A Modification of the Approach to the Evaluation of Collision Risk Using Sech Function

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Abstract

Evaluation of collision risk plays a key role in developing the expert system of navigation and collision avoidance. This paper presents a new collision risk model formula that is one modification model on the basis of one approach to the evaluation of collision risk using sech function produced by Prof. Jeong in his relevant articles[2][3][4][5]. And as a grope in collision risk evaluation field, this paper applied the new model in appraising the collision risk, suggested how to decide the safe range of own ship's action. Moreover this paper also analyzed theoretically how to determine the coefficients as describes in the new modification model formula, and suggested the appropriate values as applicable.

Keywords: collision risk, sech function, distance of closest point of approach, Approach Time,

1. Introduction

Evaluating the risk of collision quantitatively plays a key role in developing the expert system of navigation and collision avoidance. A new approach to collision risk using sech function was introduced to solve the problems other correlative researches have(Jeong,2003).
In this paper, a modification of this new approach using sech function will be introduced

2. Modification Approach

2.1 Original Approach

$$CR = p \cdot \operatorname{sech}(a \cdot dcpa) + q \cdot \operatorname{sech}(b \cdot ta) + r \cdot \phi(\theta, \alpha) \quad (1)$$

2.2 Modification Approach

$$CR = \frac{p \cdot \operatorname{sech}(a \cdot dcpa)}{ta} + r \cdot \phi(\theta, \alpha) \quad (2)$$

In equation (1) and (2), *CR* is the collision risk; *dcpa* is the distance of closest point of approach; ta is the approach time; *p, q, r* are the amplitude coefficients; *a, b* are the changing extent coefficients of *sech* function; $\phi(\theta, \alpha)$ is the own ship's state function, it has the relationship with the targets' position θ and aspect α, the magnitude of which is 0 if own ship is in the stand-on situation and 1 if she is in the give-way situation.

2.3 Some Points from Modification Approach

① When *dcpa* decreases *CR* will increase. It is inverse proportion to *CR*, but this kind of increasing of inverse proportion does not fit for intuitionistic observation. For example, if *dcpa* is 0 and 0.1 mile respectively, *CR* will almost be the same according to intuitionistic observation. And the results of *sech(dcpa)* almost are both 1, this situation fits for intuitionistic observation. So using *sech(dcpa)* is reasonable.
② In modification approach, when *ta*'s value 0, *CR* will be discontiguous. That is to say, *CR* will change from maximum to minimum. So when target is very close to own ship, for example, the distance is within 0.1 mile, the *CR* will be so low. Of course, this situation is not fit intuitionistic observation. Considering the situation that *ta*'s value is 0 seldom happens and *CR* will change from '+' to '-' when the targets pass *CPAs*, the formula to calculate *ta* should be modified and designed to a subsection function. But *ta* is inverse proportion to *CR*, using *ta* into the modification approach can clearly display the actual situation.
③ When ζ is 90°, at that time the targets pass *CPAs*, the modification approach's result will be 0. At that time relative speed's projection in the direction of approaching to own ship is 0, so *ta* can be thought it is ∞,that is to say the target vessel can not approach to own ship forever because the speed to own ship is 0 mile. Moreover when *ta* is 0 and *dcpa* is 0, that means numerator is maximum and denominator is minimum, so the *CR*'s maximum of the modification approach can be gotten, that is ∞, and only one practical situation fits for this condition, that is target really collide with own ship. And this situation is also fit intuitionistic observation.
④ Using *sech(dcpa)* and *ta* into a same formula, more important navigation factors can be considered. So the modification approach will be more scientific and effective. Put the values of *dcpa* and *ta* into the modification approach, the result of collision risk can be gotten easily.
⑤ When own ship state function $\phi(\theta, \alpha)$ is used, its appropriate value should be gotten according to own ship maintains her course and speed or alters.
⑥ Because $ta = \dfrac{R}{v_r \cos\zeta}$ (Jeong, 2003a) ①, the value of modification approach can be plus or minus, this is one characteristic of the modification approach. '+' value means targets are approaching to own ship and will pass *CPAs*, during

① $V_r \cos\zeta$ is called the approach speed according to H.Imazu's definition.

this process *CR* will increase. '-' value means targets passed *CPAs* and will leave away from own ship, so *CR* will decrease after targets passed *CPAs*. So the minimum of *CR* is not -∞,but 0. The minus value just means targets passed *CPAs* or not and can not be compared with plus value .Moreover, when the targets pass *CPAs*, *CR* will be 0. These characteristics are big differentiates to the original approach.

