Daylight background radiation modeling for the system of ocean-atmosphere with multi-layer clouds

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Abstract – A one-dimensional planar model is considered of the atmosphere with multi-layer clouds illuminated by a mono-directional parallel flux of solar radiation. A new approach is proposed to radiation transfer modeling and daylight background formation for the atmosphere with such clouds that is represented as a heterogeneous multi-layer system each layer of which is described by different optical characteristics. The influence functions of each layer are determined by solutions of the radiation transfer boundary problem with an external mono-directional wide flux while the contribution of multiple scattering and absorption in the layer is taking into account.

Keywords: radiative transfer theory, numerical models, multi-layer clouds, influence functions method, optical transfer operator

Introduction

Boundary layer problem of radiation transfer in optically thick layers is considered while describing ocean, cloudiness, aerosol particle outbreaks, dust traces and other specific effects resulted from large fires (forest, peat-lands, in steppe regions, anthropogenic). Space and angular distributions of radiation inside the relevant layer of these media as well as reflected and passed through the layer radiation are formed as a result of multiple scattering and absorption.

A planar model is considered of the atmosphere with multi-layer clouds illuminated by a mono-directional parallel flux of solar radiation. A new approach is proposed to radiation transfer modeling and daylight background formation for the atmosphere with such clouds that is represented as a heterogeneous multi-layer system each layer of which is described by different optical characteristics. The influence functions of each layer are determined by solutions of the radiation transfer boundary problem with an external mono-directional wide flux while the contribution of multiple scattering and absorption in the layer is taking into account. Such problem is analogous to the widely known problem for the plane layer with an external solar flux the solution of which is not difficult and can be found by different well-known methods.

The boundary problem for each layer can be solved depending on optical thickness, scattering and absorption characteristics by one of the following techniques: i) as a solution of transfer equation with an azimuth dependence; ii) as a solution of the problem with azimuth symmetry; iii) as a solution in two-flux approach; iv) as an approximate solution in asymptotic approach. Operators of radiation transmittance and reflectance on the boundaries between the layers are formulated based on the collision integrals and the separate layers are united in a system by these operators. To calculate the total radiation inside or on the boundaries of the system with radiation exchange between the layers a matrix-vector operation is constructed for the operators the kernels of which are given by the influence functions of the layers.

The representation of the solution to boundary-value problem as a functional is the optical transfer operator of the radiation transfer system which establishes the explicit relationship between the recording radiation and the "scenarios" (the optical image) at the dividing boundaries of media. In turn, by the use of the influence functions, the "scenarios" is described clearly through the characteristics of the reflection and transmission of the dividing boundaries at the given its illumination. The influence functions are invariant about the conditions of the illumination and the properties of the dividing boundaries.

Mathematical statement of the problem

A one-dimensional planar model is considered of the atmosphere with multi-layer clouds illuminated by a mono-directional parallel flux of solar radiation. A new approach is proposed to radiation transfer modeling and daylight background formation for the atmosphere with such clouds that is represented as a heterogeneous multi-layer system each layer of which is described by different optical characteristics. The influence functions of each layer are determined by solutions of the radiation transfer boundary problem with an external mono-directional wide flux while the contribution of multiple scattering and absorption in the layer is taking into account. Such problem is analogous to the widely known problem for the plane layer with an external solar flux the solution of which is not difficult and can be found by different well-known methods [1, 2, 3].

Boundary-value problem for multi-layer heterogeneous system is considered

$$\hat{K}\Phi = F^{in}, \qquad \Phi\big|_{t\downarrow} = F_t^{\downarrow}, \qquad \Phi\big|_{b\uparrow} = \hat{R}_b^{\uparrow}\Phi + F_b^{\uparrow},$$

at internal boundaries h_m for $m = 2 \div M$

$$\Phi\big|_{d\uparrow,m} = \varepsilon\,(\hat{R}_m^{\uparrow}\Phi + \hat{T}_m^{\uparrow}\Phi) + F_{m-1}^{\uparrow}, \qquad \Phi\big|_{d\downarrow,m} = \varepsilon\,(\hat{R}_m^{\downarrow}\Phi + \hat{T}_m^{\downarrow}\Phi) + F_m^{\downarrow};$$

integro-differential operator $\hat{K} \equiv \hat{D} - \hat{S}$; $m = 1 \div M$ – number of layer.

