Review on the Application of Statistical Methods to Maritime Traffic Safety Assessment

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Abstract

For the maritime traffic safety assessment of vessels navigating in harbor or fairway, simulation techniques by using shiphandling simulator system have been traditionally used. When designing the simulation experiments and when analyzing the simulation results, however, there has been a little systematic method. Ship-handling simulations can be regarded as a kind of statistical experiment by using ship-handling simulator system, which means that shiphandling simulation conditions should be designed statistically and that the simulation results should be statistically analyzed as well. For the safe and economic design of harbor and fairway, reasonable decisions based upon the scientific analysis of shiphandling simulation results are indispensable.

In this paper, various statistical methods, such as Bayes theorem, statistical hypothesis testing, and probability distributions, are reviewed with a view to application to maritime traffic safety assessment. It is expected that more reasonable decisions on harbor and fairway design can be made from shiphandlers' view point by using statistical methods.

Keywords: maritime traffic safety, statistical analysis method, ship-handling simulation, ship-handling simulator

1. Introduction

Statistics is a branch of applied mathematics concerned with the collection and interpretation of quantitative data and the use of probability theory to estimate population parameters. It helps us collect, organize, and summarize data obtained from various surveys or statistical experiments. It also enables us to make scientific reasoning for the characteristics of population based on the information obtained from restricted number of sample data.

Statistical experiment is an experiment all of the possible outcomes of which are already known, however, it is unknown which outcome it would be as a result of specific experiment. Examples are dice play, coin throw, and card games.

Would it be possible to regard a shiphandling simulation as a kind of statistical experiment? That is, would it be possible to predict all of the possible outcomes of a shiphandling simulation? It is possible to say that all outcomes of shiphandling simulations, such as ship trajectories, rudder angles, engine settings, drift angles, subjective difficulties felt by pilots, and etc., may fall in some predictable ranges of their values. In a broad sense, this may include groundings and collisions as well. Therefore it would be possible to regard a shiphandling simulation by using shiphandling simulator system as a statistical experiment.

If a grounding or collision accident occurs during shiphandling simulation at some specific environmental condition, it has been regarded that there might be possibilities of same accident occurrence in real situation. How much right is this reasoning? And if so, how big is the quantitative probability of this occurrence?

To bring complicated real world into shiphandling simulator system, various modeling processes are inevitable. There usually exist a number of assumptions and simplifications during modeling of complicated real world. The characteristics of a ship in a simulator may be different from those of real ship. And the mathematical models describing various hydrodynamic forces and influences of various environmental forces, such as wind, wave, tidal current, shallow water depth, and etc., acting on a ship may be different from those of real world. This means that simulator world may be different from real world, and the shiphandling simulation situations may be different from real ones. As a consequence of these differences, the simulation results may contain various inherent errors, which may deteriorate the reliability of the reasoning based on the shiphandling simulation results.

Furthermore, due to the time and economical restrictions, shiphandling simulations can usually be carried out only for very limited cases among huge combinations of environmental conditions, with which various vessels differing in sizes and types are expected to be encountered during its navigation in a harbor and fairway. To draw reasonable and scientific conclusions from these not only inaccurate but also limited numbers of simulation experiments, systematic and statistical analysis of the simulation results as well as simulation design seems to be inevitable.

In this paper, various statistical methods are briefly reviewed with a view to application to maritime traffic safety assessment. It is expected that more reasonable and scientific conclusions can be drawn for the harbor and fairway designs from shiphandlers' view point by using statistical methods.

2. Review on the Statistical Methods and Their Applications

2.1 Bayes Theorem and Validation of Simulator Experiment

Bayes theorem is a kind of probability theory, which relates the conditional and marginal probability distributions of random variables. It tells us how to update or revise probability by taking new evidences into account.

If sample space $\boldsymbol{S}$ is divided into two events, $\boldsymbol{A}$ and $\boldsymbol{A^C}$, which are mutually disjoint, and if $\boldsymbol{B}$ is an event that can arise in $\boldsymbol{S}$, then $\boldsymbol{B}$ also can be divided into two mutually disjoint events of $\boldsymbol{AB}$ and $\boldsymbol{A^C B}$, that is $\boldsymbol{B=AB \cup A^C B}$, as shown in Figure 1. In this case, the probability of occurrence of event ***B***, ***P(B)***, can be written as follows;

$$
\begin{aligned}
P(B) &= P(AB \cup A^C B) \\
&= P(AB) + P(A^C B) \\
&= P(A) \bullet P(B \mid A) + P(A^C) \bullet P(B \mid A^C)
\end{aligned} \tag{1}
$$

