

Fast Ambiguity Resolution using Galileo Multiple Frequency Carrier Phase Measurement

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Abstract

Rapid and high-precision positioning with a Global Navigation Satellite System (GNSS) is feasible only when very precise carrier-phase observations can be used. There are two kinds of mathematical models for ambiguity resolution. The first one is based on both pseudorange and carrier phase measurements, and the observation equations are of full rank. The second one is only based on carrier phase measurement, which is a rank-defect model. Though the former is more commonly used, the latter has its own advantage, that is, ambiguity resolution will be freed from the effects of pseudorange multipath.

Galileo will be operational. One of the important differences between Galileo and current GPS is that Galileo will provide signals in four frequency bands. With more carrier-phase data available, frequency combinations with long equivalent wavelength can be formed, so Galileo will provide more opportunities for fast and reliable ambiguity resolution than current GPS.

This paper tries to investigate phase only fast ambiguity resolution performance with four Galileo frequencies for short baseline. Cascading Ambiguity Resolution (CAR) method with selected optimal frequency combinations and LAMBDA method are used and compared. To validate the resolution, two tests are used and compared. The first one is a ratio test. The second one is lower bound success-rate test.

The simulation test results show that, with LAMBDA method, whether with ratio test or lower bound success rate validation criteria, ambiguity can be fixed in several seconds, 8 seconds at most even when 1 sigma of carrier phase noise is 12 mm. While with CAR method, at least about half minute is required even when 1 sigma of carrier phase noise is 3 mm. It shows that LAMBDA method performs obviously better than CAR method.

Keywords: Ambiguity Resolution, Galileo, LAMBDA method, Ratio Test, Success-rate

1 Introduction

Rapid and high-precision positioning with a Global Navigation Satellite System (GNSS) is feasible only when very precise carrier-phase observations can be used. Unfortunately, these observations are ambiguous by an unknown, integer number of cycles. These integer ambiguity parameters need to be resolved before carrier-phase observations can begin to serve as very precise pseudo-range measurements. For precise navigation, reliable real-time ambiguity resolution is necessary.

Generally, there are two kinds of mathematical models for ambiguity resolution. The first one is based on both pseudorange and carrier phase measurements, and the observation equations are of full rank. The second one is only based on carrier phase measurement, which is a rank-defect model. Though the former is more commonly used, the latter has its own advantage, that is, ambiguity resolution will be freed from the effects of pseudorange multipath.

For short-distance baseline, with current GPS, the efficiency and reliability of fast static ambiguity resolution with carrier phase-only measurements are not high and fewer available carrier-phase data is an important reason. This makes it impossible for safety-related applications.

The above problems can be attributed to several reasons, such as poor satellite geometry, fewer satellites. Fewer available carrier-phase data is also an important reason. With only two frequency carrier-phase data available, it is impossible to form frequency combinations with long equivalent wavelength, which is very important for ambiguity resolution.

In future, a totally new global positioning system – Galileo will be operational. It is a civil system, launched by Europe. One of the important differences between Galileo and current GPS is that Galileo will provide signals in four frequency bands (Luis Ruiz, 2005) with central frequencies at:

$$E1: f_1 = 1575.42\text{M Hz}, E6: f_2 = 1278.75\text{M Hz}$$

$$E5b: f_3 = 1207.14\text{M Hz}, E5a: f_4 = 1176.45\text{M Hz}$$

With more carrier-phase data available, frequency combinations with long equivalent wavelength can be formed, so Galileo will provide more opportunities for fast and reliable ambiguity resolution than current GPS.

In recent years, a lot of research works (Tiberius C. et al, 2002; Vollath U. et al. 2002; Zhang W. et al., 2003; Werner W. and Winkel J., 2003; Schlotzer S. and Martin S., 2005) have been done to investigate ambiguity resolution performance with four frequencies of Galileo system. Most of them are based on mathematical model which takes advantage of both pseudorange and carrier phase measurements. Though this mathematical model has the advantage of full rank even with single-epoch data, it also has one weakness: ambiguity resolution with this model is easily affected by pseudorange multipath and leads to wrong solution.

