

# LINEAR INTERNAL WAVES THAT FOLLOWS NONLINEAR INTERNAL WAVES

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## ABSTRACT

Nonlinear internal waves (NLIWs) are usually generated by nonlinear process on linear internal waves (IW). Near HengChun Ridge that links Taiwan and Luzon Islands, we found that there are linear internal waves following NLIW and they travel westward at different speed, about 1.5 m/s for IW and 2.9 m/s for NLIW. This phenomenon was observed on site with ship radar and echo sounders, and later verified with thermistor chain. West of Luzon Strait, the separation of NLIW are 5 km or more, while linear internal waves are lines of wave crests at nearly equal distance that is only a few hundred meters apart. The current hypothesis is that most of the energy of internal tide forms a beam that propagates upward from the eastern shoulder of ocean ridge and later interacts with sea surface and thermocline. The interaction with thermocline generates linear internal wave that propagate along the pycnocline at about 1.5 m/s. The interaction with sea surface scatters internal wave energy downward, ensonifies the water column and generates large nonlinear waves that propagate westward at 2.9 m/s as mode 1 in a waveguide.

**KEYWORDS:** Linear internal wave, Non-linear internal wave, Thermocline, SAR, South China Sea

## 1. INTRODUCTION

Near regions with steep change of bottom topography, tidal currents are forced to make large vertical excursion to flow across a shelf break, a sea mount, a sill, or submarine ridge. But, the surface water stays on the surface, therefore there is internal tidal wave (IT) generated by this process. As IT propagates away from the bathymetric feature, nonlinear process will steepen the isopycnal in the front of IT and concentrate the energy of IT at the front of IT, until the dispersive mechanism balances it to stop further concentration of wave energy. A stable non-sinusoidal nonlinear internal wave (NLIW) is thus formed. At the NLIW propagates through the open ocean, it will gradually form secondary, tertiary, .. fronts at the trailing part. Besides these large solitary fronts, one often found a packet of many short and small amplitude waves, like in Figure 1 of Radarsat Scan SAR image of Luzon Strait.

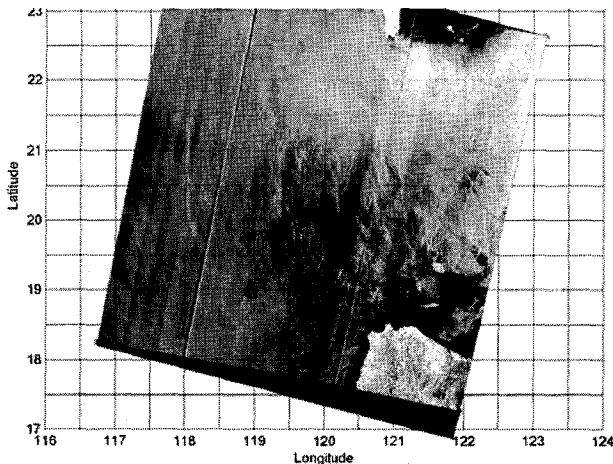


Fig. 1 Radarsat Scan SAR image of Luzon Strait. Large solitary waves (NLIW) are visible at 118E and 119.5E. There are packets of short waves east of NLIW (courtesy of Formosa Remote Sensing Center).

Fig. 2 is a blow-up of the region with a packet of short waves near (119.5E, 20N). One can easily find many fine scale bright lines behind the two thick white lines that are believed to be NLIW. These fine scale bright lines are the subject of study of this article. They can be explained by a simple two layer model of linear internal waves that travels along pycnocline.



Fig. 2 Rotated and enlarged image of the section near (119.5E, 20N).

## 2. FIELD OBSERVATION

If these small amplitude and fine scale waves are indeed linear internal waves (LIWs) that propagates along the pycnocline, then

(1) LIWs will create convergence and divergence near sea surface, that in turn concentrates or diffuse short gravity waves, changes the surface roughness, and shows up bright and dark lines in satellite radar images (like SAR images from ERS, Radarsat and Envisat), satellite visible images (like MODIS images), and ship radar images. Fig. 3 shows a packet of waves that was observed by ship radar on R/V Ocean Researcher 1, at 20:11 UTC of May 13, 2006.