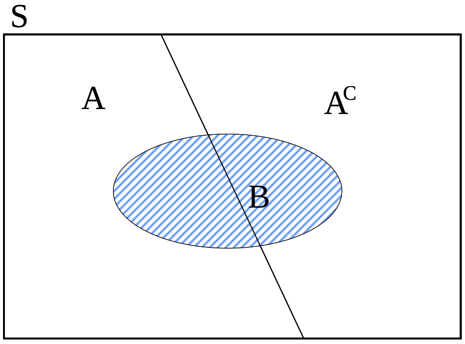

Figure 1 Ben diagram for Bayes theorem

And the conditional probability of event ***A*** with the previous occurrence of event ***B***, which is denoted as ***P(A|B)***, can be expressed as follows[1];

$$
\begin{aligned}
P(A \mid B) &= \frac{P(AB)}{P(B)} \\
&= \frac{P(A) \bullet P(B \mid A)}{P(A) \bullet P(B \mid A) + P(A^C) \bullet P(B \mid A^C)}
\end{aligned} \tag{2}
$$

This is simple form of Bayes theorem and by applying this, it is possible to obtain more detailed information about population or probability based on the partial information obtained from samples.

As an application of Bayes theorem, let's consider the probability of real occurrence of maritime accident, for example, grounding of a ship during navigation in a fairway, which has been predicted by shiphandling simulation experiment.

Let ***A*** be an event of real grounding accident of a ship at specific environmental condition, and ***P(A)*** be its probability. If the fairway has been operated for long time from the past, then ***P(A)*** may be estimated from accident statistics in that area. If it is a new fairway to be designed and constructed in the future, then ***P(A)*** may be designated as a safety criteria of the fairway.

And let ***B*** be an event that a grounding accident occurs during shiphandling simulation for the same situation of real grounding accident of event ***A***. Then ***P(B|A)*** represents the probability that grounding accident occurs during shiphandling simulation for the condition of event ***A***(real grounding accident occurs), and ***P(B|A^C)*** represents the probability that grounding accident occurs during shiphandling simulation for the condition of event ***A^C*** (real grounding accident does not occur).

Then the probability, ***P(A|B)***, of real occurrence of grounding accident of a ship in a fairway in case that it is predicted by shiphandling simulation, can be estimated by using Bayes theorem. For these, the quantities of ***P(A)***, ***P(B|A)***, and ***P(B|A^C)*** should be estimated in advance prior to the estimation of ***P(A|B)***.

Figures 2 and 3 show the variations of ***P(A|B)*** according to the variations of ***P(B|A)*** and ***P(B|A^C)*** for a fixed value of ***P(A)***=0.01 and 0.001, respectively. Each line type in Figures 2 and 3 corresponds to each ***P(B|A)*** value designated in each figure. ***P(A|B)*** value of 1.0 means that whenever there is an accident during simulation, there certainly follows a same accident in real situation. The higher the value of ***P(A|B)*** is, the more reliable the simulation results are. For relatively smaller values of ***P(A)***, it seems that ***P(A|B)*** value is highly sensitive to ***P(B|A)*** and ***P(B|A^C)***. This means that the accuracy of a model is very important to reliably predict the occurrence of an event ***A*** whose probability ***P(A)*** is extremely low.

Let ***P(A)*** be a probability of real occurrence of event ***A***, and ***P(B)*** be a probability of occurrence of event ***A*** predicted by some model. Then ***P(A|B)*** represent a probability of occurrence of event ***A*** when it is predicted by the model. For this value to be bigger than α , following condition should be satisfied.

$$
\beta \equiv \frac{P(B \mid A^C)}{P(B \mid A)} \leq \frac{(1-\alpha)\,p}{\alpha(1-p)} \tag{3}
$$

That is, ***P(B|A)*** should be as big as possible and ***P(B|A^C)*** should be as small as possible. If the prediction model is perfect, then ***P(B|A^C)*** = 0.0 and ***P(B|A)*** = 1.0 will follow.

