Solution and Estimate to the Angular Velocity of INS Formed only by Linear Accelerometers

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Abstract

At present, most efforts tend to develop a INS which is only based linear accelerometers, because of the low cost micro-machining gyroscopes lack of the accuracy needed for precise navigation application and possible achieving the required levels of precise for micro-machining accelerometer.

Although it was known in theory that a minimum of six accelerometers are required for a complete description of a rigid body motion, and any configuration of six accelerometers (except for a "measure zero " set of six-accelerometer schemes) will work.

Studies on the feasible configuration of GF-INS indicate that the errors of angular velocity resolved from the six accelerometers scheme are diverged with time or have multi solutions. The angular velocity errors are induced by the biases together with the position vectors of the accelerometers, therefore, in order to treat with the problem just mentioned, researchers have been doing many efforts, such as the extra three accelerometers or the magnetometers may be taken as the reference information, the extended Kalman filter (EKF) involved to make the angular velocity errors bound and be estimated, and so on.

In this paper, the typical configurations of GF-INS are introduced; for each type GF-INS described, the solutions to the angular velocity and the specific force are derived and the characteristic is indicated; one of the corresponding extend Kalman filters are introduced to estimate the angular errors; parts of the simulation results are presented to verify the validity of the equations of angular velocity and specific force and the performance of extend Kalman filter.

Keywords: angular velocity, extended Kalman filter, linear accelerometer, INS.

1. Introduction

In general, most of inertial navigation systems use accelerometers to sense linear acceleration and gyroscope to sense angular velocity. But at present, most efforts tend to develop a INS which is based only linear accelerometers, because of the low cost micro-machining gyroscopes lack of the accuracy needed for precise navigation application and impossible achieving the required levels of precise in the near future, the fundamental physical constraints inhibit the precision of micro-machining accelerometer being less than that of gyroscopes and the cost more affordable.

Although it was known in theory that a minimum of six accelerometers are required for a complete description of a rigid body motion, six-accelerometer schemes were not realized until a cube type GF-INS was proposed by J. Chen. In fact, except for a "measure zero " set of six-accelerometer schemes, any other configuration of six accelerometers will work. They all have the same computational simplicity as the cube type GF-IMU.

Studies on the feasible configuration of GF-INS indicate that the errors of angular velocity resolved from the six accelerometers scheme are diverged with time or have multi solutions. The angular velocity errors are induced by the biases together with the position vectors of the accelerometers, therefore, most work is on the scheme of gyroscope-free inertial navigation system in order to treat with the problem just mentioned. The extra three accelerometers or the magnetometers may be involved and taken as the observation to make the angular velocity errors bound and be estimated.

It is known that the standard Kalman filter provides an optimal or minimum error estimate of the state vector of linear system in the case of both the system noise and the measurement noise being zero-mean and white, i.e. Gauss process. Because the differential equation of angular and linear velocity is nonlinear, the extended Kalman filter (EKF) will be involved to estimate the angular velocity errors. The EKF is the optimal least squares estimators for the system described by the nonlinear state-space model and well adaptive to our problem.

In this paper, the typical configurations of GF-INS are introduced; for each type GF-INS described, the solutions to the angular velocity and the specific force are derived and the characteristic is indicated; one of the corresponding extend Kalman filters are introduced to estimate the angular errors; parts of the simulation results are presented to verify the validity of the equations of angular velocity and specific force and the performance of extend Kalman filter.

2. Cube-Type GF-INS Presented by J. Chen

The scheme includes six accelerometers placed respectively at the center of each of the six cube faces, and its sensing direction along the respective cube face diagonal form a regular tetrahedron, the extra three orthogonal accelerometers are involved and placed at the center of the cube, and taken as the observation to estimate and bound the angular velocity error, as shown in Fig. 1.

(1) Angular velocity differential equation and the centroid specific force[1].