3. Application of Modification Approach to Verify Its Effect

Before research the detailed coefficients of modification approach, the verification of the approach should be researched first. Meanwhile the characteristic of the coefficients also can be observed.

For simplicity, the coefficients of modification approach are supposed as $p=1$, $r=0$ and $a=1.15$. Here $r=0$ means ignoring the effect of own ship state function temporarily.

3.1 When Different Targets Approaching Own ship :(Own ship Does Not Take an Action)

In table 1, own ship and the targets' initial positions are given and in Figure 1 the approaching targets' *dcpa* are 0.5 miles and 1.5 miles respectively. Figure 2 expresses the two targets' *CR*.. When they pass the *CPAs* respectively, the *CR* is 0 and *CR* values are minus after they pass the *CPAs*. At a short time before the targets pass *CPAs* the collision risk value reaches its maximum. The target which *dcpa* is 0.5 mile, its *CR* maximum value is 0.6065, and the target which *dcpa* is 1.5 mile, its *CR* maximum is 0.0716.

But, just as Figure 2 shows before targets pass *CPAs* the *CR* value reaches its maximum and after that it decreases sharply to 0. This is not fit for our aim to design the modification approach. *ta* should be modified in order to let *CR* value reaches its maximum just before targets pass *CPAs*.

Table 1. Own ship with no Action and Targets' Situation

	Own ship	Target 1	Target 2
dcpa(mile)	--	0.5	1.5
Range(mile)	--	8.0	8.0
Bearing(°)	--	023.9	050
Course(°)	000	235	258
Speed(mile/min)	0.4	0.4	0.4

3.2 The Collision Risk When Own ship Alters Her Course or Speed

In table 2 own ship and the target's information is shown, and when the target is as near as 4.2 miles, own ship makes the course or the speed changed actions, and the *dcpa* will be expected from 0.5 mile to 1.5 mile. If course is altered, it will be changed from 000° to 035° and if speed is changed ,it will be changed from 0.4 mile/min to 0.18 mile/min.

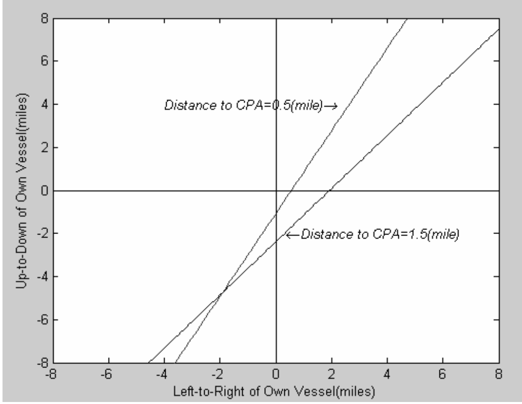

Figure 1. Two Targets with Different *dcpas* Approaching to Own ship

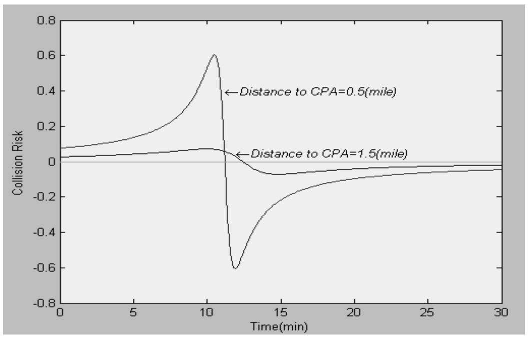

Figure 2. Comparison of Collision Risk of Two targets with Different *dcpas*

Table 2. Own ship with Action and Target's Situation

	Own ship	Target 1	Avoidance Action
Range(mile)	--	8.0	4.2
Bearing(°)	--	023.9	050
Course(°)	000	235	035
Speed(mile/min)	0.4	0.4	0.18