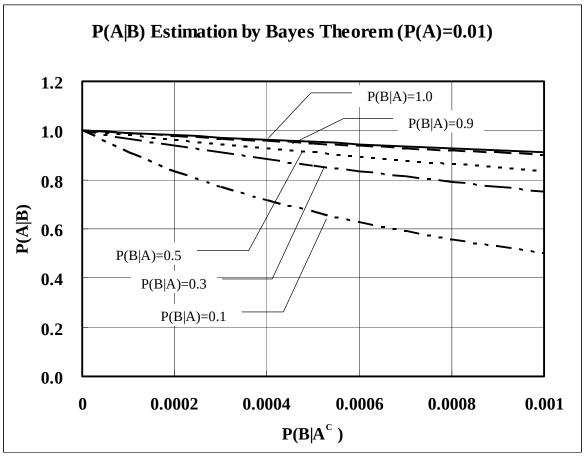

Figure 2 Variation of ***P(A|B)*** with ***P(B|A)*** and ***P(B|A^C)*** for ***P(A)***=0.01

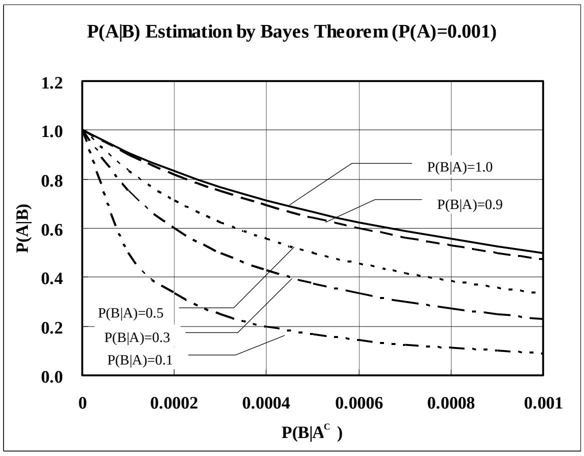

Figure 3 Variation of ***P(A|B)*** with ***P(B|A)*** and ***P(B|A^C)*** for ***P(A)***=0.001

For example, in order to predict the occurrence of an event whose probability p is 0.001, by its prediction model with more than 90% of accuracy (α =0.9), β should be smaller than 0.00011. This value becomes smaller as p decreases or as α increases.

This implies that the value of ***P(B|A)***, a probability that simulation accident occurs for the condition of real accident, should have big value. And, on the contrary, ***P(B|A^C)***, the probability that simulation accident occurs for the condition of no real accident, should have small value.

If $P(B|A^C)$ is 0.0, $P(A|B)$ becomes 1.0 regardless of $P(B|A)$. This means if it is confirmed that no simulation accident occurs for the condition of no real accident ($P(B|A^C)$=0), then the probability of real accident occurrence when it is predicted by simulation becomes 100% ($P(A|B)$=1.0). This is because $P(B|A^C)$=0 and $P(A|B)$=1.0 are logically in contraposition. In this case, however, there still remains probability of real accident even though it is not predicted during simulation.

Out major concern would be whether the shiphandling simulator systems which are generally used in maritime traffic safety assessment have any qualitative and quantitative validation results of this kind. The probability of maritime accident along a fairway or in a harbor is usually very small value, and as described above, the prediction model for this accident should be very accurate in order that its prediction results are reliable.

For these kind of validations, shiphandling simulations need to be carried out for the conditions of maritime accident, from which the probability of $P(B|A)$ can be estimated. Shiphandling simulations for the conditions of real safe navigation cases need to be carried out as well to estimate the probability of $P(B|A^C)$. By estimating the magnitude of β, as defined in equation (3), it would be possible to discriminate whether the simulation model has enough accuracy or not for the prediction of specific accidents in real world. This is not, however, a simple job and may require a huge number of simulations. In order to develop more effective validation tool for the shiphandling simulator system, more study on this subject would be necessary.

2.2 Binomial Distribution

If there exist only two possible outcomes as a result of a trial, which are usually referred to as a success (true, probability is p) or a failure (false, probability is $q = 1 - p$), it is called a Bernoulli Trial. If successes occur x times during n repeated Bernoulli trials, this probability follows Binomial distribution and is denoted as $B(n,p)$, whose probability, $f(x)$, is expressed as;

$$f(x) = \begin{cases} {}_nC_x p^x q^{n-x}, & x = 1, 2, \cdots, n \\ 0 \quad , & x = \text{else} \end{cases} \tag{4}$$

If random variable X follows binomial distribution $B(n,p)$, then its mean value and variation are np and npq, respectively.

Let p be a probability of a specific ship to enter a specific harbor safely at specific environmental conditions. This p value can be estimated by repeated simulations under same conditions or it can be designated as safety criteria of a harbor.