This paper tries to investigate carrier phase only fast static ambiguity resolution performance with four Galileo frequencies for short baselines.

In this paper, first, optimal frequency combinations are selected according to appropriate success-rates based on ADOP. Then, CAR method and LAMBDA method are used and

compared. To validate ambiguity resolution, two kinds of tests are used and compared. The first is lower bound success rate test based on apriori information (Teunissen, P.J.G., 1998), the other is ratio test based on posteriori information (Frei, E. and Beutler G., 1990; Euler H.J. and Schaffrin B., 1992; Landau H. and Euler H.J., 1992; Abidin, H.A., 1993).

To investigate the effects of carrier-phase observation noise on ambiguity resolution performance, the data are simulated under different noise levels. Finally, test results are presented and analyzed.

2 Combinations of Galileo frequencies

Galileo system will provide four frequency bands for navigation (Luis Ruiz, 2005) with central frequencies at:

$$E1: f_1 = 1575.42\text{M Hz}, E6: f_2 = 1278.75\text{M Hz}$$

$$E5b: f_3 = 1207.14\text{M Hz}, E5a: f_4 = 1176.45\text{M Hz}$$

The general form for a combination of the four frequencies is:

$$N_c = iN_1 + jN_2 + kN_3 + mN_4 \quad (1)$$

$$\lambda_c = \frac{\lambda_1 \lambda_2 \lambda_3 \lambda_4}{i\lambda_2 \lambda_3 \lambda_4 + j\lambda_1 \lambda_3 \lambda_4 + k\lambda_1 \lambda_2 \lambda_4 + m\lambda_1 \lambda_2 \lambda_3} \quad (2)$$

$$f_c = if_1 + jf_2 + kf_3 + mf_4 \quad (3)$$

$$L_c = \alpha L_1 + \beta L_2 + \gamma L_3 + \delta L_4 \quad (4)$$

Here, N_1, N_2, N_3, N_4 are ambiguities of E1, E6, E5b and E5a. $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are wavelengths of E1, E6, E5b and E5a.

$L_1, L_2, L_3,$ and L_4 are carrier phase measurements of E1, E6,

E5b and E5a (Unit: meter). $N_c, \lambda_c,$ and f_c are ambiguity, wavelength and frequency of the combination respectively. And $\alpha = i\lambda_c / \lambda_1, \beta = j\lambda_c / \lambda_2, \gamma = k\lambda_c / \lambda_3, \delta = m\lambda_c / \lambda_4, \alpha + \beta + \gamma + \delta = 1.$

In above equations, i, j, k, m are integers so that the integer property of the ambiguity for the combination is also reserved.

For the combination, tropospheric delay remains the same as that of E1, E6, E5b, or E5a. But ionospheric delay I_c and observation noise σ_c are different:

$$I_c = R_{i,j,k,m} I_1 \quad (5)$$

$$\sigma_c = A_{i,j,k,m} \sigma_0 \quad (6)$$

Where

$$R_{i,j,k,m} = \frac{i + jf_1/f_2 + kf_1/f_3 + mf_1/f_4}{i + jf_2/f_1 + kf_3/f_1 + mf_4/f_1}$$

$$A_{i,j,k,m} = \sqrt{\alpha^2 + \beta^2 + \gamma^2 + \delta^2}$$

σ_0 is the measurement noise level of an individual frequency (suppose E1, E6, E5b, or E5a have the same observation noise).

Table 1 lists top ten combinations with larger wavelength to noise ratio. They have one common feature: $i + j + k + m = 0$. Therefore, only three independent ones can be selected from them.

Table 2 lists top five combinations with larger wavelength to noise ratio and uncorrelated with those in Table 1. They have one common feature: $i + j + k + m = 1$.

Except those combinations listed in Table 1 and 2, there are some other combinations with long wavelength and reasonable wavelength to noise ratio. They are listed in Table 3.