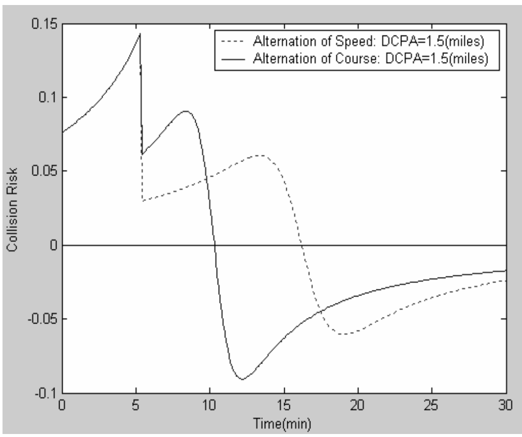

Figure 3. After Avoiding Action with Alternation of Course and Speed Respectively

In Figure 3, the approach time *ta* of the situation course altered is smaller than that of the situation speed altered. The reason is the relative speed of course altered situation is bigger than that of speed changed situation. And in these situations the collision risk sometimes can be bigger than *CR* threshold again, so in those situations we should consider the re-avoidance actions. Moreover the problem can be solved by adjust the appropriate coefficients *p* and *a*, and sufficiently considering the own ship state function $\phi(\theta, \alpha)$ into modification approach.

3.3 Own ship's Safety Action Range

1) Single Target Situation

Using the modification approach, own ship's safety action range can be gotten.

In table 3 own ship and the target's information is shown. But safety action range only can be calculated by considering course altered only or speed altered only respectively. Own ship's initial course is 000°, speed is 0.4 mile/min. Moreover the expected *dcpa* after avoidance action should be at least 1.5 miles.

Table 3. Own ship with Full Range of Action and Target's Situation

	Own ship	Target	Range of Action
Range(mile)	--	4.2	Course: 0~360° Speed: -0.4~0.4 mile/min
Bearing(°)	--	040	
Course(°)	000	230	
Speed(mile/min)	0.4	0.3	

When one appropriate threshold is given, the safety course range with speed maintaining or safety speed range with course maintaining can be gotten easily when own ship takes corresponding avoidance actions. In Table 3, the target's initial position is course 000°, speed is 0.3 mile/min, and the distance to own ship is 4.2 mile. When own ship takes avoidance actions by taking course changed from 0° to 360° with speed maintaining, the collision risk values according courses is shown in Figure 4. And when own ship takes avoidance actions by taking speed changed from -0.4~0.4mile/min(-24~ 24 knot) with course maintaining, the collision risk values according speed are shown in Figure 5. In the two figures, 0.04 is both used as the collision risk threshold. Then own ship's safety course range and speed range can be gotten respectively. Here according to Figure 4, the safety course range is the course value under *CR* threshold, that is 101.5° ~338.5°; and according to Figure 5. the safety course range is -0.4~0.249 (mile/min), that is -24~14.94 knot. Moreover, here the safety threshold is just a supposed value, a useful threshold should be gotten by analyzing from practical data and be verified by many vessel experiments on the sea.

2) Multi-Targets Situation

Sometimes the targets are more than one and that time this method should also can be used to get the kind of safety range. Using the data in the table 4, safety course ranges with speed maintaining can be gotten. Own ship's initial course is 000° and speed is 0.4 mile/min. And the expected *dcpa* after avoidance action should be at least 1.5 miles.