It is known by above that the angular velocity differential equation is linear and the specific force equation non-linear to the angular velocity, described as follow:

$$\dot{\omega}_{ib}^{b} = \frac{1}{2l^2} J_1 \tilde{A} \tag{1}$$

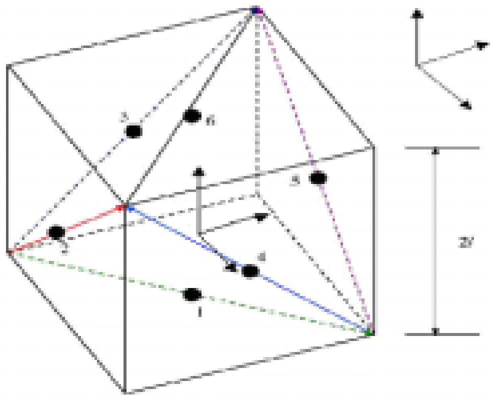


Fig1. Cube-type GF-INS (nine-accelerometer)

$$f_c^b = \frac{1}{2} J_2 \tilde{A} + l\hat{\omega} \tag{2}$$

Where

$$J_1 = \frac{l}{\sqrt{2}} \begin{bmatrix} 1 & -1 & 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & -1 & 0 & -1 \\ 0 & 1 & -1 & -1 & 1 & 0 \end{bmatrix}$$

$$J_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 & -1 & -1 \\ 1 & 0 & 1 & -1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\tilde{A} = \begin{bmatrix} \tilde{A}_1 & \cdots & \tilde{A}_6 \end{bmatrix}^{\mathrm{T}}$$

$$\hat{\omega} = \begin{bmatrix} \omega_y \omega_z & \omega_z \omega_x & \omega_x \omega_y \end{bmatrix}^T$$

The angular velocity ω_{ib}^b will diverge rapidly in the case of the accelerometer biases and noises being considered.

(2) Angular velocity observation and estimation

The extra three accelerometers at the center of cube are involved to measure the centroid specific force and make the observation equation based on the left side of equation (2) being replaced by the outputs of the extra three accelerometers, the principle of the estimation $\hat{\omega}_{ib}^b$ is shown in Fig.2.

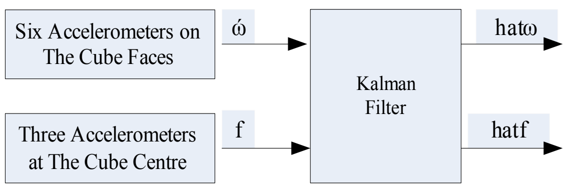


Fig. 2. Estimation of angular velocity ω_{ib}^b

The discrete-time forms of the angular velocity equation a nd observation equation are as follow:

$$\omega_{ib}^b(k+1) = \omega_{ib}^b(k) + \frac{T}{2l^2} J_1 \tilde{A}(k) + w(k) \tag{3}$$

$$f_o^b = \frac{1}{2} J_2 \tilde{A}(k) + l\hat{\omega}(k) + v(k) \tag{4}$$

Here the $w(k)$ and $v(k)$ are with the power spectral densities Q and R. The angular velocity estimate algorithm is stated as:

$$\hat{\omega}_{ib}^b(k/k-1) = \hat{\omega}_{ib}^b(k-1/k-1) + \frac{T}{2l^2} J_1 \tilde{A}(k-1) \tag{5}$$

$$P(k/k-1) = P(k-1/k-1) + Q \tag{6}$$

$$S(k) = H(k)P(k/k-1)H^T(k) + R \tag{7}$$

where

$$H(k) = \left[\frac{\partial \hat{\omega}}{\partial \omega_{ib}^b} \right]_{\omega_{ib}^b = \omega_{ib}^b(k/k-1)} \tag{11}$$

$$\hat{f}^b(k) = \frac{1}{2} J_2 \tilde{A}(k) + l\hat{\hat{\omega}}(k/k-1) \tag{12}$$

In the case of the reference angular velocity and the specific force, the angular velocity solved by conventional method and involving extra accelerometers are shown as the following Figures[1]:

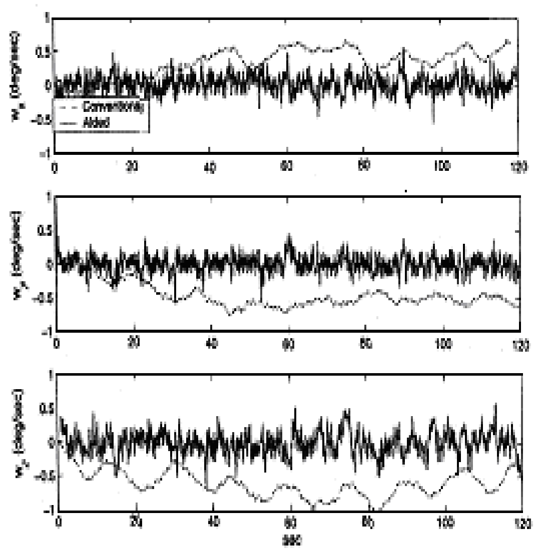


Fig. 3. Angular velocity estimation with same initial conditions

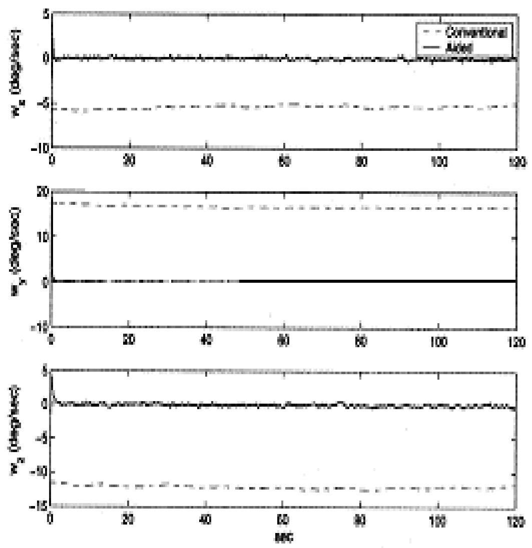


Fig. 4. Angular velocity estimation with same initial mismatches

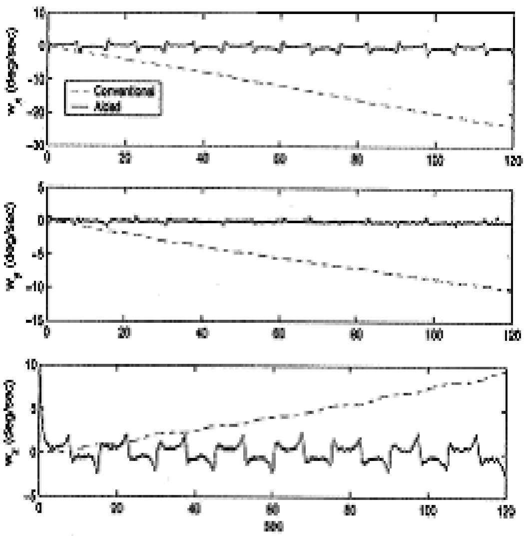


Fig. 5. Angular velocity estimation with biased accelerometers

3. Six Accelerometers on the Unit Sphere Presented by Hsin-Yuan Chen

The six accelerometers are positioned at the unit sphere, the sensing direction is shown as Fig.6.

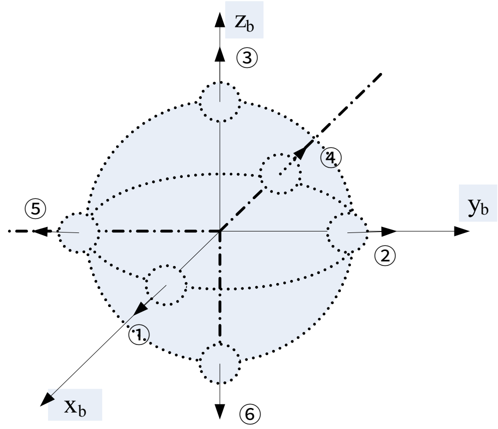


Fig. 6. Scheme of six accelerometers on the unit sphere

When the installation errors are neglected, there are the f ollowing:

$$r = \begin{bmatrix} r_1 & \cdots & r_6 \end{bmatrix} = l \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix}$$

$$\theta = \begin{bmatrix} \theta_1 & \cdots & \theta_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix}$$

Substituting r and θ given above, the relations of angular velocity and specific force with the accelerometer outputs may be get as the equations (13) and (14):

$$\begin{bmatrix} \omega_x^2 \\ \omega_y^2 \\ \omega_z^2 \end{bmatrix} = \frac{1}{4l} \begin{bmatrix} -1 & 1 & 1 & -1 & 1 & 1 \\ 1 & -1 & 1 & 1 & -1 & 1 \\ 1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix} \tilde{A} \tag{13}$$

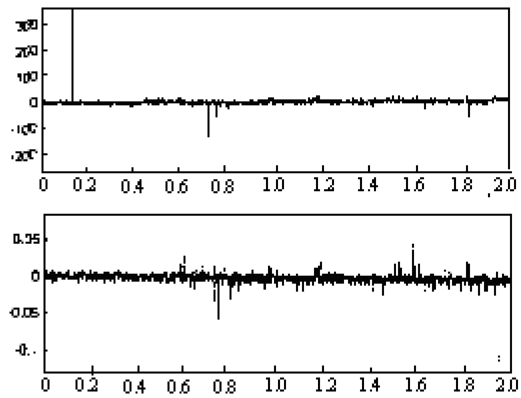
$$\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \tilde{A} \tag{14}$$

It is can be known that the angular velocity error is bounded, but the angular velocity has the multiple values and the solutions.