Table 1 Top ten combinations with larger wavelength to noise ratio

No.	i	j	k	m	λ_c (m)	$R_{i,j,k,m}$	$A_{i,j,k,m}$	$\lambda_c / A_{i,j,k,m}$
1	0	0	1	-1	9.76	-1.74	54.92	0.18
2	0	1	-1	0	4.18	-1.6	24.55	0.17
3	0	1	0	-1	2.93	-1.64	16.98	0.17
4	1	-1	0	0	1.01	-1.23	6.84	0.15
5	1	0	-1	0	0.81	-1.31	5.39	0.15
6	1	0	0	-1	0.75	-1.34	4.93	0.15
7	1	-1	-1	1	1.13	-1.17	9.92	0.11
8	1	-1	1	-1	0.92	-1.28	8.06	0.11
9	1	1	-1	-1	0.64	-1.38	5.61	0.11
10	0	1	1	-2	2.25	-1.67	22.08	0.10

Table 2 Top five combinations with larger wavelength to noise ratio uncorrelated with those in Table 1

No.	i	j	k	m	λ_c (m)	$R_{i,j,k,m}$	$A_{i,j,k,m}$	$\lambda_c / A_{i,j,k,m}$
11	-1	0	1	1	0.37	3.21	2.85	0.130
12	-2	1	1	1	0.59	5.77	7.41	0.078
13	-2	0	1	2	0.73	7.63	10.05	0.072
14	-2	1	0	2	0.62	6.25	8.60	0.072
15	-2	0	2	1	0.68	6.98	9.41	0.072

Table 3 Other combinations with long wavelength and reasonable wavelength to noise ratios

No.	i	j	k	m	λ_c (m)	$R_{i,j,k,m}$	$A_{i,j,k,m}$	$\lambda_c / A_{i,j,k,m}$
1	0	1	-3	2	29.3	-0.77	440.27	0.066
2	-1	4	0	-3	29.3	-13.77	626.69	0.047
3	0	1	-2	1	7.32	-1.5	72.69	0.10
4	1	-4	2	1	5.86	0.66	117.07	0.05

3 Select optimal combinations of Galileo frequencies for CAR method

3.1 Optimal combinations for the first step

From Table 1, 2 and 3, we can find that there are two possible optimal combinations for the first step as listed in Table 4. For the second one as named Com1 in Table 4 has the longest wavelength and the first one as named Com0 has the largest wavelength to noise ratio. Besides, wavelength of Com0 is larger than the rest.

Table 4 Possible optimal combinations for the first step

Name	i	j	k	m	λ_c (m)	$R_{i,j,k,m}$	$A_{i,j,k,m}$	$\lambda_c / A_{i,j,k,m}$
Com0	0	0	1	-1	9.76	-1.74	54.92	0.18
Com1	0	1	-3	2	29.3	-0.77	440.27	0.066

To determine which to select, their appropriate success-rates based on ADOP by the following formula (Teunissen, 2003) are

calculated when using code measurement with different noises to fix them under different phase noise levels.

$$P_s \approx (2\Phi(\frac{1}{2ADOP}) - 1)^n$$

See Figure 1 for the results.

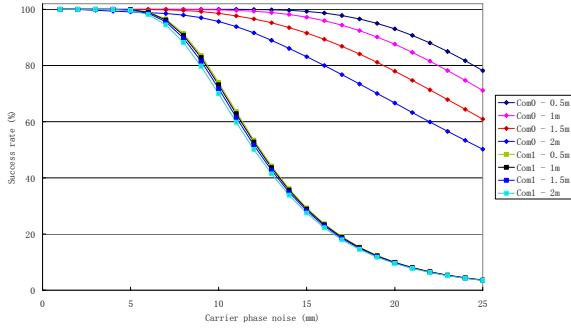


Figure 1 Success-rates of Com0 and Com1 for the first step

The success-rate of Com0 is always bigger than that of Com1 under same code and phase noise level. So, the optimal combination for the first step of CAR method should be Com0 combination.

3.2 Optimal combinations for the second step

After the optimal combinations for the first step is determined, for the same reason as stated in the first step, we can find that there are three possible optimal combinations for the second step as listed in Table 5.