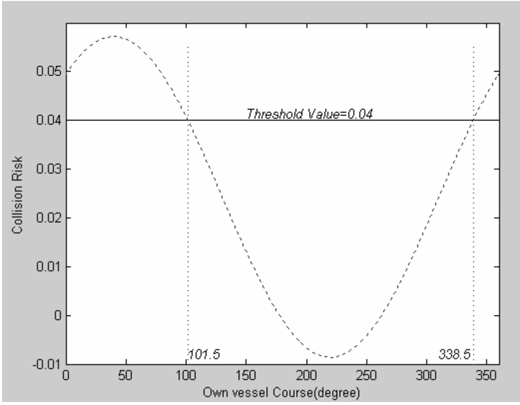

Figure 4. Collision Risk of a Target against Alternation of Own ship's Course

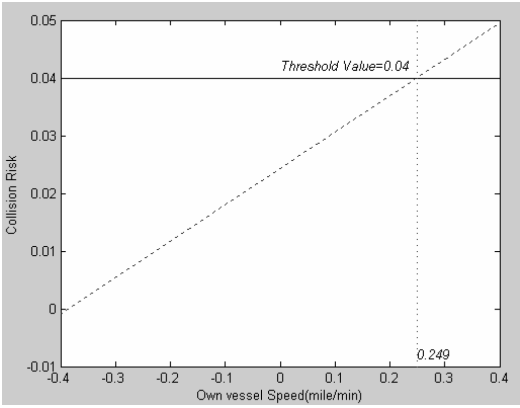

Figure 5. Collision Risk of a Target against Alternation of Own ship's Speed

Table 4. Own ship with Full Range of Action and Targets' Situation

	Own ship	Target 1	Target 2	Target 3	Range of Action
Range(mile)	--	5.0	8.0	5.0	Course: 0~360°
Bearing(°)	--	040	170	300	
Course(°)	000	260	012	120	
Speed (mile/min)	0.4	0.3	0.6	0.4	

Here are 3 targets, and own ship changes course from 0°~360°. Any time own ship's course collision risk value is the maximum of the 3 targets'. The 3 targets' collision risk values according course can be gotten and shown in Figure 6. Also,0.04 is as the course threshold. Then the safety course range can be gotten. Just as shown in figure 6, the safety course range with speed maintaining is 3.5° ~ 10.8°, 69.2° ~ 147.7°, 192.4°~ 236.5°. Moreover the threshold is also effected by the own ship state function $\phi(\theta, \alpha)$. So all referred important factors should be considered synthetically when the threshold value be decided.

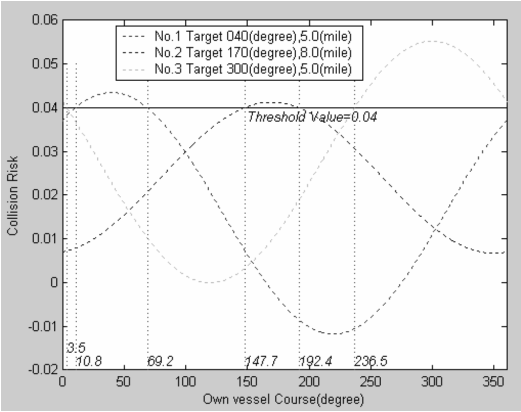

Figure 6. Collision Risk of Multi-Targets with Own ship's Course Alternation

4. Coefficient Decision of Modification Approach

4.1 Modification of *ta*

For solving the problem in 2.3 ②, *ta* should be modified. Some basic factor relationships are as follows:

$$dcpa = R \cdot \sin\zeta \tag{3}$$

$$\zeta = \left| Cr - (\theta + 180) \right| \tag{4}$$

$$ta = \frac{R}{v_r \cos\zeta} \tag{5}$$

Here, R is the distance between targets to own ship; ζ is target's relative moving direction angle, It's range is $0 \leqslant \zeta \leqslant 180^{o}$; Cr is target's relative moving course; θ is target's bearing.

According to (5), when ζ=90º, *ta*'s value is ∞, the modification approach *CR* value is 0.

Substituting (3) into (5),

$$\begin{aligned} ta &= \frac{R}{V_r . \cos\zeta} \\ &= \frac{2 \cdot R \cdot \sin\zeta}{V_r \cdot 2\sin\zeta\cos\zeta} \quad \text{when } \zeta \neq 0^{\circ} \text{or } \zeta \neq 180^{\circ} \\ &= \frac{2dcpa}{V_r \cdot \sin 2\zeta} \end{aligned} \tag{6}$$

From (6), when ζ=45º or 135º, *ta* will be minimum, so *1/ta* will be maximum.

Moreover, when ζ=90º, the value of *1/ta* is 0,and *CR*=0. So ζ=90º should be separated to be a single point as a case of a subsection function to satisfy design requests and this point is also a connection from '+' to '-', so the *1/ta*'s modification is as follows:

$$\begin{aligned} \frac{1}{t_a} &= \frac{V_r}{2 \cdot dcpa} = \frac{\sqrt{2} \cdot V_r}{2 \cdot R} \quad 45^{\circ} < \zeta < 90^{\circ} \\ &= 0 \qquad \zeta = 90^{\circ} \\ &= -\frac{V_r}{2 \cdot dcpa} = -\frac{\sqrt{2} \cdot V_r}{2 \cdot R} \quad 45^{\circ} < \zeta < 90^{\circ} \\ &= \frac{V_r \cdot \cos\zeta}{R} \qquad \text{otherwise} \end{aligned} \tag{7}$$

