Preliminary Orbit Determination For A Small Satellite Mission Using GPS Receiver Data

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Abstract

The deviations in the injection orbital parameters, resulting from launcher dispersions, need to be estimated and used for autonomous satellite operations. For the proposed small satellite mission of the university there will be two GPS receivers onboard the satellite to provide the instantaneous orbital state to the onboard data handling system. In order to meet the power requirements, the satellite will be sun-tracking whenever there is no imaging operation. For imaging activities, the satellite will be maneuvered to nadir-pointing mode. Due to such different modes of orientation the geometry for the GPS receivers will not be favorable at all times and there will be instances of poor geometry resulting in no output from the GPS receivers. Onboard the satellite, the orbital information should be continuously available for autonomous switching on/off of various subsystems. The paper presents the strategies to make use of small arcs of data from GPS receivers to compute the mean orbital parameters and use the updated orbital parameters to calculate the position and velocity whenever the same is not available from GPS receiver. Thus the navigation message from the GPS receiver, namely the position vector in Earth-Centered-Earth-Fixed (ECEF) frame, is used as measurements. As for estimation, two techniques - (1) batch least squares method, and (2) Kalman Filter method are used for orbit estimation (in real time). The performance of the onboard orbit estimation has been assessed based on hardware based multi-channel GPS Signal simulator. The results indicate good converge even with short arcs of data as the GPS navigation data are generally very accurate and the data rate is also fast (typically 1Hz).

Keywords: Low Earth Orbit, Preliminary Orbit Determination, GPS Measurements.

1. Introduction

Accurate, time stamped orbital positions of a satellite is required for annotating the payload data onboard the satellite. In recent times, many satellite missions have started using onboard GPS receivers. Since selective availability is off, the navigation data from a standard positioning receiver is already quite accurate for such purposes. The proposed micro-satellite XSAT (Figure-1) of Nanyang Technological University of Singapore will be equipped with GPS receivers for providing the navigation data during imaging operations. In order to meet the power requirements, the satellite will be nadir pointing only during imaging operations and at other time, the satellite will be in 'sun-pointing' mode (Figure-2).

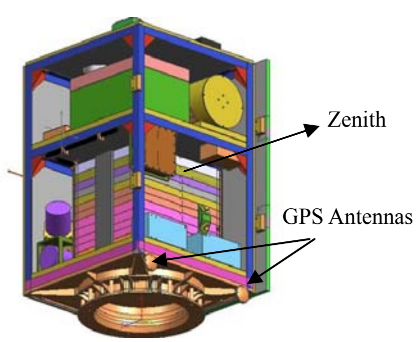


Figure 1. XSAT Satellite

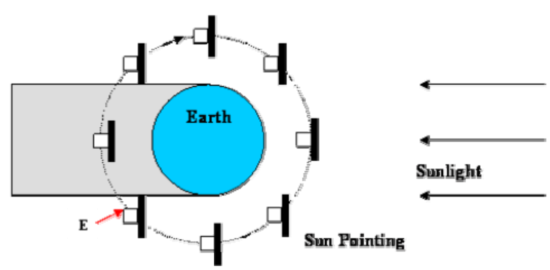


Figure 2. Sun-Pointing Attitude

Due such orientation geometry and the location of antenna on the zenith side it is obvious that the GPS satellites' visibility will not be favorable at all time; Similarly during the initial phase, when the satellite is in sun-acquired mode continuously for a few orbits, the visibility of GPS satellites may pose a problem. Also, the a-priori knowledge of the orbit may be quite inaccurate during the initial phase. *The objective of this paper is to assess how the orbit knowledge can be improved during small arcs of GPS data, particularly during the initial phase.* Of course such constraints have been there for other small satellites missions and for the Fedsat. mission Yanming Feng et al [1] address the problem of orbit determination using 10-15 orbital arc in every orbit and uses such arcs of data sets collected over several days using a Blackjack receiver onboard Fedsat. The paper establishes that for FedSat orbits, the orbit determined from code measurements can be propagated forward for about 72 hours with a maximum orbit error of ±120 meters. Based on the experimental data from other satellites, the same authors show that shorter data arcs based estimation allows for simplifications of both physical and observational models [2]. In our work the emphasis is on orbit determination with small data spans of about 15-30 minutes. This paper is organized as follows. Following introduction, the measurement equations and estimation parameters are formulated using the Keplerian mean elements and the geodetic position data from GPS receivers. The experimental setup involving the GPS signal simulator, the high dynamic GPS receiver of the LEO satellite and the results obtained are discussed subsequently. Then follows a section describing a simplified analytical formulation to assess the sensitivity of the orbital parameters to the navigation data from GPS receiver. This section explains the trends observed from the simulator results. The paper concludes by ascertaining that even a small arc of data slightly away from the nodes provides a reasonably good estimate of the orbital parameters that can serve to predict the orbit during data outages, particularly during the initial phase.

