theta,\sigma^2)\underline{\underline{\Delta}}\ln p(\theta,\sigma^2)-N\ln\sigma - \frac{r+1}{2}\sum_{i=1}^{N}\ln[1+(r\sigma^2)^{-1}(y_i-\varphi^T(i)\theta)^2] \quad (21)$$

Next consider how chose least information Bayesian prior distribution on the condition of finite observation data. From literature [8], information measure of prior density was defined as:

$I(p(\theta))$ =Information content of θ in

$p(\theta|Y)-$ information content of θ in

$p(\theta)$ =

$$\iint p(\theta|Y)\log\{p(\theta|Y)\}p(Y)d\theta dY-\int p(\theta)\log\{p(\theta)\}d\theta \quad (22)$$

Maximize (22) is the optimum least information distribution. Maximize (22) distribution is different to prior distribution of maximum entropy, namely different to prior distribution with minimum second item in (22). The influence of first item is decrease along with increasing amount of data, then, least information distribution become prior distribution gained by maximum entropy method.

In the condition of small sample, confirm the least information distribution concern optimum of infinite dimensional. So we use optimum convex-combination which prior distribution is clear as least information distribution method. Any density function may approximate by combination of several $Gauss$ prior distribution very well. Foundation Gauss prior distribution defined:

$$p(\theta)=N(\theta-\mu_i,\Sigma_i),(i=1,2,\cdots l)$$

$$U=\{a=(a_1,a_2,\cdots,a_{l-1})^T,\ \sum_{j=1}^{l-1}a_j\le 1,a_j\ge 0\}$$

Seeking convex-combination $p_a^*(\theta)$ for

$$I(p_a^*(\theta))=\max_{a\in u}\{I(p_a(\theta))$$
$$p_a(\theta)=\sum_{i=1}^{l}a_iN(\theta-u_i,\Sigma_i)\} \quad (23)$$

Where $a_l=1-\sum_{j=1}^{l-1}a_j$ is confirmed one and only.

4. Distinguish Rule and Example

Different estimation methods deliver different result, so we should set up a standard to measure the estimation result. Here, we regard relative mean square error as standard, namely

$$RMSE=[\sum_i(\hat{\theta}_i-\theta_i)^2]/[\sum_i\theta_i^2] \quad (24)$$

where θ_i is the true value of parameters, $\hat{\theta}$ denote estimation value. Set up a time-dependence bilinear system model base on objective factor of deformation with $p=1,q=2,d=0,l=1,m=3.$

$$\begin{aligned}&y(t)+A_1y(t-1)\\&=A+\sum_{i=1}^{3}[B_{0i}u_i(t)+B_{1i}u_i(t-1)+B_{2i}u_i(t-2)]+\\&\sum_{i=1}^{3}[r_{11}^iy(t-1)u_i(t)+r_{12}^iy(t-1)u_i(t-1)\\&\quad+r_{13}^iy(t-1)u_i(t-2)]+e(t)+c_1e(t-1)+\\&\sum_{i=1}^{3}[a_{11}^ie(t-1)u_i(t)+a_{12}^ie(t-1)u_i(t-1)+a_{13}^ie(t-1)u_i(t-2)]\end{aligned}$$

case

$$\theta^T=[A,A_1,B_{01},...,B_{23},r_{11}^1,r_{11}^2,...,r_{13}^3,c_1,a_{11}^1,a_{11}^2,...,a_{13}^3]$$

$$\begin{aligned}\varphi^T&=[1,-y(t-1),u_1(t),...,u_3(t-2),y(t-1)\\&u_1(t),y(t-1)u_2(t),...,y(t-1)u_3(t-2),\\&e(t-1),e(t-1)u_1(t),e(t-1)u_2(t),...,e(t-1)u_3(t-2)]\end{aligned}$$

Then (25) denoted by:

$$y(t)=\varphi^T(t)\theta+e(t) \quad (26)$$

where $e(t)=y(t)-\varphi^T(t)\theta=y(t)-\hat{y}(t|\theta)$. regard as forecast error, denoted by $e(t,\theta)$. The variance of forecast error is $\sigma^2(t)\underline{\underline{\Delta}}c(t)=E[(y(t)-\hat{y}(t|\theta))^2]$