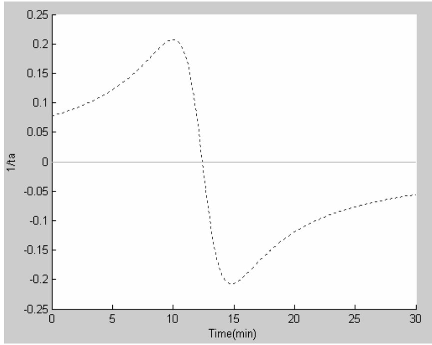

Figure 7. *1/ ta* with no modification

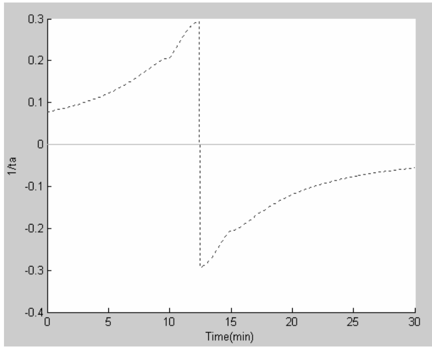

Figure 8. *1/ ta* with modification

Figure 7 is *1/ ta* with no modification, and Figure 8 is that with modification. Obviously, after modification the value of 1/ *ta* increase until targets just before passing *CPA*s, and *CR* value changes from '+' to '-', all of these fit the aim of 1/ *ta*'s modification design and can display actual situation well.

When targets leave *CPAs*, this time the value of *CR* is minus. When this distance from target to own ship is quite far, for example 200 miles, the *CR* value using the modification approach is almost 0(the minimum of modification approach), this situation fits actual situation.

4.2 Determining Coefficient *a*

For determining coefficient *a* simply, the amplitude coefficients of modification approach are supposed as *p* is 1 and *r* is 0.

Commonly, when target approaches to own ship, target's *CR* value's difference between before action's *CR* value and that of after action is a criterion to judge the action is effective or not. If the difference is minus, that is to say *CR* value of after avoidance action taken is bigger that of before action taken, it is so say the action is not effective and should be adjusted. Obviously, if the difference can get to its maximum, the action taken is the most effective one. Here, the difference can be defined as follows:

$$F = CR_1 - CR_2 = \frac{\sec h(a \cdot dcpa_1)}{ta_1} - \frac{\sec h(a \cdot dcpa_2)}{ta_2} \tag{8}$$

Here, $dcpa_1$ and t_{a1} mean target's *dcpa*'s value and value of approaching time before action taken, and $dcpa_2$ and t_{a2} are the values when the approach time is maximum after action taken. The result of *F* in (8) should bigger than 0.

When $dcpa_1$, $dcpa_2$, ta_1, ta_2 are all known, *F* is a function of *a*. That is as follows:

$$F(a)=\frac{\sec h(a.dcpa_1)}{ta_1}-\frac{\sec h(a.dcpa_2)}{ta_2} \qquad (9)$$

According to the known mathematic derivative knowledge, when the derivative of *F(a)* is 0, that time a value of *a* can be gotten, using the value of *a* in *F(a)*, the maximum of *F(a)* can be gotten. So the value of *a* is just the expected value of coefficient *a*. *F(a)*'s derivative formula is as follows:

$$\begin{aligned}\frac{dF}{da}&=\frac{-\sec h(a\cdot dcpa_1)\tanh(a\cdot dcpa_1)}{ta_1}dcpa_1\\&+\frac{\sec h(a\cdot dcpa_2)\tanh(a\cdot dcpa_2)}{ta_2}dcpa_2 \qquad (10)\\&=0\end{aligned}$$

The value of *a* getting from (10) is when the value of *dF/da* is 0, and $dcpa_1 < dcpa_2$, then *F(a)* can reaches its maximum. *F(a)*'s value is bigger is to say the effect of the avoidance action is better.

