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이동평균 코시 잡음에서의 약한 다진 신호 검파

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Detection of Weak M-ary Signals in Moving-Average of Cauchy Noise

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Abstract: In first-order moving-average Cauchy noise, the maximum likelihood (ML) and suboptimum ML (S-ML) detectors are analyzed in terms of the bit-error-rate in impulsive environment. Despite reduced complexity and simpler structure, the S-ML detector exhibits practically the same performance as the ML detector.

1. Introduction

Detectors optimized for the independent noise are often not guaranteed to be optimum in practical signal detection systems. Many dependent observation models have been proposed and investigated to address such a situation. Typical examples of dependent noise models include the transformation noise ϕ-mixing, and m-dependent models [1-3]. Dependent noise models have also been applied in deriving locally optimum detectors [3, 4]. When the dependence of noise is weak, we can model the dependent noise components as a simple first-order moving-average (FOMA).

In this paper, detection of weak signals is considered in weakly-dependent impulsive noise described by the FOMA model. More specifically, we compare the performance of the maximum likelihood (ML) and suboptimum ML (S-ML) detectors under impulsive Cauchy noise environment.

2. Observation Model

Assume that a set $\{s_k(t)\}_{k=1}^{M}$ of M signals is used to transmit information in a communication system. The received signal $x(t)$ can be written as

$$x(t) = s_k(t) + w(t) \tag{1}$$

where $w(t)$ is a sample function of the additive noise process. After a sampling procedure, the model (1) can be expressed as

$$X_i = s_{k,i} + W_i \tag{2}$$

where $\{X_i\}$ are the observations (data), $\{s_{k,i}\}$ are the transmitted signal components, and $\{W_i\}$ are the random noise components.

We assume that the transmitted signal vector $\underline{s_k} = (s_{k,1}, s_{k,2}, \cdots, s_{k,n})$ in (2) can be expressed as $\underline{s_k} = \Theta\varepsilon_k \underline{\tilde{s}_k}$. Here, Θ is the common factor of the signal strength, ε_k is the non-negative proportionality constant for the strength of signal vector $\underline{s_k}$, and $\underline{\tilde{s}_k} = \underline{s_k}/\|\underline{s_k}\|_2$.

Detection of M-ary signals can be modelled as an M-ary hypothesis testing problem in which the hypotheses $\{H_k\}_{k=1}^{M}$ are described by

$$H_k:\ \underline{X} = \Theta\varepsilon_k \underline{\tilde{s}_k} + \underline{W}. \tag{3}$$

In (3), $\underline{X} = (X_1, X_2, \cdots, X_n)$ is the observation vector and $\underline{W} = (W_1, W_2, \cdots, W_n)$ is the random noise vector.

The dependent noise components $\{W_i\}$ in this paper is modelled by the FOMA of independent and identically distributed (i.i.d.) random variables. Specifically, let us assume that

$$W_i = \Lambda_i + \rho_d \Lambda_{i-1} \tag{4}$$

where $\{\Lambda_i\}$ is a zero-mean i.i.d. random process with $\Lambda_0 = 0$ and univariate probability density function (pdf) $f_\Lambda(\cdot)$ and ρ_d is the dependence parameter. Clearly, the noise components $\{W_i\}$ are independent when $\rho_d = 0$. Defining the transformed observations

$$Y_i = \sum_{j=0}^{i-1} (-\rho_d)^j X_{i-j} \tag{5}$$

the joint pdf of $\underline{Y} = (Y_1, Y_2, \cdots, Y_n)$ can be obtained from the pdf of $\underline{X}$.

3. Discussion

The S-ML criterion [5] tells us that, when the signal strength approaches zero, the probability $P_e(\Theta)$ of error is minimized if the decision region D_k^S for the hypothesis H_k is determined as

$$D_k^S = \{\underline{y}: \sum_{i=1}^{n} q_{k,m,i}\, g_\Lambda(y_i) \geq 0,\ \forall m\} \tag{6}$$

where $g_{\Lambda}(y) = -\frac{f'_{\Lambda}(y)}{f_{\Lambda}(y)}$, $q_{k,m,i} = \varepsilon_k b_{k,i} - \varepsilon_m b_{m,i}$, and $b_{k,i} = \sum_{j=0}^{i-1} (-\rho_d)^j \tilde{s}_{k,i-j}$. With the ML criterion, we obtain the decision region

$$D_k^{ML} = \{\underline{y}: \sum_{i=1}^{n} \ln \frac{f_{\Lambda}(y_i - \Theta \varepsilon_k b_{k,i})}{f_{\Lambda}(y_i - \Theta \varepsilon_m b_{m,i})} \geq 0, \ \forall m\}. \quad (7)$$

To assess the performance in $S\alpha S$ environment [6, 7], we employ the geometric SNR (G-SNR). The G-SNR for the transformed observation space described by $\underline{Y}$ with $S\alpha S$ noise can be defined as

$$G-SNR = \frac{A_k^2}{2C_g^{-1+2/\alpha}\gamma^{2/\alpha}} \quad (8)$$

where $A_k^2 = \Theta^2 \varepsilon_k^2 \, ||\underline{b_k}||_2^2$ is the power of the transformed signal $\Theta \varepsilon_k \underline{b_k}$, γ is the dispersion parameter related to the spread of the SαS distribution, and $C_g \approx 1.78$ is the exponential of the Euler constant. Note that the G-SNR represents the standard SNR when $\alpha = 2$.

Fig. 1 shows the performance of various detectors in FOMA Cauchy noise. The S-ML and ML detectors optimized for i.i.d. noise are obviously not optimum in FOMA noise. The performance of the FC S-ML and IC S-ML detectors is slightly inferior to that of the FC ML and IC ML detectors, respectively: the performance gap between the S-ML and ML detectors becomes negligible as the sample size n increases. When n increases, the performance of the FC and IC detectors gets better, while the BER curves of the FG and IG detectors are almost flat.

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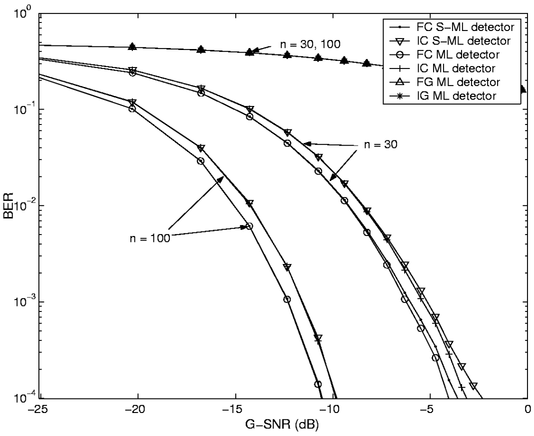

Fig. 1. Performance characteristics of various detectors in FOMA Cauchy noise