For convenience, we use limit variance σ^2 replace $\sigma^2(t)$. From (25), we know $e(t,\theta)$ is linear function of $A_1,B_{0i},B_{1i},B_{2i},r_{11}^i,r_{12}^i,r_{13}^i$, is non-linear function of $c_1,a_{11}^i,a_{12}^i,a_{13}^i$ so we use iteration method to settle. We adopt change of scale algorithm for maximize (21) in order to estimate θ. Where $\hat{\theta}(0)$ is least square estimator,

$\sigma^2(0)=1$. We obtain the deformation model of earth's crust in this area

$$
\begin{aligned}
& y(t)-0.431y(t-1)=0.731+0.056u_1(t)-0.121u_2(t) \\
& +0.038u_3(t)+0.045u_1(t-1)-0.089u_2(t-1) \\
& +0.031u_3(t-1)+0.043u_1(t-2)-0.036u_2(t-2) \\
& +0.025u_3(t-2)-0.432y(t-1)u_1(t)+ \\
& 0.417y(t-1)u_2(t)-0.358y(t-1)u_3(t) \\
& +0.315y(t-1)u_1(t-1)-0.242y(t-1)u_2(t-1) \\
& +0.138y(t-1)u_3(t-1)-0.083y(t-1)u_1(t-2) \\
& +0.025y(t-1)u_2(t-2)-0.059y(t-1)u_3(t-2)
\end{aligned}
\tag{27}
$$

Use 8th data to test the model above-mentioned. The model fits the reality observation data very well, so the model is effective and applied.

5. Conclusion

We obtain conclusion through theory and example as follow:

1) The time-dependence system model mentioned in this paper fits deformation study of dam, building as well as abnormal movement of earth' crust. We regard the change law of deformation body as whole thereby avoid the large error of general model, meanwhile, the model own the advantage of primely reflect the change law of entirety.

(2) The external factor taken into account for set up the system model in this paper. The influences by the external factor exist for a certainty although it is inconspicuous. Thus we must consider external factor when inspect dam, building and so on.

(3) In the condition of finite observation data, the estimation method in this paper for example: least squares method, maximum likelihood method and so on is better than others. But in this condition, the statistical character of noise didn't exhibit adequately, so parameters estimation affected by observation noise and stochastic approximate error require more research.

Reference

[1] C.Y., Xi. The Pistinguish and the Estimation of Parameters for Deformation Model. Editorial Board of Geomatics and Information Science of Wuhan University, 1990, (2)

[2] Z. O., Wang, W. G. Bao. A maximum Likelihood Estimation Method of a Bilinear System under Small Samples. Journal of Tianjin University, 1990, (2)

[3] W. P., Wang, D. H., Pan. Two Adaptive Recursive Algorithms for Estimating Time-varying Parameters. Control Theory & Applications, 1990, (1)

[4] Z.C., Cheng. Data fitting. Xian Xi'an Jiaotong University Press, 1979

[5] J. Aitchison. Goodness of Prediction Fit. Biomertrika, 1975,62 (3)

[6] C.D Murry. The Estimation of Multivariate normal Density functions Using Inlomplete Data. Eiometrika, 1979,66(2)

[7] J. Aitchison & I.K Dansmure. Statistical Prediction Analysis. Cambridge University press, 1975

[8] Least information Bayesian prior distribution for Finite Sample Based on Information Theory. IEEE Trans.Automat. contr., 1990, 35(5)

[9] D.L. Alspach & H.W.Sorenson. Nonlinear Bayesian Estimation using Gaussian Sum Approximations. IEEE Trons . Automat . contr, vol.AC-17, 1972, (4)

The study is supported by Special Project Fund of Taishan Scholars of Shandong Province, Natural Science Fund of Qingdao (04-2-JZ-101), Natural Science Fund of Shandong Province (Y2003E01), Fund of Key Laboratory of Geospace Environment and Geodesy Ministry of Education, China (04-01-04)

Address:

College of Geo-information Science & Engineering,
Shandong University of Science and Technology
Qianwangang Road 579,
QingDao Economic & Technological Development Zone,
QingDao, Shandong Province, P.R.China
Email: gliu@sdust.edu.cn