The linear operators: transfer operator and the collision integral

$$\hat{D} \equiv \mu\frac{\partial}{\partial z} + \sigma_{tot}(z); \qquad \hat{S}\Phi \equiv \sigma_{sc}(z)\int_{\Omega}\gamma(z,s,s')\Phi(z,s')ds' \ , \quad ds' = d\mu' d\varphi' .$$

Radiation propagation through the internal boundaries between the layers is described by equal-measure restricted operators of reflectivity $\hat{R}_m^{\downarrow}$, $\hat{R}_m^{\uparrow}$ and transmittance $\hat{T}_m^{\downarrow}$, $\hat{T}_m^{\uparrow}$.

We seek solution in the form of a regular perturbation series

$$\Phi = \sum_{n=0}^{\infty}\varepsilon^n\Phi^{(n)} .$$

Let`s introduce the algebraic vectors with the dimension $2M$:

n - approximation of the solution $\mathbf{\Phi}^{(n)} = \{\Phi_1^{\downarrow(n)}, \Phi_1^{\uparrow(n)}, \Phi_2^{\downarrow(n)}, \Phi_2^{\uparrow(n)}, \dots\Phi_m^{\downarrow(n)}, \Phi_m^{\uparrow(n)}, \dots, \Phi_M^{\downarrow(n)}, \Phi_M^{\uparrow(n)}\}$;

the complete solution $\mathbf{\Phi} = \{\Phi_1^{\downarrow}, \Phi_1^{\uparrow}, \Phi_2^{\downarrow}, \Phi_2^{\uparrow}, \dots\Phi_m^{\downarrow}, \Phi_m^{\uparrow}, \dots, \Phi_M^{\downarrow}, \Phi_M^{\uparrow}\}$;

n - approximation of the sources $\mathbf{F}^{(n)} = \{F_1^{\downarrow(n)}, F_1^{\uparrow(n)}, F_2^{\downarrow(n)}, F_2^{\uparrow(n)}, \dots F_m^{\downarrow(n)}, F_m^{\uparrow(n)}, \dots F_M^{\downarrow(n)}, F_M^{\uparrow(n)}\}$;

the influence functions of the layers $\mathbf{\Theta} = \{\Theta_1^{\downarrow}, \Theta_1^{\uparrow}, \Theta_2^{\downarrow}, \Theta_2^{\uparrow}, \dots\Theta_m^{\downarrow}, \Theta_m^{\uparrow}, \dots, \Theta_M^{\downarrow}, \Theta_M^{\uparrow}\}$;

the initial approximation of the sources $\mathbf{E} = \{E_1^{\downarrow}, E_1^{\uparrow}, E_2^{\downarrow}, E_2^{\uparrow}, \dots E_m^{\downarrow}, E_m^{\uparrow}, \dots E_M^{\downarrow}, E_M^{\uparrow}\}$;

the "scenario" on boundaries $\mathbf{Z} = \{Z_1^{\downarrow}, Z_1^{\uparrow}, Z_2^{\downarrow}, Z_2^{\uparrow}, \dots Z_m^{\downarrow}, Z_m^{\uparrow}, \dots Z_M^{\downarrow}, Z_M^{\uparrow}\}$.

We produce decomposition of the original problem at $2M$ problems with their own boundary conditions. The initial approximation – radiation from sources without radiation interchange between layers for $m = 1 \div M$:

$$\hat{K}\Phi_m^{\downarrow(0)} = F_m^{\downarrow in}, \ \ \Phi_m^{\downarrow(0)}\Big|_{d\downarrow,m} = F_m^{\downarrow}, \ \ \Phi_m^{\downarrow(0)}\Big|_{d\uparrow,m+1} = 0; \qquad \hat{K}\Phi_m^{\uparrow(0)} = F_m^{\uparrow in}, \ \ \Phi_m^{\uparrow(0)}\Big|_{d\downarrow,m} = 0, \ \ \Phi_m^{\uparrow(0)}\Big|_{d\uparrow,m+1} = F_m^{\uparrow} .$$