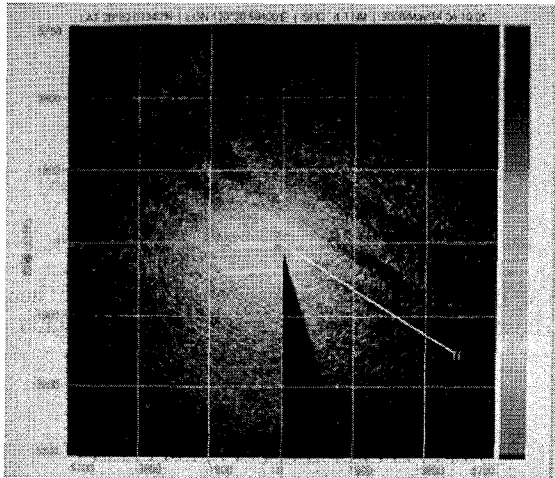


Fig. 3. Lines of wave crest were visible in a photo of radar screen on R/V Ocean Researcher 1, at 20:11 UTC of May 13, 2006. The waves propagate westward (i.e. towards upper right)

The wavelength of these LIWs in Fig. 3 is estimated to be about 0.8 km. The propagation speed of these waves that were propagating westward from Luzon Strait towards Dongsha Atoll, may be estimated from contiguous radar images and GPS location of the ship.

(2) LIWs will move the isotherms up and down as it propagates. Fig. 4 is a plot of a Thermister chain (T-chain) that was towed behind R/V OR1.

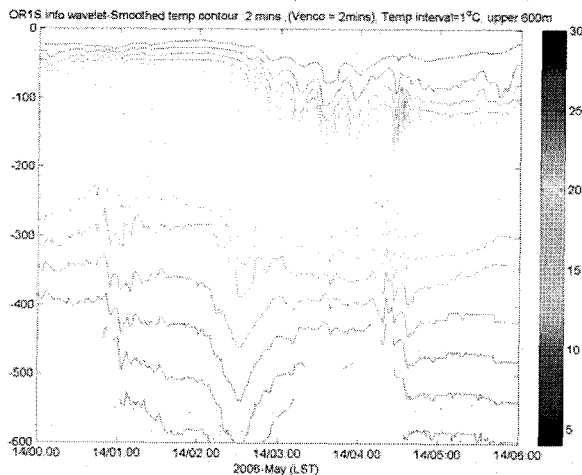


Fig. 4a The movement of isotherms as LIW passing a T-chain that was towed behind R/V OR1.

Fig. 4b is an enlargement of Fig. 4a for the period of 02:10 ~ 04:55 (i.e. 0.09 ~ 0.205 day) of May 14. As the isotherms deepens with the passing of internal tide (with period about 12 hour), there are fine scale LIW that moves isotherm at time scale of 10 minutes.

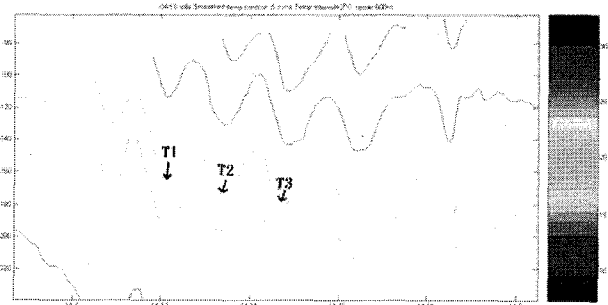


Fig 4b An enlarged figure of Fig. 4a.

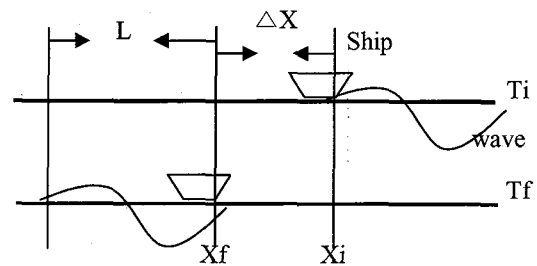


Fig. 5 Estimation of the propagation speed of LIW when the ship is also drifting.

Table.1 Assumptions and Symbols

1. Wave moving along x axis.
2. Ship encountered IWjr at  $T_i, X_i$
3. Ship left IWjr at  $T_f, X_f$
4. Wave length =  $L$
5. Wave phase speed =  $C$
6. ship displacement =  $X_f - X_i$
7. wave propagating distance =  $C*(T_f - T_i)$
8. number of waves:  $n$

From fig.5 and table.1:  $(X_f - X_i) + n \times L = C*(T_f - T_i)$   
And wave phase velocity  $C$ :

$$C = \frac{(X_f - X_i + n \times L)}{(T_f - T_i)} \dots (1)$$

Table.2 Phase Speed is estimated  $3.3 \pm 0.2$  knot.

$\Delta T$ (hour)	0.283	0.583	0.916	1.316
$\Delta X + nL$ (km)	1.7	3.68	5.24	8.54
$C$ (km/ hr)	6	6.31	5.72	6.49
$C$ (knot)	3.24	3.41	3.10	3.51

The estimated phase velocity of LIW was  $3.3 \pm 0.2$  knot, or  $1.65 \pm 0.1$  m/s.