Table 5 Possible optimal combinations for the second step

Name	i	j	k	m	λ_c (m)	$R_{i,j,k,m}$	$A_{i,j,k,m}$	$\lambda_c / A_{i,j,k,m}$
Com1	0	1	-3	2	29.3	-0.77	440.27	0.066
Com2	0	1	-1	0	4.18	-1.60	24.55	0.17
Com3	0	1	-2	1	7.32	-1.50	72.69	0.10

To determine which to select, their success-rates are calculated when using unambiguous Com0 measurement to fix them under different phase noise levels. See Figure 2 for the results.

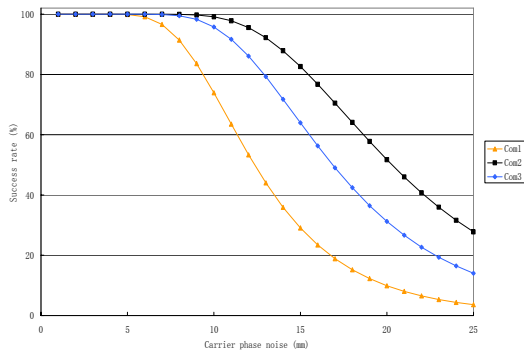


Figure 2 Success-rates of Com1, Com2 and Com3 for the second step

We can see that, the success-rate of Com2 is always bigger than those of Com1 and Com3 under same phase noise level. So, the optimal combination for the second step of CAR method should be Com2 combination.

3.3 Optimal combinations for the third step

In research papers about CAR method, generally, the combination fixed in the second step is used as unambiguous measurement for the third step. But this is not optimal for the combination fixed in the second step is always has big noise.

There are a lot of combinations which are correlated with Com0 and Com2. After Com0 and Com2 are fixed, ambiguities of these combinations can be calculated from their linear relationships with Com0 and Com2. Among them, the combination with minimum noise is listed in Table 6, which is the transferring combination from the second step to the third and named Trans0.

Table 6 Transferring Combination from the second step to the third

Name	i	j	K	m	λ_c (m)	$R_{i,j,k,m}$	$A_{i,j,k,m}$
Trans0	0	5	-1	-4	0.62	-1.64	16.7

For the same reason as for first and second step, from Table 1, 2 and 3, we can see that there are three possible optimal combinations as named Com4, Com5 and Com6 in Table 7.

Table 7 Possible optimal combinations for the third step

Name	i	j	k	m	λ_c (m)	$R_{i,j,k,m}$	$A_{i,j,k,m}$	$\lambda_c / A_{i,j,k,m}$
Com4	1	-1	0	0	1.01	-1.23	6.84	0.15
Com5	1	-4	2	1	5.86	0.66	117.07	0.05
Com6	-1	4	0	-3	29.3	-13.77	626.69	0.047

To determine which to select, their success-rates are calculated when using unambiguous Trans0 measurement to fix them under different phase noise levels. See Figure 3 for the results.

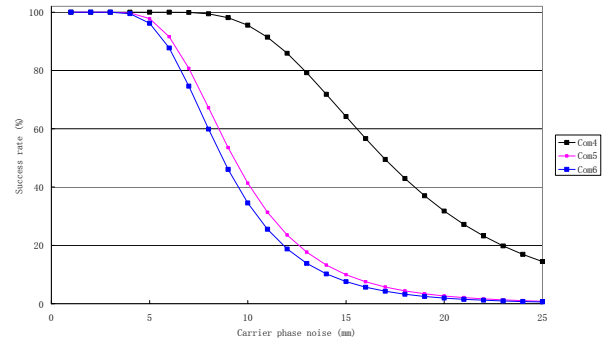


Figure 3 Success-rates of Com4, Com5 and Com6 for the third step

The success-rate of Com4 is always bigger than those of Com5 and Com6 under same phase noise level. So, the optimal combination for the third step of CAR method should be Com4 combination.

3.4 Optimal combinations for the fourth step

From third step to the fourth step, a transferring combination is also found as named Trans1 in Table 8, which has the minimum noise among combinations correlated to Com0, Com2 and Com4.

Table 8 Transferring Combination from the third step to the fourth

Name	i	j	k	m	λ_c (m)	$R_{i,j,k,m}$	$A_{i,j,k,m}$
Trans1	5	0	-2	-3	0.155	-1.32	4.64

For the same reason as for the above steps, from Table 1, 2 and 3, we can see that there are four possible optimal combinations as named Com7, Com8, and Com9 in Table 9. In addition to these three, there is another choice, E5a, which is also listed in Table 9. We can see that E5a has the largest wavelength to noise ratio.