The approximation $n \geq 1$ – system of $2M$ equations for layers $m = 1 \div M$:

$$\hat{K}\Phi_m^{\downarrow(n)} = 0,\ \Phi_m^{\downarrow(n)}\Big|_{d\downarrow,m} = F_m^{\downarrow(n-1)},\ \Phi_m^{\downarrow(n)}\Big|_{d\uparrow,m+1} = 0;\quad \hat{K}\Phi_m^{\uparrow(n)} = 0,\ \Phi_m^{\uparrow(n)}\Big|_{d\downarrow,m} = 0,\ \Phi_m^{\uparrow(n)}\Big|_{d\uparrow,m+1} = F_m^{\uparrow(n-1)}$$

with sources at the boundaries h_m for $m = 1 \div M+1$:

$$F_m^{\downarrow(n)} = \hat{T}_m^{\downarrow}\Phi_{m-1}^{\downarrow(n)} + \hat{T}_m^{\downarrow}\Phi_{m-1}^{\uparrow(n)} + \hat{R}_m^{\downarrow}\Phi_m^{\downarrow(n)} + \hat{R}_m^{\downarrow}\Phi_m^{\uparrow(n)};\quad F_m^{\uparrow(n)} = \hat{R}_{m+1}^{\uparrow}\Phi_m^{\downarrow(n)} + \hat{R}_{m+1}^{\uparrow}\Phi_m^{\uparrow(n)} + \hat{T}_{m+1}^{\uparrow}\Phi_{m+1}^{\downarrow(n)} + \hat{T}_{m+1}^{\uparrow}\Phi_{m+1}^{\uparrow(n)}.$$

Solution is assumed in the form of linear functionals for each layer with $m = 1 \div M$:

$$\Phi_m^{\downarrow(n)} = \left(\Theta_m^{\downarrow}, F_m^{\downarrow(n-1)}\right);\qquad \Phi_m^{\uparrow(n)} = \left(\Theta_m^{\uparrow}, F_m^{\uparrow(n-1)}\right).$$

Functionals kernels – influence functions of layers with $m = 1 \div M$ are determined from boundary-value problems

$$\hat{K}\Theta_m^{\downarrow} = 0,\ \Theta_m^{\downarrow}\Big|_{d\downarrow,m} = f_{\delta,m}^{\downarrow},\ \Theta_m^{\downarrow}\Big|_{d\uparrow,m+1} = 0;\ f_{\delta,m}^{\downarrow} = \delta\left(s - s_m^{\downarrow}\right);\ \hat{K}\Theta_m^{\uparrow} = 0,\ \Theta_m^{\uparrow}\Big|_{d\downarrow,m} = 0,\ \Theta_m^{\uparrow}\Big|_{d\uparrow,m+1} = f_{\delta,m}^{\uparrow};\ f_{\delta,m}^{\uparrow} = \delta\left(s - s_m^{\uparrow}\right).$$

The components of vectorial linear functional are computed for $z \in [h_m, h_{m+1}]$ by formulas:

$$\left(\Theta_m^{\downarrow}, f_m^{\downarrow}\right)(z,s) = \frac{1}{2\pi}\int_{\Omega^+}\Theta_m^{\downarrow}\left(s_m^{\downarrow}; z, s\right) f_m^{\downarrow}\left(h_m, s_m^{\downarrow}\right) ds_m^{\downarrow};\quad \left(\Theta_m^{\uparrow}, f_m^{\uparrow}\right)(z,s) = \frac{1}{2\pi}\int_{\Omega^-}\Theta_m^{\uparrow}\left(s_m^{\uparrow}; z, s\right) f_m^{\uparrow}\left(h_{m+1}, s_m^{\uparrow}\right) ds_m^{\uparrow},$$

where $f_m^{\downarrow}\left(h_m, s_m^{\downarrow}\right)$ is the brightness source distribution at boundary $z = h_m$ when $s_m^{\downarrow} \in \Omega^{\downarrow}$; $f_m^{\uparrow}\left(h_{m+1}, s_m^{\uparrow}\right)$ is the brightness source distribution at boundary $z = h_{m+1}$ when $s_m^{\uparrow} \in \Omega^{\uparrow}$.