If this p value is known, it is possible to estimate the probability of annual occurrence of maritime accident based on the estimated maritime traffic volumes of the vessel and on the probability of occurrence of the specific environmental conditions in the area.

As an application example of Binomial distribution, let's assume that the maximum allowable accident probability of a ship entering a harbor at some environmental conditions is given as p =0.0001. It is also assumed that the annual number of calls of the ship is 300 times a year and the occurrence probability of the environmental condition is about 15%. Then, the probability that the ship enters the harbor without any accident for one year is estimated as 99.55% by using Binomial distribution.

On the other hand, Figure 4 shows probability of at least one accident occurrence of the ship until each year, which can be estimated by using negative binomial distribution. According to this figure, the probability that at least one accident occur within next 25 years is expected to be about 10.6%.

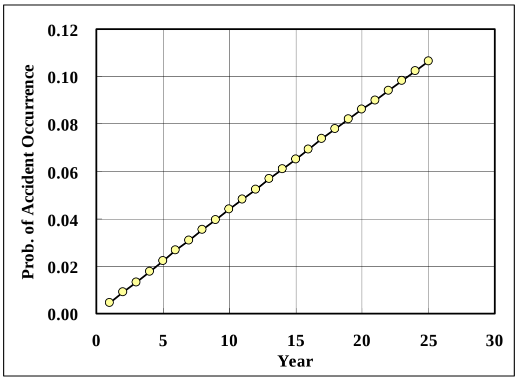

Figure 4 Probability of at least One Accident Occurrence until Each Year

2.3 Poisson and Exponential Distribution

The Poisson distribution is usually used to model the number of occurrence of events within a given time interval. The formula for the Poisson probability mass function is

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \text{ for } x = 0,1,2. \dots \tag{5}$$

where λ is a shape parameter which represents the average number of events in the given time interval. It expresses the probability of the number of events occurring in a fixed period of time if these events occur with a known average rate, and are independent of the time since the last event.

For example, if the average number of ships passing some location of a fairway is λ, then the number of ships, x, which pass the location during a unit time would follow Poisson distribution of parameter λ.

On the other hand, if random variable X has following probability density function, it is said that X follows exponential distribution of parameter λ

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } 0 < x < \infty \\ 0 & \text{else} \end{cases} \tag{6}$$

While a Poisson distribution represents the number of occurrence of an event during a unit time, exponential distribution represents a time interval distribution between two consecutive events.

If random variable X follows exponential distribution, its mean value and variance are $1/\lambda$ and $1/\lambda^2$, respectively. This distribution can be used, for example, to estimate the time interval distribution between vessel arrivals at specific location of a harbor, which have been used in maritime traffic flow simulations and fairway capacity assessment to generate a ship at specific location.

2.4 Confidence Interval Estimation

Interval estimation is a kind of statistical inference technique, which estimates the confidence interval where the true values of population parameters are expected to be included with some confidence level.

The trajectory distribution of vessels across the width of fairway is known to be approximated by normal distribution [2,3]. It is assumed that the distribution of trajectories of vessels follows normal distribution with fairway center as its mean position in case of one way traffic fairway. In case of two way traffic fairway, the mean position may be some another position as schematically shown in Figure 5.

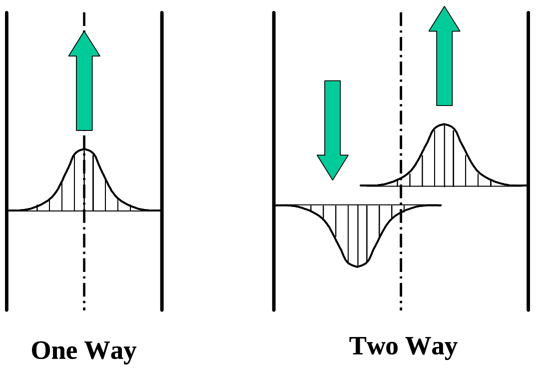

Figure 5 Trajectory Distribution of Vessels across Fairway.

If shiphandling simulations along a fairway are carried out for n times, then the sample mean position $\overline{X}$ and sample variance S^2 can be obtained. By using student t-distribution, we can obtain intervals within which the true values of mean trajectory position, μ , and its variation, σ^2 , are expected to be included with some confidence level. The confidence interval where the population mean value, μ , is expected to be included with confidence level of 100(1- α)% can be estimated by using student t-distribution as follows,

$$\left(\overline{X} - t(n-1, \alpha/2)\frac{S}{\sqrt{n}} \;,\; \overline{X} + t(n-1, \alpha/2)\frac{S}{\sqrt{n}} \right) \quad (7)$$

Student t-distribution is used for confidence interval estimation and hypothesis testing for population parameters especially when the sample size extracted from population, which has normal distribution, is relatively small.