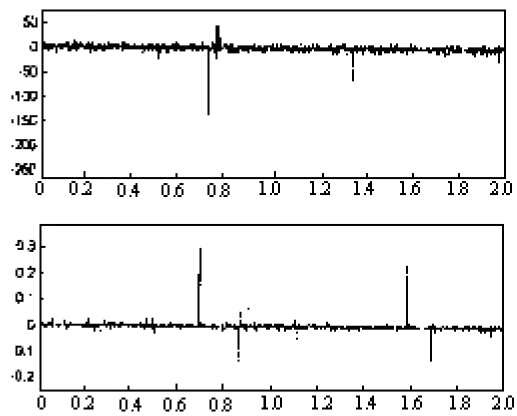
In order to determine the sign of the angular velocity in (13), the magnetometer measurements of angular velocity may be involved and located at the center of the sphere, the angular velocity used to the strapdown navigation algorithm may be the absolute value solved by (13) and with the same sign as that of magnetometer outputs. The magnetometer measurement equation is:

$$y(t) = \omega_{ib}^b(t) + v(t) \tag{15}$$

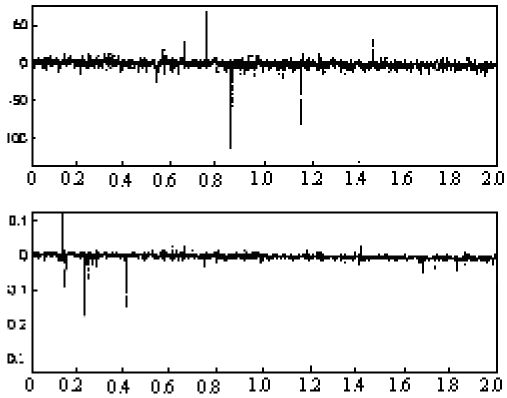
For the given conditions: $l = 1$, $\tilde{A}$ taken the random values within 0 and 1, the linear accelerometer errors and the angular velocity errors are convergent and shown in Fig. 7[2] .



(a). x axis



(b). y axis



(c). z axis

Fig.7 Linear acceleration error and angular velocity error

4. Nine Accelerometers Paralleling to the Frame Axes Presented by Wang Qi

In this scheme, the nine accelerometers are placed as shown in Fig. 8.

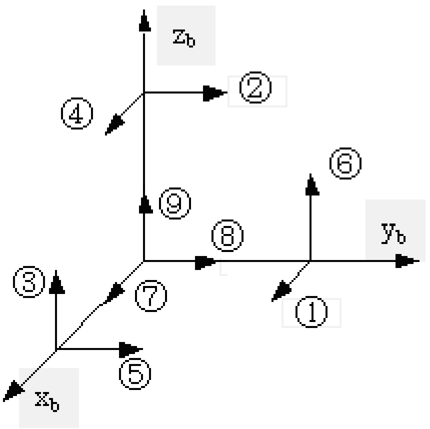


Fig.8. Nine accelerometers paralleling to the frame axis

It is convenient to mount the accelerometers for this configuration, but the angular velocity no longer linear to the outputs of the accelerometers. Six of accelerometers are respectively placed at the three axes and three at the original point of the frame. The angular velocity and the specific force calculated by the six accelerometers from the original (i.e. at the three axes) are described by the equations (16) and (17).

$$\dot{\omega}_{ib}^{b} = J_{\omega 1}\tilde{A} + J_{\omega 2}\hat{\omega} \tag{16}$$

$$f_c^{b} = J_{a1}\tilde{A} + J_{a2}\hat{\omega} \tag{17}$$

$$J_{\omega 1} = \frac{1}{2l}\begin{bmatrix} 1 & -1 & -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 & 1 & -1 \end{bmatrix}$$

$$J_{a1} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 & -1 & 1 \end{bmatrix}$$

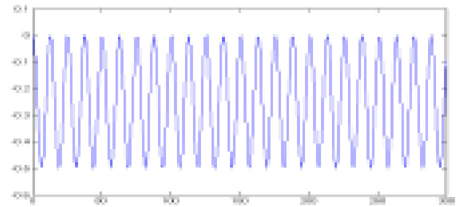
$$J_{\omega 2} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$J_{a2} = l\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