Table 9 Possible optimal combinations for the fourth step

Name	i	j	k	m	λ_c (m)	$R_{i,j,k,m}$	$A_{i,j,k,m}$	$\lambda_c / A_{i,j,k,m}$
Com7	-1	0	1	1	0.37	3.21	2.85	0.130
Com8	-2	1	1	1	0.59	5.77	7.41	0.078
Com9	-2	0	1	2	0.73	7.63	10.05	0.072
E5a	0	0	0	1	0.255	1.79	1	0.255

To determine which to select, their success-rates are calculated when using unambiguous Trans1 measurement to fix them under different phase noise levels. See Figure 4 for the results.

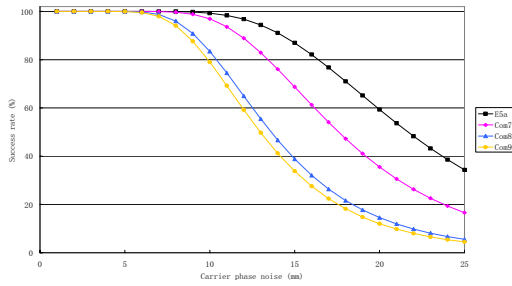


Figure 4 Success-rates of Com7, Com8, Com9 and E5a for the fourth step

The success-rate of E5a is always bigger than those of other choices under same phase noise level. So, the optimal choice for the fourth step of CAR method should be E5a.

Table 10 Optimal Combinations

Name	i	j	k	m	λ_c (m)	$R_{i,j,k,m}$	$A_{i,j,k,m}$
Com0	0	0	1	-1	9.76	-1.74	54.92
Com2	0	1	-1	0	4.18	-1.6	24.55
Com4	1	-1	0	0	1.01	-1.23	6.84
E5a	0	0	0	1	0.25	1.79	1

Table 11 Two transferring Combinations for the third and fourth steps

Name	i	j	k	m	λ_c (m)	$R_{i,j,k,m}$	$A_{i,j,k,m}$
Trans0	0	5	-1	-4	0.62	-1.64	16.7
Trans1	5	0	-2	-3	0.155	-1.32	4.64

Therefore, the optimal combinations for CAR method according to lower bound success-rate based on bootstrapping are ones listed in Table 10. And two transferring combinations for the third and fourth steps are listed in Table 11.

4. CAR method and LAMBDA method

4.1 CAR method

In the past research work, when using CAR method, for the third and fourth steps, fixed ambiguities of previous step is used directly as unambiguous measurements. But in this paper, the transferring combinations listed in Table 11 are used to improve the performance for these two combinations have minimum noises among all combinations linearly correlated with combinations fixed in previous step.

Mathematical model of geometry-based CAR method is composed of the following four steps:

$$AX + B_1 N_{Com1} = L_{Com1} \quad (7)$$

$$\begin{cases} AX = L_{Com1} - B_1 N_{Com1} \\ AX + B_2 N_{Com2} = L_{Com2} \end{cases} \quad (8)$$

$$\begin{cases} AX = L_{Trans1} - B_{Trans1} N_{Trans1} \\ AX + B_3 N_{Com3} = L_{Com3} \end{cases} \quad (9)$$

$$\begin{cases} AX = L_{Trans2} - B_{Trans2} N_{Trans2} \\ AX + B_4 N_{Com4} = L_{Com4} \end{cases} \quad (10)$$

Where, N_{Com1} , N_{Com2} , N_{Com3} , N_{Trans1} , N_{Trans2} and N_{Com4} are double-differenced ambiguity vectors of the combinations; L_{Com1} , L_{Com2} , L_{Com3} , L_{Trans1} , L_{Trans2} and L_{Com4} are double-differenced measurement vectors of the combinations, B_1 , B_2 , B_3 , B_4 , B_{Trans1} and B_{Trans2} are corresponding coefficient matrix of ambiguity vectors.

Weight matrix is applied according to observation noises.

