FDFIB Network를 이용한 광대역 DOA 추정

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# Wideband DOA Estimation with FDFIB Network

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### Abstract

In this paper, we presented an extension of the broadband DOA estimation method using frequency-domain frequency-invariant beamforming (FDFIB). The technique uses FDFIB instead of conventional frequency invariant beamforming (FIB) methods. And different narrowband DOA estimation methods, MUSIC, ESPRIT, and MPM, are used respectively. A comparison is made to demonstrate that the FDFIB-MPM not only offers a better resolution than the FDFIB-MUSIC, FDFIB-ESPRIT, but also it is computationally very efficient.

# I. Introduction

Now several methods of broadband DOA estimation have been proposed, most of which can be categorized into two sets: incoherent broadband DOA estimation and coherent DOA estimation. Incoherent methods are based on averaging the narrowband DOA estimation for different frequency bins to obtain the incoherent estimation. It can be seen that as the signal frequency band increases, the computations of incoherent methods become prohibitively expensive. The coherent estimation methods use interpolation or focusing techniques to obtain one steering vector for the whole frequency band of interest, most such methods use the coherent signal subspace (CSS) approach and interpolation array. These coherent methods are more computationally efficient than the incoherent methods, but many problems still exist in the implementation. In [4], a new coherent estimation method is proposed using frequency-invariant beamformers (FIB) and demonstrated to have a superior resolution performance to the conventional estimation methods, CSS and beamspace CSS.

In this paper, we extend the FIB methods with a new frequency-domain frequency-invariant beamformer(FDFIB) [3] and an input pencil matrix based narrowband DOA estimation method--matrix pencil method (MPM) [2]. We refer to the proposed method as FDFIB-MPM. A comparison is made among the FDFIB-MUSIC, FDFIB-ESPRIT and FDFIB-MPM to demonstrate that the FDFIB-MPM has superior resolution performance.

## **II. Beamspace DOA estimation**

Consider an arbitrary antenna array composed of M identical

elements and assume that D wideband signal sources are located in the far-field of the array, in a known sector. The signals, impinging on the array from directions  $\Theta = [\theta_1, \dots, \theta_d, \dots, \theta_D]$ , are in the frequency range  $[\omega_1, \omega_K]$ , where  $\theta_d$  is the direction to the *d*-th source measured relative to the array axis. The structure of the wideband DOA estimator using FDFIBs is shown in Fig.1. The beamforming network, designed to cover the known spatial sector, consists of J FDFIBs, where  $D \leq J \leq M$ . For the *j*-th FDFIB, j=1...J, the weights is given by [3]

$$w_m(\omega) = a_m e^{-j\frac{\omega}{c}d_m^T\theta + j\frac{\omega}{c}d_m^T\theta}$$

Assume the signals in the outputs of the array are in a finite time period of K time samples. The time series received at the m-th sensor are

$$X_{m}[k] = \sum_{d=1}^{D} S_{d}[k - \tau_{m}(\theta)] + N_{m}[k]$$

Where  $S_d[k]$  is the *d*-th source signal,  $N_m[k]$  is the additive white noise at the array, and  $\tau_m(\theta)$  is the time delay of the *d*-th source signal from the reference sensor to the *m*-th sensor. Here we consider the first sensor as the reference sensor. The frequency response of  $X_m[k]$  is

$$X_{m}[\omega] = \sum_{d=1}^{D} e^{j\omega\tau_{m}(\theta)} S_{d}[\omega] + N_{m}[\omega]$$
$$= \sum_{d=1}^{D} a(\theta_{d}, \omega) S_{d}[\omega] + N_{m}[\omega]$$

Let us denote  $\mathbf{X}[k] = [X_1[k], \dots, X_M[k]]$  as the stacked array data, with a frequency response

 $\mathbf{X}[\boldsymbol{\omega}] = \mathbf{A}(\boldsymbol{\Theta}, \boldsymbol{\omega})\mathbf{S}[\boldsymbol{\omega}] + \mathbf{N}[\boldsymbol{\omega}]$ 

Where  $\mathbf{S}[\omega] = [S_1(\omega), \dots, S_D(\omega)]$  is the  $D \times I$  source signal vector,  $\mathbf{N}[\omega] = [N_1(\omega), \dots, N_M(\omega)]$  is the  $M \times I$  additive noise vector, and  $\mathbf{A}(\Theta, \omega) = [a(\theta_1, \omega), \dots, a(\theta_D, \omega)]$  is the  $M \times D$  source direction matrix.