4.3 Flow Chart of Getting Coefficient *a*

Commonly, using the below method to get the value of coefficient *a*. Its flow chart is as follows:

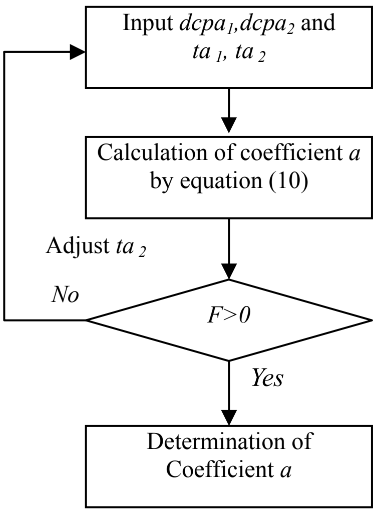

Figure 9. Procedure of getting of Coefficient *a*

There are two points should be explained:

(1) The value of coefficient *a* should let the value of *F* bigger than 0. Otherwise, ta_2 should be calculated and adjusted again. The situation that the value of *a* can not let *F* bigger than 0 means the action taken can not decrease target's *CR*, so some other actions should be taken. This process will continue until *F* bigger than 0.

(2) Usually, $ta_2 < ta_1$ because only when the vessel's speed is very low, $ta_2 > ta_1$ could happen. But this situation almost has no effects to *F>0*.

4.4 The Meaning of Getting Value of Coefficient *a*

The meaning of the formula (10) is shown in Figure.10 as follows:

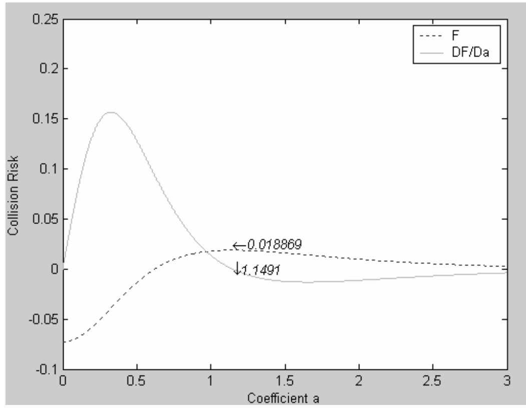

Figure 10. Collision Risk Difference F by coefficient *a*

In Figure 10, for getting coefficient *a* using the $dcpa_1$ is 1.5 mile and $dcpa_2$ is 2.3 mile, ta_1 is 6.998 minutes and ta_2 is 4.635 minutes substitute into formula (9) and formula(10), the graphics of *F* and *dF/da* can be gotten in Figure 10. In Figure 10, the point where *dF/da* is 0, the value of *a* to that point is $a = 1.1491$. And to this point, $F(\xi)=0.018869$ is the maximum of the graphics of *F*. So the value of coefficient *a* is determined to that value $a = 1.1491$.

4.5 Validating Coefficient *a*

The upper parts discuss the method to get the coefficient *a*, and this part some examples will be used to verify the coefficient *a*. Here *dcpa* before the avoidance action is 1.5 miles and the *dcpa* after the avoidance action is expected to be at least 2.3 miles. And after the avoidance action the *CR* maximum will be considered as the collision risk value.

Own ship's initial course is 000° and the target's initial course is 180°.

In the situations with the relative speed is 1.0(mile/min) the speeds of own ship and the target are 0.5(mile/min) respectively. And the target's initial position is 9.0 mile and its bearing is 000°.

In the situation with the relative speed is 0.1(mile/min) the speed of own ship and the target is 0.05(mile/min) respectively. The target's initial position is 5.0 mile and bearing is 000°.

When the approach time is smaller than 7.0 minutes the avoidance action will be taken.

In the second situation if the avoidance action is taken when the approach time is smaller than 7.0 minutes, that time the target is too close to own ship, it is not appropriate. So in the second situation the avoidance action is taken when the range is less than 3.25 mile.

The avoidance action of own ship is supposed to change course from 000° to 035° in the two situations. And the value of coefficient *a* is 1.1491.

Figure 11 is *CR* figure of the situation of relative speed 1.0(mile/min), *dcpa* before avoidance action is 1.5 mile and when the approach time is less then 7.0 minutes(approach distance is 7.0 miles), own ship changes its course(takes avoidance action), and the *dcpa* this time is 2.3 miles. And the collision risk value is 0.0494 before the avoidance action, and it is bigger than 0.0435 which is the maximum value of collision risk after the action.