On the other hand, the confidence interval where the population variance, σ^2 , is expected to be included with confidence level of 100(1- α)% can be estimated by using chi-square distribution as follows,

$$\left(\frac{(n-1)S^2}{\chi^2(n-1, \alpha/2)} \;,\; \frac{(n-1)S^2}{\chi^2(n-1, 1-\alpha/2)} \right) \quad (8)$$

As a sample calculation, let's assume that we obtain $\overline{X}$ =25.0 and S=75.0 from 50 shiphandling simulations (n =50) for navigation along one way traffic fairway. Then the confidence intervals for population parameters, μ and σ , with 90% confidence level (α =0.1) can be estimated as $7.2 \le \mu \le 42.8$ and $64.5 \le \sigma \le 90.1$, respectively. As the number of shiphandling simulation increases, the width of confidence interval decreases, which means that the accuracy of the prediction for the population parameters is improved. This kind of specific information on population parameters can be used, for instance, when carrying out more detailed risk assessment by using Monte Carlo experiment [4].

2.5 Determination of Sample Size

If sample size increases during sampling from population, the width of confidence interval can be reduced, which means that the accuracy predicted from the samples may increase. Since the increase of sample size, however, usually entails increases of cost and time, it should be determined taking the maximum allowable error into account.

In order to determine the optimum sample size, the confidence level should be determined first. And then the maximum allowable error between the estimator and parameter should be determined. In case that a population follows a normal distribution of variance σ^2 , the sample size n which is necessary to estimate the population mean μ with confidence level 100(1- α)% and within maximum allowance error d is determined as a minimum integer satisfying following equation.

$$n \ge \left(\frac{z_{\alpha/2} \bullet \sigma}{d} \right)^2 \quad (9)$$

where $z_{\alpha/2}$ is upper percentile of normal distribution.

The equation (9), however, requires population variance σ^2 , which is usually unknown. In this case, the value σ^2 obtained from past experience may be used. Or, the minimum integer satisfying following equation can be iteratively obtained by using t-distribution and sample variance S of relatively small size of samples.

$$n \ge \left(\frac{t(n-1, \alpha/2) \bullet S}{d} \right)^2 \quad (10)$$

By using this equation, the minimum required number of simulations for maritime traffic safety assessment to maintain desired accuracy can be obtained.

2.6 Hypothesis Testing

In case that a hypothesis is suggested for some social or natural phenomena, complete survey of population may be necessary to discriminate whether the hypothesis is true or false. Since this complete survey is practically unrealistic, usually samples are extracted from the population and tested for the hypothesis, which provides statistical clues to the rightness of the hypothesis. This kind of statistical technique is called hypothesis testing.

As an example, if navigational environment of existing fairway is changed due to some reason (fairway straightening, fairway widening, construction of structures around fairway, and etc.), its influence on the navigational safety of a ship can be assessed by using hypothesis testing technique.

Let's assume that minimum approach distance of vessels to some point of a fairway follows normal distribution $N(\mu_0, {\sigma_0}^2)$. The parameters of this distribution can be estimated from survey of VTS records of vessel navigations in this area or from shiphandling simulation results for the existing fairway. To assess the influences of changes of navigational environments on vessels, shiphandling simulations can be carried out for the changed environmental conditions for n sampled cases, from which we can obtain $N(\overline{X}, S^2)$. Now, it is possible to establish a hypothesis that the navigational safety of a ship in a changed navigational environment is unchanged or changed (enhanced or deteriorated). The former is null hypothesis, H_0, and the latter is alternative hypothesis, H_1, and which one to accept or to reject can be determined by t-test. In this case, the test statistics is

$$T = \frac{\overline{X} - \mu_0}{S / \sqrt{n}} \qquad (11)$$

and the rejection region for null hypothesis, H_0, with confidence level α are summarized as shown in Table 1.