2. Orbit Estimation Problem

In the present work the observations are the geodetic position $[\, x, y, z\,]^T$ output by the GPS receiver and the parameters to be refined are the Keplerian orbital elements at a *given epoch* (T_E). Using the a-priori orbital parameters, the position is predicted for a given instant of a measurement. Then the residue is related to the correction in the orbital parameters by the following equation, which becomes the 'measurement equation', for the Least Square or the Kalman Filter based estimation.

$$\partial P = \begin{bmatrix} \partial x \\ \partial y \\ \partial z \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial x}{\partial a} & \frac{\partial x}{\partial e} & \frac{\partial x}{\partial i} & \frac{\partial x}{\partial w} & \frac{\partial x}{\partial \Omega} & \frac{\partial x}{\partial M} \\ \frac{\partial y}{\partial a} & \frac{\partial y}{\partial e} & \frac{\partial y}{\partial i} & \frac{\partial y}{\partial w} & \frac{\partial y}{\partial \Omega} & \frac{\partial y}{\partial M} \\ \frac{\partial z}{\partial a} & \frac{\partial z}{\partial e} & \frac{\partial z}{\partial i} & \frac{\partial z}{\partial w} & \frac{\partial z}{\partial \Omega} & \frac{\partial z}{\partial M} \end{bmatrix}}_{H} \begin{bmatrix} \partial a \\ \partial e \\ \partial i \\ \partial w \\ \partial \Omega \\ \partial M \end{bmatrix} \quad (1)$$

$$= \frac{\partial P}{\partial X} \partial X$$

$$\partial P = H \partial X \,, \; where \;\; H = \frac{\partial P}{\partial X}$$

The above equation is of the form **Z** = **CX** where **Z** = ∂P, **C**=H and **X** = $[\partial a \quad \partial e \quad \partial i \quad \partial w \quad \partial \Omega \quad \partial M]^T$. While using the Batch Least Squares approach, **X** is calculated after processing all the measurements collected over the time span. Collecting the measurements for n sampling instants, we have an $3n \times 1$ observation vector and a $3n \times 6$ sensitivity matrix as shown in Figure-2.

$$\begin{bmatrix} Z_1 \\ Z_2 \\ . \\ . \\ Z_n \end{bmatrix}_{3n \times 1} = \begin{bmatrix} C_1 \\ C_2 \\ . \\ . \\ C_n \end{bmatrix}_{3n \times 6} X_{6 \times 1} \quad (2)$$

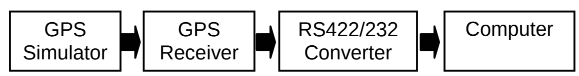
For real-time applications, a Kalman Filter formulation is applied based on the following approach. The state of the Kalman Filter is the deviation between the 'true' orbital elements at a given epoch and the recent knowledge of the same. Treating this as time invariant the state dynamics is represented by,

$$X_{k+1} = \mathbf{I}\, X_k \quad (3)$$

Therefore equations (1) and (3) form the measurement and state equations of Kalman Filter which is in linear form. The estimated state (i.e. corrections to the orbital elements) then refines the epoch parameters.

3. Experiments Using GPS Simulator

The primary navigation data for XSAT is the geodetic position output from the high dynamic GPS receiver made by Accord S/W Inc. In order to test the receiver as well as the orbit estimation algorithm an experiment was setup using the WelNavigate GPS signal simulator (Figure-3).