4.2 LAMBDA method

Mathematical model of LAMBDA method is:

$$AX + BN = L \quad (11)$$

Where, X is real-valued parameter vector which includes coordinate parameters; N is double-differenced ambiguity vector of E1, E6, E5b and E5a; L is difference vector formed by subtracting computed values from the double-differenced carrier phase observations of E1, E6, E5b and E5a.

The weight matrix of observations is P and same weight is applied to all carrier phase observations.

5 Validation criteria used for ambiguity resolution

The ambiguity solution should only be used when one can have enough confidence in it. Therefore, it is important to validate the ambiguity solution. Currently, there are two kinds of validation methods. The first one is success rate, which is based on apriori information of measurement models and provides the expected probability of the ambiguity solution. Unfortunately, instead of exact success rate, only approximations can be obtained. One lower bound approximation is provided by the following formula (Teunissen P.J.G. and Odijk D., 1997; Teunissen P.J.G., 1998 ;S. Verhagen, 2005):

$$P_s \geq P_{s,B} = \prod_{i=1}^n (2\Phi(\frac{1}{2\sigma_{i|I}}) - 1) \quad (12)$$

Where, $\sigma_{i|I}$ is the standard deviation of the i -th ambiguity obtained through a previous $I = 1, \dots, (i - 1)$ ambiguities, and

$$\Phi(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} \exp\{-\frac{1}{2}v^2\} dv$$

The other kind of validation method is based on posteriori information, where practical measurements are needed. A lot of discrimination tests can be included in this kind. Though without sound theoretical foundation, one ratio test is often used to discriminate the best and the second best ambiguity solution as given by the following formula (Euler, et al. 1991):

$$\frac{R_S}{R_{\min}} \geq k \quad (13)$$

Where,

$$R_{\min} = (\hat{a} - \bar{a})^T Q_{\hat{a}} (\hat{a} - \bar{a})$$

$$R_S = (\hat{a} - \bar{a}_S)^T Q_{\hat{a}} (\hat{a} - \bar{a}_S)$$

and k is the empirically chosen critical value.

To compare these two kinds of validation methods, both validation tests by formula (12) and (13) are used. For the former, the critical value is set to 99.9%. As regard to the latter, for LAMBDA method, k is set to 2; for CAR method, because of severe singularity, k becomes very unstable. In this paper, it is set to 4.

6 Data simulation

To investigate ambiguity resolution performance of Galileo, Galileo measurements are simulated on two stations A (-2420466.778, 5388173.100, 2398086.812) and B (-2420906.778, 5387712.100, 2398673.812). Both stations are around Hong Kong and the distance of formed baseline AB is about 860m.

The simulated multipath and noise error levels of carrier phase measurements are listed in Tables 12. Tropospheric and ionospheric delays are simulated with Hopfield and Klobuchar models.

Table 12 Carrier phase multipath + noise level

Carrier phase observation noise and multipath error	1 sigma (mm)
Level 1	3
Level 2	6
Level 3	12

Twenty-four hour Galileo measurement data are simulated with epoch interval of one second.

7 Test results

The simulated data are processed every 20 seconds with LAMBDA and CAR methods with two validation tests: success rate test and ratio test.

The following two figures show LAMBDA method results under different carrier phase noises. And there is no misfixed case.

Figure 5 is the result of LAMBDA method with ratio validation test and Figure 6 is the result of LAMBDA method with lower bound success rate validation test.

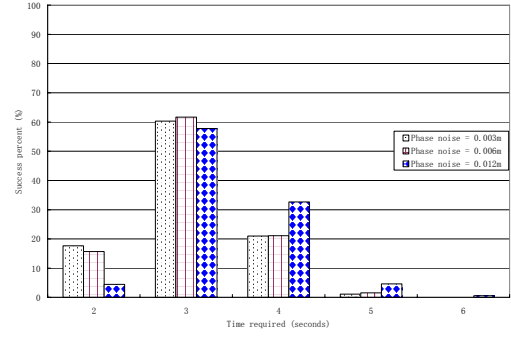


Figure 5 LAMBDA method with ratio test

From these two figures, we can see that for LAMBDA method with both validation criteria, ambiguity can be fixed in several seconds, at most 8 seconds even when carrier phase noise is as big as 12mm. The results also show that with the increase of carrier phase noise, the time required also increases.