We refer to  $\mathbf{Y}[\omega] = [y_1(\omega)\cdots y_j(\omega)\cdots y_J(\omega)]$  as the outputs of the beamforming network and let  $\mathbf{C}[\omega] = [\omega_1(\omega)\cdots\omega_j(\omega)\cdots\omega_J(\omega)]$  be the  $M \times J$  beamforming matrix, which transforms the received element-space

data  $\mathbf{X}[k]$  to a reduced dimension beam-space  $\mathbf{Y}[k]$ . Apply FDFIBs to the received array data

 $\mathbf{Y}[\omega] = \mathbf{C}^{H}(\omega)\mathbf{A}(\Theta,\omega)\mathbf{S}(\omega) + \mathbf{C}^{H}\mathbf{N}(\omega) = \mathbf{A}_{c}(\Theta,\omega)\mathbf{S}(\omega) + \mathbf{N}_{c}(\omega)$ Where  $\mathbf{A}_{c}(\Theta,\omega) = \mathbf{C}^{H}(\omega)\mathbf{A}(\Theta,\omega)$  is the  $J \times D$  source direction matrix in the beam-space, and  $\mathbf{N}_{c}(\omega) = \mathbf{C}^{H}\mathbf{N}(\omega)$  is the  $J \times I$ vector of additive white noise in the beam-space.

Since the FDFIBs are designed to equalize the variance of frequency with an assumption of a known spatial sector, the source direction matrix in the beam-space is approximately constant for all frequencies within the design band, i.e.  $\mathbf{A}_{c}(\Theta, \omega) \cong \mathbf{A}_{c}(\Theta, \omega_{0})$ ,  $\forall \omega \in [\omega_{1}, \omega_{K}]$ . Therefore, the broadband DOAs are completely characterized by a single beam-space source direction matrix  $\mathbf{A}_{c}(\Theta, \omega_{0})$ , and the narrowband DOA estimators can be used on the output data of beamforming network to get the estimation of broadband signals.

If we use the classical subspace-based methods, like MUSIC and ESPRIT [1], which are based on the eigenstructure of the input covariance matrix, we have to make an approximation of the covariance matrix. By assuming the signals are wide-sense stationary, the covariance matrix can be approximately formed, given by

$$\hat{R}_{Y} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{Y}[k] \mathbf{Y}^{H}[k]$$

where  $\mathbf{Y}[k] = [y_{1,k} \cdots y_{j,k} \cdots y_{J,k}]$ , the Fourier transform of  $\mathbf{Y}[\omega]$ , is the *J*-dimension observation vector of the *k*-th

time sample at the outputs of the beamforming network.

Different from the conventional methods, MPM is a variation of the eigen-structure approach and it uses the input pencil matrix instead of the input covariance matrix. Therefore, we directly apply MPM on each snapshot  $\mathbf{Y}[k]$ , then average all the estimated DOAs from *K* data samples to obtain the DOA estimation.

$$\widehat{\Theta} = \frac{1}{K} \sum_{k=1}^{K} MPM \left( \mathbf{Y}[k] \right)$$

#### **III.** Simulations and conclusion

A comparison of performance of FDFIB-MUSIC, FDFIB-ESPRIT, and FDFIB-MPM is performed. The resulting root-mean-squared errors (RMSE) of DOA estimations are shown for various SNR levels in Fig.2. And as the increase of the bandwidth of signals, the estimation performance will deteriorate. The performance of each method, when the fractional bandwidth increases, is shown in the Fig.3. The simulation results show that FDFIB-MPM exhibits a better resolution performance than conventional eigen-structure based method FDFIB-MUSIC, FDFIB-ESPRIT.

# Reference

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Fig.1 The structure of the wideband DOA estimator using FDFIB



Fig.2 Comparison of the average RMSEs for FDFIB-MUSIC, FDFIB-ESPRIT, and FDFIB-MPM for different SNR values



Fig.3 Comparison of the RMSEs for FDFIB-MUSIC, FDFIB-ESPRIT, and FDFIB-MPM, as the fractional bandwidth increases