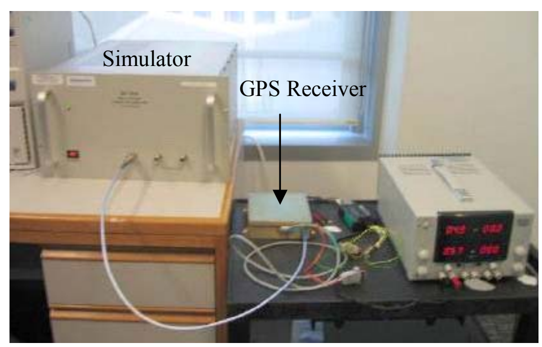


Figure 3. Simulator Setup

The simulator is first started up and once the simulated orbit is stabilized, the GPS receiver is switched on and approximately 1540 measurements are obtained from the simulator spanning a duration of nearly 4 hours. The measurement recording interval is 10 seconds over this period. The actual parameters and the initial guess for the estimator are shown below.

Table 1. Orbital parameters for testing

a	e	*i*	ω	Ω	M	Remarks
7203	0.001	98.8	213	16.5	340	Actual Orbit
7030	0.0	94.1	207	12.9	336	Initial Guess

Using the measurements thus obtained from the simulator, the orbit estimation was performed based on a Kalman Filter formulation. Figure-4 shows the convergence of the three primary orbital elements *a, i, Ω*. The initial guess is nearly

170km off in 'a ', 5 deg in i and 4 deg in Ω. From Figure-4 it can be observed that semi major axis is estimated very quickly within about two minutes of data, while the inclination i is better estimated only after about 1000 (sec) and for the right ascension of node Ω it even takes longer time (2500 sec).

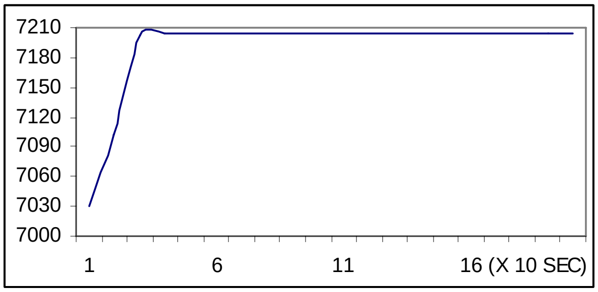


Figure 4a. Semi-major axis (Kms) estimate Vs Time

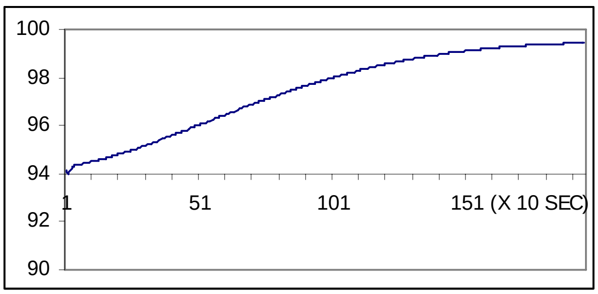


Figure 4b. Inclination estimate Vs Time

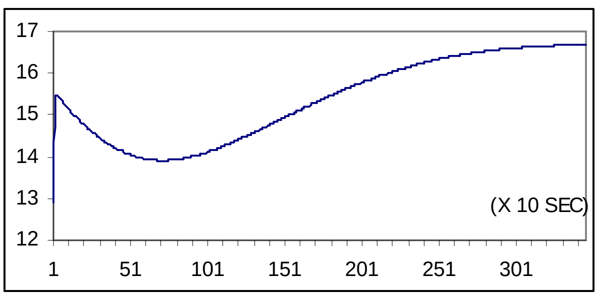


Figure 4c. RAAN (Ω) estimate Vs Time

The simulations results were further analyzed to understand the slow convergence in i and Ω and a second order system behavior is seen in the convergence of i (Figure-5) when observed over long time. This is because the inclination sensitivity, as can be seen intuitively also (later shown analytically), is very poor near the nodes and in the present simulation the satellite's starting position is near the descending node. The following section provides an analytical explanation for this behavior.