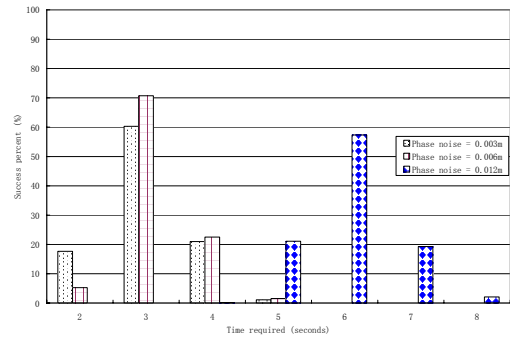


Figure 6 LAMBDA method with success rate test

The following two figures show CAR method results under different carrier phase noises.

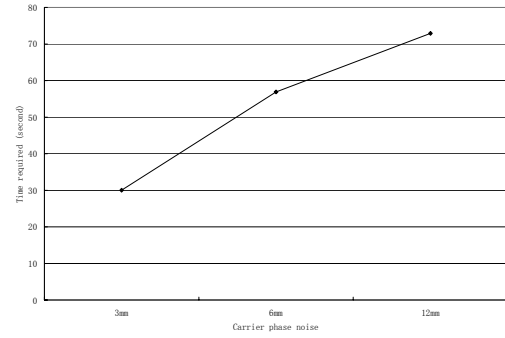


Figure 7 Average time required with CAR method and success rate test

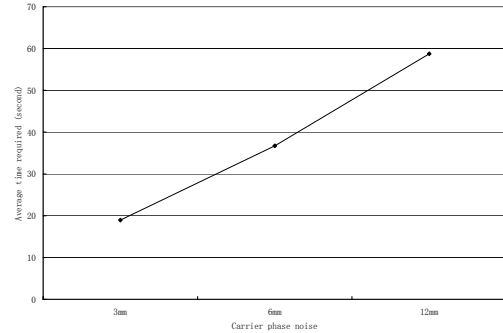


Figure 8 Average time required with CAR method and ratio test

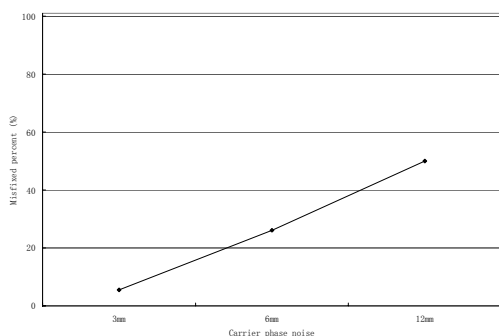


Figure 9 Misfixed percent with CAR method and ratio test

Figure 7 is the average time required to fix ambiguity with lower bound success rate validation test and there is no misfixed case. Figure 8 is the average time required to fix ambiguity with ratio validation test and Figure 9 shows the misfixed percent under different carrier phase noises.

From Figure 7 and 8, we can see that, the average time required to fix ambiguity is much longer than that of LAMBDA method, at least half one minute with success rate test even when 1 sigma of carrier phase noise is 3mm.

And due to severe singularity, ratio test becomes unreliable for CAR method. There is misfixed cases with ratio values greater than 14.

So, LAMBDA method performs obviously better than CAR method.

8 Conclusions

In this paper, ambiguity resolution performances with four Galileo frequencies are investigated under different carrier phase noise levels. LAMBDA and CAR methods are used and compared. Two ambiguity validation methods, F ratio test (critical value = 1.2) and lower bound success rate (>99.9%), are used and compared.

From the results, the following conclusions can be drawn about carrier phase-only fast static ambiguity resolution performance:

- When using LAMBDA method, whether with ratio test or lower bound success rate validation criteria, ambiguity can be fixed in several seconds, at most 8 seconds even when carrier phase noise is as big as 12mm.
- With the increase of carrier phase noise, the time required also increases.
- Comparing the results with these two methods, it shows that LAMBDA method performs obviously better than CAR method.

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