Vectorial optical transfer operator

Let $\hat{P}$ – matrix of band type with layers reflection and trasmission characteristics. Matrix-vector operation, which describe one act of radiation interaction with internal and external boundaries and take into account multiply scattering in the layers through influence functions, is determined:

$$\hat{G}\mathbf{F} = \hat{P}(\mathbf{\Theta}, \mathbf{F}) =$$

$$\begin{bmatrix} 0 \\ \hat{R}_2^{\uparrow}(\Theta_1^{\downarrow}, F_1^{\downarrow}) + \hat{R}_2^{\uparrow}(\Theta_1^{\uparrow}, F_1^{\uparrow}) + \hat{T}_2^{\uparrow}(\Theta_2^{\downarrow}, F_2^{\downarrow}) + \hat{T}_2^{\uparrow}(\Theta_2^{\uparrow}, F_2^{\uparrow}) \\ \vdots \\ \hat{T}_m^{\downarrow}(\Theta_{m-1}^{\downarrow}, F_{m-1}^{\downarrow}) + \hat{T}_m^{\downarrow}(\Theta_{m-1}^{\uparrow}, F_{m-1}^{\uparrow}) + \hat{R}_m^{\downarrow}(\Theta_m^{\downarrow}, F_m^{\downarrow}) + \hat{R}_m^{\downarrow}(\Theta_m^{\uparrow}, F_m^{\uparrow}) \\ \hat{R}_{m+1}^{\uparrow}(\Theta_m^{\downarrow}, F_m^{\downarrow}) + \hat{R}_{m+1}^{\uparrow}(\Theta_m^{\uparrow}, F_m^{\uparrow}) + \hat{T}_{m+1}^{\uparrow}(\Theta_{m+1}^{\downarrow}, F_{m+1}^{\downarrow}) + \hat{T}_{m+1}^{\uparrow}(\Theta_{m+1}^{\uparrow}, F_{m+1}^{\uparrow}) \\ \vdots \\ \hat{T}_M^{\downarrow}(\Theta_{M-1}^{\downarrow}, F_{M-1}^{\downarrow}) + \hat{T}_M^{\downarrow}(\Theta_{M-1}^{\uparrow}, F_{M-1}^{\uparrow}) + \hat{R}_M^{\downarrow}(\Theta_M^{\downarrow}, F_M^{\downarrow}) + \hat{R}_M^{\downarrow}(\Theta_M^{\uparrow}, F_M^{\uparrow}) \\ \hat{R}_b^{\uparrow}(\Theta_M^{\downarrow}, F_M^{\downarrow}) + \hat{R}_b^{\uparrow}(\Theta_M^{\uparrow}, F_M^{\uparrow}) \end{bmatrix}$$

The n-approximation of solution is stated in form of vectorial linear functional: $\mathbf{\Phi}^{(n)} = \left(\mathbf{\Theta}, \mathbf{F}^{(n-1)}\right)$, where sources in $(n-1)$-approximation $\mathbf{F}^{(n-1)} = \hat{P}\mathbf{\Phi}^{(n-1)}$. Two sequential n-approximations are connected by recursion relation $\mathbf{\Phi}^{(n)} = \left(\mathbf{\Theta}, \hat{P}\mathbf{\Phi}^{(n-1)}\right)$. The notion can be found $\mathbf{\Phi}^{(n)} = \left(\mathbf{\Theta}, \hat{G}^{n-1}\mathbf{E}\right)$, where $\mathbf{E}$ – initial approximation.

Asymptotical exact solution is obtained in the form of vectorial linear functional – optical transfer operator:

$$\mathbf{\Phi} = (\mathbf{\Theta}, \mathbf{Z}).$$

"Scenario" – vector **Z** distribution of brightness on internal and external boundaries $m = 1 \div M + 1$:

$$\mathbf{Z} \equiv \hat{Z}\mathbf{E} \equiv \sum_{n=0}^{\infty} \hat{G}^n \mathbf{E} = \sum_{n=0}^{\infty} \hat{P}\mathbf{\Phi}^{(n)}$$

is sum of Neyman series by multiplicity passing of radiation through boundaries with consideration of multiply scattering contribution by the help of influence functions of each layer.

Conclusions

The boundary problem for each layer can be solved depending on optical thickness, scattering and absorption characteristics by one of the following techniques: i) as a solution of transfer equation with an azimuth dependence; ii) as a solution of the problem with azimuth symmetry; iii) as a solution in two-flux approach; iv) as an approximate solution in asymptotic approach.

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This approach enables to simulate radiation fields in a wide range of variations of the optical characteristics of these layers and to analyze mechanisms of radiation characteristics formation inside and outside the layers as well as to estimate any contribution of each region. Reflected radiation and distributions of radiation characteristics inside the layer near to the border illuminated by an external flux are thus calculated with higher accuracy than in common-used techniques.

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