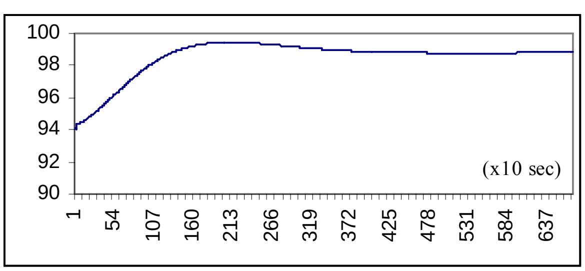


Figure 5. Inclination convergence

4. Analytical Investigation for Inclination Convergence

The analytical formulation assumes a circular orbit. At any given instant of time, the position vector $[x_g, y_g, z_g]^T$ in the Greenwich Coordinate System (GCS) is given by,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_g = r \begin{bmatrix} C_u C_{\lambda_a} + S_u C_i S_{\lambda_a} \\ -C_u S_{\lambda_a} + S_u C_i C_{\lambda_a} \\ S_u S_i \end{bmatrix} \quad (4)$$

where C and S refer to the $Sin(.)$ and $Cos(.)$ functions, r the radius of the orbit and other variables are shown Figure-6.

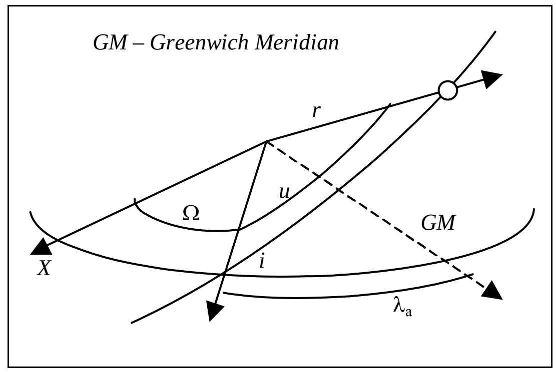


Figure 6. Orbit Geometry

Differentiating equation (4) with respect to the orbit parameters r, i, Ω and simplifying we get,

$$\Delta \bar{x}_g = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_g = \begin{bmatrix} H_a & H_i & H_\Omega \end{bmatrix} \begin{bmatrix} \Delta r \\ r\Delta i \\ r\Delta\Omega \end{bmatrix} \quad (5)$$

where,

$$H_a = \begin{bmatrix} C_u C_{\lambda_a} + S_u C_i S_{\lambda_a} - \frac{3}{2}\eta\tau\left(-S_u C_{\lambda_a} + C_u C_i S_{\lambda_a}\right) \\ -C_u S_{\lambda_a} + S_u C_i C_{\lambda_a} - \frac{3}{2}\eta\tau\left(S_u S_{\lambda_a} + C_u C_i C_{\lambda_a}\right) \\ S_u S_i - \frac{3}{2}\eta\tau C_u S_i \end{bmatrix}$$

$$H_i = \begin{bmatrix} -S_u S_i S_{\lambda_a} \\ -S_u S_i C_{\lambda_a} \\ S_u C_i \end{bmatrix} \quad \text{and} \quad H_\Omega = \begin{bmatrix} C_u S_{\lambda_a} - S_u C_i C_{\lambda_a} \\ C_u C_{\lambda_a} + S_u C_i S_{\lambda_a} \\ 0 \end{bmatrix}$$

Choosing only the terms linked to Δi,

$$\Delta \bar{x}_g = H_i (r\Delta i) = S_u \begin{bmatrix} -S_i S_{\lambda_a} \\ -S_i C_{\lambda_a} \\ C_i \end{bmatrix} (r\Delta i) \quad (6)$$

The above equation is further simplified as,

$$\Delta\bar{x}_g = H_i(r\Delta i) = S_u \hat{n}_g (r\Delta i) \qquad (7)$$

The term $\hat{n}_g$ in the above equation, by referring to Figure-6 can be interpreted as the orbit normal expressed in GCS and S_u is the *Sine* function of the argument of latitude *u*. Further, the product $S_u \hat{n}_g$ is the vector cross product of $\left[\hat{N} \times \hat{S}\right]_g$, $\hat{N}$ being the ascending node unit vector and $\hat{S}$ the unit vector of satellite position in GCS. Summarizing the derivations, the sensitivity of Δi is given by the simple relation,