Table 1 Rejection Region for Null Hypothesis H_0 (Confidence Level : α)

Testing	H_0	H_1	Rejection Region
Right-sided testing	$\mu = \mu_0$	$\mu > \mu_0$	$T \geq t(n-1, \alpha)$
Left-sided testing	$\mu = \mu_0$	$\mu < \mu_0$	$T \leq -t(n-1, \alpha)$
Two-sided testing	$\mu = \mu_0$	$\mu \neq \mu_0$	$\lvert T \rvert \geq t(n-1, \alpha/2)$

2.7 Multiple Populations

Sometimes it is necessary to determine which one is better among various layouts of harbors or fairways from the viewpoint of navigational safety of a ship. In order to assess the influence of these differences on ship navigational safety, paired comparison technique for two populations may be used. After carrying out shiphandling simulations for each layout, paired comparison may be carried out to assess whether there exist any significant difference between the parameters of the two populations.

On the other hand, analysis of variance (ANOVA) is a kind of statistical technique which compares the mean values of population when there are more than three populations. Depending on the number of factors which influence the measured variables, there may be one-way or two-way ANOVA.

This technique can be used to discriminate whether there exist any significant differences between the layouts from the navigational view point, when there are multiple layout plans of fairways or harbors.

2.8 Correlation Analysis and Testing

When examining social or natural phenomena, it is sometimes necessary to investigate the mutual relations between involved variables. When two or more variables vary with mutual relation, correlation analysis can be used to analyze their linear relationship. If the measured values are population itself, the population correlation coefficient is defined as follows,

$$\rho = \frac{Cov(X,Y)}{\sqrt{V(X) \bullet V(Y)}} = \frac{E(XY) - E(X)E(Y)}{\sqrt{V(X) \bullet V(Y)}} \qquad (12)$$

where, E and V are expected value and variance, respectively.

If measured values are for samples rather than population, it is called sample correlation coefficient and is defined as follows,

$$r = \frac{S_{xy}}{S_x S_y}, \quad S_{xy} = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{n-1} \qquad (13)$$

$$S_x = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1}}, \quad S_y = \sqrt{\frac{\sum_{i=1}^{n} (Y_i - \overline{Y})^2}{n-1}}$$

where S_{xy} is sample covariance and S_x, S_y are sample standard variation of X and Y, respectively.

In order to discriminate whether two variables ***X, Y*** have any linear relationship, testing can be carried out for population correlation coefficient ρ by using sample correlation coefficient r. Our major concern for the population correlation coefficient ρ is usually whether their exists any linear relationship or not between the variables, that is, whether $\rho = 0$ or not. When two variables ***X, Y*** follow normal distributions and when $\rho = 0$, the statistic value, T^*, as defined in equation (14) is known to follow t-distribution with (n-2) degrees of freedom. By using this T^*, hypothesis testing for the population correlation coefficient ρ =0 can be carried out with confidence level α as shown in Table 2.

$$T^* = r\sqrt{\frac{n-2}{1-r^2}} \qquad (14)$$

Table 2 Rejection Region for Null Hypothesis H_0 (Confidence Level : α)

Testing	H_0	H_1	Rejection Region
Right-sided testing	$\rho=0$	$\rho>0$	$T^* \geq t(n-2, \alpha)$
Left-sided testing	$\rho=0$	$\rho<0$	$T^* \leq -t(n-2, \alpha)$
Two-sided testing	$\rho=0$	$\rho\neq0$	$\lvert T^* \rvert \geq t(n-2, \alpha/2)$

If it is necessary to test whether ρ has some specific value ρ_0, Fisher's Z statistic variable can be used.

3. Concluding Remarks

Once a harbor and its approach channels are constructed, a number of vessels of various sizes and types may navigate in the area. There also exist a number of factors influencing the safe navigation of vessels, such as wind, tidal current, wave, fog, maritime traffic volume, and etc. This means that there may exist a huge number of combinations of vessels and their navigational environmental conditions, which makes it very difficult to judge about the navigational safety level of the area.

Maritime traffic safety assessment by shiphandling simulation is a technique assessing the navigational safety of a ship in harbor or fairway based on restricted number of sampled simulations among huge number of environmental conditions subsisting in the population. The complete survey for the population is inherently unrealistic in this case and we have to draw conclusions anyway from very limited number of sampled simulation results. This implies that systematic and proper application of statistical approach is inevitable to draw reliable and reasonable conclusions from shiphandling simulation results.

In this paper, various basic concepts and techniques used in statistics are briefly reviewed as a preliminary study of its application to maritime traffic safety assessment. Besides the contents of this paper, there exist a number of statistical methods which can be applied to maritime traffic safety assessment topics. And further research is necessary to incorporate these various statistical techniques into maritime traffic safety assessment. It is strongly expected that these statistical techniques would be applied to real maritime traffic safety assessment project in the near future.

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