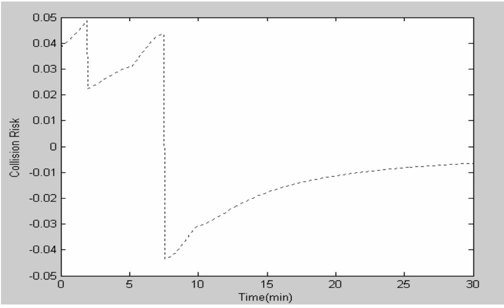

Figure 11. Collision Risk in case of Relative Speed 1.0 (mile/min)

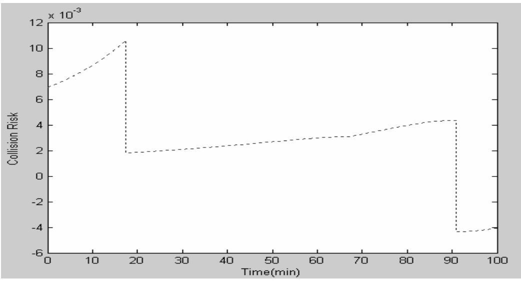

Figure 12. Collision Risk in case of Relative Speed 0.1 (mile/min)

Figure 12 is *CR* figure of the situation of relative speed 0.1(mile/min), *dcpa* before avoidance action is 1.5 mile and when the approach distance is less then 3.25 miles, own ship changes its course, and the *dcpa* this time is 2.3 miles. And the collision risk value is 0.010608 before the avoidance action, and it is bigger than 0.004353 which is the maximum value of collision risk after the action.

Another two situations of relative speeds are 0.7 and 0.5 mile/min are also be used to validate coefficient *a*. And in the two situations the same result that *CR* value before action taken is bigger than *CR* maximum after action taken can be gotten.

From these situations, the value of coefficient *a* is 1.1491 is fit these situations, the *CR* values are smaller than that before the avoidance action. When the relative speed is high the timing to take the avoidance action should select the time when the approach time is less than 7.0 minutes. And when the relative speed is low the timing to take the avoidance action should select the time when the approach distance is less than 3.25 miles.

The value of coefficient *a* should be determined carefully, because the value has close relationship with the threshold of collision risk.

5. Conclusion

This paper introduced a modification approach using *sech* function to evaluate collision risk. The modification approach introduced in this paper is to evaluate collision risk using *sech* function and is to solve the problems the other existing approaches have.

As a result, some conclusions can be concluded as follows:

① *sech(dcpa)* and *ta* are both used into the modification approach, the approach can consider more important factors and be more scientific and effective.

② When target passes *CPA*, that is to say ζ is 90º, *CR* value of modification approach will be 0. It is because the speed on the direction to own ship is 0 , *ta* is ∞, the target can not approach own ship any more. So *CR* value is 0 is reasonable in this situation. Moreover, when collision really happened, ta and *dcpa* will both be 0, denominator is minimum, numerator is maximum, so *CR* value can reach maximum ∞ obviously. The result of this situation is also reasonable.

③ When target approaches own ship, modified ta makes *CR* value can increase until just before it will pass *CPA*. This is fit intuitionistic observation.

④ Using the method in the flow chart, the value of coefficient *a* can be gotten. Coefficient *a* is 1.1491 is gotten and used in modification approach. And after different situations' data verifying, the modification approach works well and gets good effects. So coefficient *a* is 1.1491 can be used in the common situations.

⑤ The result of modification can be plus or minus, the sign of result just means the target are approaching to own ship or not. The minimum of the CR is 0, not $-\infty$.

⑥ When the *dcpa* is less then 1.5 miles and the target's relative speed is around 1.0 mile/min approaching to own ship, the timing to take avoidance action is when the approach time is less than 7 minutes; and when the target's relative speed is around and below 0.5 mile/min approaching to own ship, the timing to take avoidance action is when the approach time is less than 3.25 miles.

The modification approach can solve some problems and get good results. However, such results should be checked by many vessel experiments on the sea. And in the further study, own ship state function $\phi(\theta, \alpha)$should also be considered and researched. Also, the threshold should be decided carefully because it decides the safety range of own ship and makes the avoidance actions can be taken easily. All of these will be dealt with in the future study.

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