$$\Delta\bar{x}_g = r\left[\hat{N} \times \hat{S}\right]_g \Delta i \qquad (8)$$

It is now very clear from the above equation (and also from Figure-6) that a given error in the inclination (i.e.Δi) does not influence the measurement residue $\Delta\bar{x}_g$ around the equator (u = 0 or π). Conversely, the measurements around equator are not sensitive to refine the inclination. When such measurements are used in a Kalman Filter, depending upon the initial covariance (P0), the measurement variance R and the process noise Q, there could be erroneous estimations of Δi. As the satellite moves away from the equator, the sensitivity H_i becomes 'well-defined' and the estimation improves. The equations (4) through (8) were programmed in MATLAB and a scalar Kalman Filter form was used to just estimate inclination alone. The initial guess in inclination was off by 5 deg and the MATLAB simulation was started with satellite at the ascending node. Figure-7 shows how the inclination gets estimated as the satellite moves away from the equator. The different curves in the figure correspond to different values of P0. When P0 is very high, the filter gives high weight to the measurements and as the measurements near the equator have very poor sensitivity to Δi corrections are improper resulting in transients. On the other hand, when P0 is small, the filter 'assumes' that the initial guess is accurate and hence gives low weight to measurements. This results in only a small correction to the inclination estimation and hence a slow and delayed improvement in estimation. As a consequence, wild or erroneous corrections are avoided. This is seen in Figure-7 very clearly. Figure-8 shows the Kalman gain.

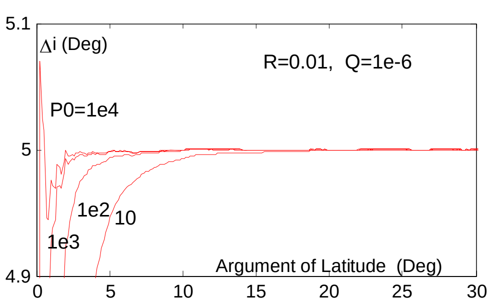


Figure 7. MATLAB Simulation for Δi estimation

For the MATLAB simulation the initial offset in inclination was 5 deg. The measurements were assumed to have a noise of 100 m (1σ). Hence R = 0.01 Km^2. Irrespective of the value for P0, the inclination correction Δi is correctly estimated at about 20 deg of argument of latitude (about 6 minutes from equator).

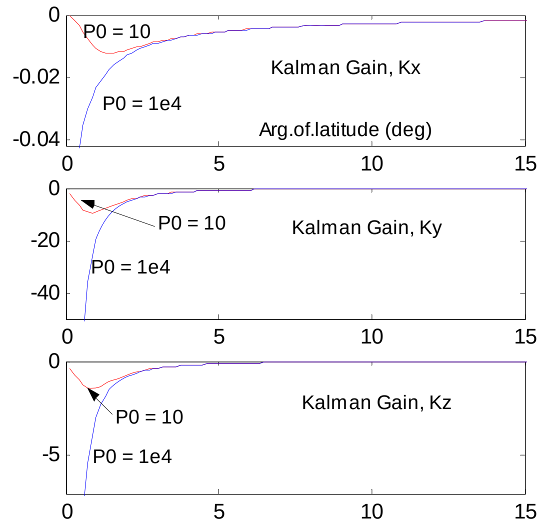


Figure 8. Kalman gain variation

5. Conclusion

The power requirements of a typical micro-satellite mission calls for sun pointing most of times and during imaging the satellite needs to be nadir pointing. Due to such constraints, the geometry for the GPS receiver changes unfavorably resulting in data outages. Also, during the initial phase the satellite attitude may be in Sun acquisition mode and as a result the GPS receiver data may not be continuously available. At the same time, it is essential to update the orbital elements to carry out the initial phase operations. Considering these situations, preliminary orbit determination with short arcs of GPS receiver measurements has been examined by setting up an experiment with the WelNavigate GPS signal simulator and the high dynamic GPS receiver proposed for the mission.

Using the GPS receiver output (position in ECEF system) as measurement, the epoch orbit determination was performed as a Kalman Filter problem. While the convergence in semi-major axis is quite fast, a relative slow convergence is observed for inclination and right ascension of ascending node. This has been analytically examined and shown to be due to the poor sensitivity of the measurements with respect to inclination. Nevertheless, within about 15 to 20 minutes (i.e less than one quarter of an orbit) convergence in the elements could be realized.

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