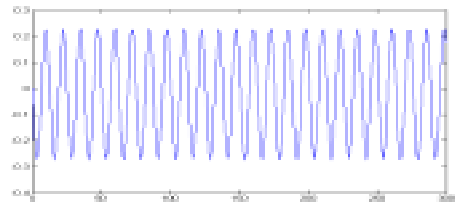
Both the angular velocity differential equation and the specific force of the original are non-linear. The outputs of three accelerometers positioned at the original are taken as the measurement to estimate the angular velocity then the measurement equation may be:

$$f_o^{b} = J_{a1}\tilde{A} + J_{a2}\hat{\omega} \tag{18}$$

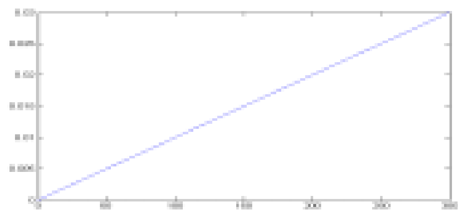
Giving the simulation conditions: ω_{ibx}^{b} and ω_{iby}^{b} being period functions, ω_{ibz}^{b} quadric function, f_c^{b} period functions with two periods, accelerometer biases neglected, then the angular velocity errors are shown in Fig. 9.



$\Delta\omega_x(t)$



$\Delta\omega_y(t)$



$\Delta\omega_z(t)$

Fig.9 Angular velocity errors

It is obvious that the angular velocity errors will diverge with time in the case of accelerometer biases considered because of integration, so the extra three accelerometers (at the coordinate original) are involved and be taken as the measurement to guarantee the estimate of ω_{ib}^{b} bounded.

5. One of the Extended Kalman Filter Algorithm

For the non-linear angular velocity differential equation, the Extended Kalman Filter (EKF) is required, one of the recommendatory method is stated by the following.

Assuming that the general discrete non-linear system is presented by:

$$x_k = f_{k-1}(x_{k-1}) + \Gamma_{k-1}(x_{k-1})w_{k-1} \tag{18}$$

$$z_k = h_k(x_k) + v_k \tag{19}$$

where f_{k-1} and h_k are nonlinear function of x_{k-1} and x_k, w_{k-1} and v_k are zero-mean Gaussian white noise sequences with covariances R and Q, x_0 is a Gaussian vector with a constant mean value and initial covariance matrix P_0. Let x_k^0 is the ideal value corresponding to the real state x_k, and $\Delta x_k = x_k - x_k^0$, $\Delta z_k = z_k - h_k(x_k^0)$, then the linearization equations of (18) and (19) are:

$$\Delta x_k = \frac{\partial f_{k-1}}{\partial x_{k-1}^0}\Delta x_{k-1} + \Gamma_{k-1}(x_{k-1}^0)w_{k-1} \tag{20}$$

$$\Delta x_k = \frac{\partial f_{k-1}}{\partial x_{k-1}^0}\Delta x_{k-1} + \Gamma_{k-1}(x_{k-1}^0)w_{k-1} \tag{21}$$

where

$$\frac{\partial f_{k-1}}{\partial x_{k-1}^0} = \left.\frac{\partial f_{k-1}(x_{k-1})}{\partial x_{k-1}}\right|_{x_{k-1}=x_{k-1}^0} \tag{22}$$

$$\frac{\partial h_k}{\partial x_k^0} = \left.\frac{\partial h_k(x_k)}{\partial x_k}\right|_{x_k=x_k^0} \tag{23}$$

Then the estimate of x_k is

$$\hat{x}_k = \Delta\hat{x}_k + x_k^0 \tag{24}$$

Taking the system stated by formula (16) and (17) for example, when the linearization has been done, the EKF above may be used to estimate the angular velocity.

It is should be indicated that the method just mentioned above only suited to the less filter error and the one-step predict error, otherwise the additional estimate algorithm must be investigated.

6. Conclusion

The scheme of accelerometer configurations are not unique to get the angular velocity of body frame relative inertial frame and may be solved only by six accelerometers allocated from the centroid, but the calculation error is unavoidable diverged rapidly with time. In order to bound the calculation error of angular velocity error, the measurement must be introduced, for example, the magnetometer. Because of the non-linear angular velocity differential equation, the extend Kalman filter is required. The linearized differential equation discussed above corresponds to the smaller error angular and invalids for the case of larger error angular, additional estimate algorithm should be investigated in the near future work.

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