The passing of NLIW at 02:30 of May 14, and LIW at 02:50~04:20 of May 14 were also observed by the ship board Acoustic Doppler Current Profiler (ADCP) that measures the water velocity, as shown in Fig. 6.

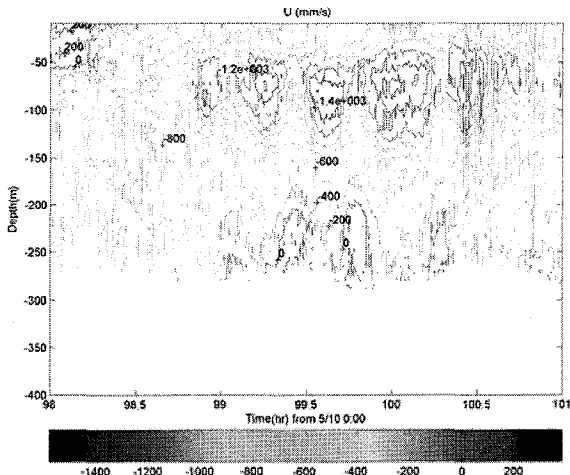


Fig. 6 Eastward velocity component  $U(z, t)$  that was observed by R/V OR1 ADCP

A 2-layer type LIW will have upper layer moving in the direction opposite to the lower layer. In Fig. 6 of  $U(z, t)$ , one can find this 2-layer type LIW that passed the ship at 02:50 ~ 04:20 (about 99.9~100.3 hour from 0:00 of May 10). This matches the T-chain observation in Fig. 4b, and radar image at 20:11 UTC of May 13 (100.18 hour from LST 0:00 May 10).

A simple two-layer model (Fig. 7) requires the knowledge of hydrographic data of both upper and lower layers. From the density profile (Fig. 8), the parameters were chosen as in Table 3. The phase velocity of internal wave in a two layer case may be derived by:

$$C_p = \sqrt{g \times h_{\text{eff}}} \dots (2)$$

where reduced gravity  $g' = g \frac{\rho_2 - \rho_1}{\rho_{\text{mean}}}$

$$\text{effective water depth } h_{\text{eff}} = \frac{(h_1 + h_2)}{h_1 h_2}$$

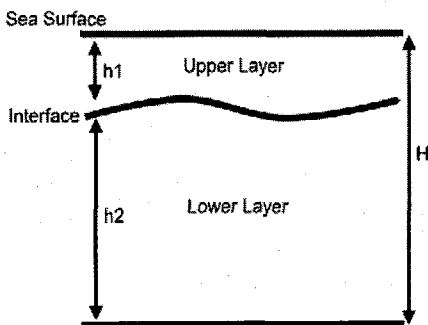


Fig. 7 Schematic plot of a two layer model of LIW.

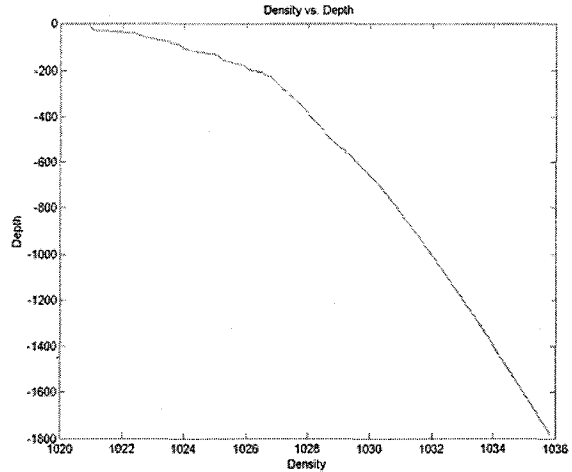


Fig. 8 Density profile of west of Luzon Strait

Table 3. Parameters for a two-layer model of LIW. There were chosen according to the density profile in the region.

$g$	m/s/s	9.8
$H$	m	1800
$h_1$		30
$h_2$		1770
$h' = h_1 * h_2 / (h_1 + h_2)$		29.5
1	kg/m <sup>3</sup>	1020.97
2		1028.45
$g'$	m/s <sup>2</sup>	0.071
$C$	m/s	1.45

### 3. CONCLUSION

In the northeastern South China Sea and near Luzon Strait, there are large amplitude (over 100 m), long wavelength (>3 km) and fast moving (about 2.9 m/s) nonlinear internal waves (NLIW) moving from Luzon Strait westward. In the SAR and visible images, one can often find a packet of fine scale (~ 1 km wavelength) waves trailing these NLIW. There are less visible, probably due to their final scale and smaller amplitude (~ 40 m). From ship radar image on their spatial pattern, from thermister chain observation of isotherm movement, and from ship board observation of their vertical structure, they seem to be a linear internal wave traveling along thermocline at phase velocity of 1.65 m/s.

### Acknowledgment

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